EFFICIENT CHANNEL DESCRIPTION IN TIME-FREQUENCY DOMAIN WITH APPLICATION TO FLEXIBLE RADIO

Thomas Hunziker, Ziyang Ju, and Dirk Dahlhaus

Communications Laboratory, University of Kassel
Wilhelmshöher Allee 73, D-34121, Kassel, Germany
phone: +49 561 804-6471, fax: +49 561 804-6008, email: {hunziker,ju,dahlhaus}@uni-kassel.de
web: www.comlab.uni-kassel.de

ABSTRACT

The handling of dispersive channels poses one major challenge in the design of reconfigurable multi-standard radios, since very different techniques are established in this area for conventional single carrier, OFDM, and CDMA receivers. We discuss an approximate, efficient description of doubly dispersive discrete-time channels which is adaptable to different signal types and channel conditions. The proposed model maps the channel input onto the output via time-frequency signal representations. Optimized prototype functions for the encompassing Gabor transform and signal expansion are aimed at which minimize the model error for the case of WSSUS channels. We formulate an adequate objective function and employ numerical methods to obtain optimized prototypes on the basis of the long-term channel statistics. The paper concludes with the discussion of a possible flexible receiver architecture incorporating the channel model.

1. INTRODUCTION

Over recent years a variety of new wireless standards like UMTS, WiMAX, Bluetooth, and DVB-T have been introduced to provide users more advanced services. The coexistence of an increasing number of mutually incompatible air interfaces inspires the design of terminals with multi-standard capabilities. Transceivers incorporating two or three subsystems, each dedicated to a different air interface, are already on the market. If this trend continues, however, reusability of resources through flexible transceiver components will very soon become a crucial issue. Reconfigurable devices, able to comply with many different air interfaces and possibly even with future standards, are the objective within the emerging research area of flexible radio [1].

Receivers often need to devote a major effort into dealing with signal distortions in the radio channel. Handling dispersive channels poses a particular challenge in the design of reconfigurable radios since conventional receivers for single carrier, orthogonal frequency-division multiplexing (OFDM), and direct sequence code-division multiple access (DS-CDMA) signals employ very different schemes for channel description, channel estimation, equalization and demodulation. While the tap delay line model typically adopted in single carrier and DS-CDMA systems leads to equalizer or MLSE (maximum-likelihood sequence estimation) structures, the single-tap frequency domain channel model makes OFDM a superior technique in terms of efficiency and scalability.

In this paper we discuss a general model for the signal transformation by doubly dispersive channels by means of the Gabor transform [2]. The scheme can be seen as a generalization of the OFDM frequency domain channel model, like the Gabor transform represents a generalization of the discrete Fourier transform (DFT). The single-tap structure of the proposed model facilitates complexity-limited, scalable signal processing in reconfigurable receivers. And the degrees of freedom in terms of the prototype (or window) functions and lattice constants associated with the involved Gabor families enable an adaptation to the signal format of a particular air interface and to the encountered channel characteristics.

The accuracy of the channel description depends on the shape of the prototype functions. Formerly being linked to radar systems, prototype optimization for Gabor families has recently become an issue in the context of generalized multicarrier schemes, which are sometimes called Gabor-based or filter bank-based multicarrier transmission [3, 4, 5]. These are similar to OFDM in that they divide the band into multiple sub-bands, however, they eliminate the rigid framework of rectangular windows and cyclic prefixes. The problem of finding the optimal prototype pair of transmit pulse and matched filter for wide-sense stationary uncorrelated scattering (WSSUS) channels with known second-order statistics has been addressed in [6, 7, 8]. A related optimization problem is encountered in this paper, however, we employ the Gabor families for the analysis and synthesis operations encompassing the single-tap mapping rather than for the signal design.

The rest of the paper is organized as follows. In Sect. 2 we define the generalized single-tap channel model, which has the form of a so-called Gabor multiplier as discussed in Sect. 3. An objective function for the prototype function optimization on the basis of second-order statistics is derived in Sect. 4 for WSSUS channels, and numerically computed functions for a number of channels are presented in Sect. 5. An architecture of a reconfigurable baseband receiver incorporating the channel model is outlined in Sect. 6, and conclusions are drawn in Sect. 7.

2. EFFICIENT CHANNEL DESCRIPTION

In the following squared brackets are used to represent functions with arguments from \( Z \), and \( \langle \cdot, \cdot \rangle \) denotes the inner product in the \( l^2(\mathbb{Z}) \) space, i.e., \( \langle x, y \rangle = \sum_{n \in \mathbb{Z}} x[n] y^*[n] \). The asterisk in the superscript stands for complex conjugation, and \( x[n] y[n] \) represents the convolution of \( x[n] \) and \( y[n] \).

Let \( g_{kl} \) with the index set \( A = \mathbb{Z} \times \{0, \ldots, M-1\} \) represent a Gabor family of elementary functions \( \mathbb{Z} \rightarrow \mathbb{C} \). The element with the index \( (k, \ell) \) results from translating and
modulating the prototype function $g_0[n]$ according to

$$g_k[n] = g_0[n - kn] \exp(i2\pi(n - kn)f/M)$$

with $N, M \in \mathbb{N}$ constants. For a discrete-time signal $x[n]$, the so-called Gabor transform $x \mapsto (b_{k\ell}(x), g_{k\ell}(x)) \in \mathbb{C} \times \mathbb{C}$ renders a time-frequency representation. Conversely, the sequence $(b_{k\ell}, g_{k\ell})_{(k,\ell) \in \Lambda}$ of Gabor coefficients can be mapped onto a discrete-time signal by means of another Gabor family $(w_{k\ell})_{(k,\ell) \in \Lambda}$ as

$$y[n] = \sum_{(k,\ell) \in \Lambda} b_{k\ell}w_{k\ell}[n].$$

If $(g_{k\ell})_{(k,\ell) \in \Lambda}$ and $(w_{k\ell})_{(k,\ell) \in \Lambda}$ constitute a frame and a dual frame, respectively, the signal $x[n]$ is perfectly reconstructed.

3. GABOR MULTIPLIERS

Linear operators of the form (3), concatenating a Gabor transform, an element-wise multiplication by a certain sequence, and a Gabor expansion are known as Gabor multipliers [10]. In the following we are interested in the Gabor multiplier approximating a given linear mapping with minimal error. Since this derivation is simpler using matrix expressions, we consider discrete $L$-periodic signals and functions within this section, where $L/N \in \mathbb{N}$. Besides, $(\cdot)^\dagger$ and $(\cdot)^T$ denote Hermitian transpose and transpose, respectively, by $\operatorname{diag}(X)$ we mean the column vector representing the diagonal of the matrix $X$, and by $\operatorname{Diag}(x)$ the diagonal matrix generated from the vector $x$.

Let the $L \times K$-matrices $G$ and $W$ contain the elementary functions derived from $g_0[n]$ and $w_0[n]$, respectively, with the translation being a cyclic operation [11]. There are $K = LM/N$ unique translated/modulated versions of either prototype, defining the columns of $G$ and $W$ in a corresponding fashion. The space $\mathcal{F}_G W = \{\operatorname{WDiag}(h)G^T : h \in \mathbb{C}^K\}$ of Gabor multipliers is a closed subspace of all $L \times L$-matrices, i.e., the space of the linear operators. Given an argument $x$ and $y = \operatorname{WDiag}(h)G^Tx$, the error resulting from the approximation of a matrix $C$ by $\operatorname{WDiag}(h)G^T$ equals $(y-C)x$. If $x$ is a random vector with independent, zero-mean elements with unit variance,

$$E[\|y - Cx\|^2] = \|\operatorname{WDiag}(h)G^T - C\|_F^2,$$

where $E[\cdot]$ denotes the expectation and $\|\cdot\|_F$ and $\|\cdot\|_F$ represent the 2-norm and the Frobenius norm, respectively. The Frobenius norm of a matrix $X$ is given as $\|X\|_F = \sqrt{\operatorname{tr}(XX^T)}$ with $\operatorname{tr}(\cdot)$ the trace. Hence, the mean squared error corresponds to the squared distance between $C$ and its approximation in the space of the $L \times L$-matrices with distance induced by the Frobenius norm.

The element of $\mathcal{F}_G W$ minimizing the error (4) is given as $\operatorname{Wdiag}(h_0)G^T$ with

$$h_0 = \arg \min_{h \in \mathbb{C}^K} \|\operatorname{Wdiag}(h)G^T - C\|_F^2.$$

Methods for numerically computing the optimal Gabor multiplier in the special case of $G = W$ are discussed in [10]. In the more general case considered here, $h_0$ can be found in a straightforward fashion via the derivation of the expression to be minimized w.r.t. $h$. Before doing so we rewrite (5) as

$$h_0 = \arg \min_{h \in \mathbb{C}^K} \operatorname{tr}(\operatorname{WDiag}(h)G^T \operatorname{Diag}(h)W^T)$$

$$-2\mathbb{R}[\operatorname{tr}(\operatorname{WDiag}(h)G^T C^T)]$$

$$= \arg \min_{h \in \mathbb{C}^K} h^T((W^T W) \circ (G^T G)^T) h$$

$$-2\mathbb{R}[\operatorname{Diag}(W^T C G) h],$$

where $\mathbb{R}[\cdot]$ denotes the real part operator and $\circ$ element-wise multiplication of two matrices. The solution of the standard problem (7) reads

$$h_0 = ((W^T W) \circ (G^T G)^T)^{-1} \operatorname{Diag}(W^T C G).$$
And for the error (4) at $h = h_0$ we find

$$E \left[ \left\| y - Cx \right\|^2 \right] = \|C\|^2_i,$$

$$-\text{diag}(W^T CG) \left( \left( W^T W \right) \odot (G^T G)^T \right)^{-1} \text{diag}(W^T CG).$$

(9)

In summary, given $W, G$, and a random vector $x$ with i.i.d. entries, $y = W \text{diag}(h_0) G x$ with $h_0$ given in (8) represents the optimal approximation of $C x$ by means of $\Sigma_G W$ in the sense of minimal mean squared Euclidean distance between $y$ and $C x$.

### 4. PROTOTYPE OPTIMIZATION

The error resulting from the approximation of a linear operator - representing a channel - crucially depends on the chosen Gabor families. Since adaptation to instantaneous channels in real time is usually impractical, prototypes can only be optimized on the basis of statistical long-term channel information. WSSUS channels, for instance, are characterized by a channel coefficient subject to a delay power profile $S_c[m]$ and the time correlation function $\phi[n]$. Provided a separable scattering function, the time-impulse response is subject to $E[c_n[m]] = 0 \forall n, m$, and

$$E[c_n^T[\ell]c_n[m]] = \begin{cases} \phi_n - k S_c[\ell], & \ell = m \geq 0, \\ 0, & \text{otherwise.} \end{cases}$$

(10)

We assume additionally that $S_c[m]$ and $\phi[n]$ are normalized such that $\sum_{m=0}^{\infty} S_c[m] = 1$ and $\phi[0] = 1$.

Our objective is to find the two prototypes minimizing the mean-squared sample error (MSSE) resulting from applying the model (3) at the output of a random channel when the random input samples are zero-mean i.i.d.. Finding optimized prototypes under the assumption of always employing the nearest Gabor multiplier (8) is involved. We shall thus choose the channel coefficients $(h_{k,\ell})_{k,\ell} \in \Lambda$ in a suboptimal way. Note that under the assumptions of a not excessively dispersive channel and reasonably time-frequency localized $g_0[n]$ and $w_0[n]$ around $(0, 0)$, the channel coefficients reflect the gain of the doubly selective channel at the time-frequency points $(kN, 2\pi t/M)$. The actual channel gain in time and frequency is given by the time-variant transfer function $C_n(\omega) = \sum_{m=0}^{\infty} c_n[m] e^{-j\omega m}$, and a natural choice for the channel coefficients would thus be

$$h_{k,\ell} = C_n(2\pi t/M).$$

(11)

Given (11) and fixed $N, M$, the MSSE as a function of the prototypes equals

$$\epsilon_{\text{MSSE}}(g_0, w_0) = E \left[ \sum_{m=0}^{\infty} c_n[x[n-m] - \sum_{(k,\ell) \in \Lambda} C_n(2\pi t/M) \langle x, g_{k,\ell} \rangle w_{k,\ell}[n]]^2 \right],$$

(12)

with the expectation $E[\cdot]$ w.r.t. $x[n]$ and the random channel.

Using (10) and $E[x^*[n]x[n]] = \delta_n$, we find after some algebra

$$\epsilon_{\text{MSSE}}(g_0, w_0) =$$

$$1 + \frac{M}{N} \left( \sum_{(k,\ell) \in \Lambda} \phi[k \delta_n] \langle g_{k,\ell}[2\pi t/M], \langle g_0, w_0 \rangle \right) - 2\Re \left[ \langle \langle g_0, S_c \rangle \phi, w_0 \rangle \right].$$

(13)

where $\phi(\omega) = \sum_{m=0}^{\infty} S_c[m] e^{-j\omega m}$ represents the frequency correlation function. The expression (13) can be taken as the basis for finding optimized prototypes for given channel statistics. We note that the sum term in the first line depends on the cross-correlations between different Gabor family elements, weighted by the time and frequency correlation functions. The term in the second line depends only on the shape of the prototypes and the second-order statistics of the channel.

Before numerically computing optimized prototypes in Sect. 5, we focus on two special cases.

**Flat fading channel:**

If $\phi[n] = \phi(\omega) = 1$, we have

$$\epsilon_{\text{MSSE}}(g_0, w_0) =$$

$$1 + \frac{M}{N} \left( \sum_{(k,\ell) \in \Lambda} \langle g_0, g_{k,\ell} \rangle \langle w_{k,\ell}, w_0 \rangle - 2\Re \left[ \langle g_0, w_0 \rangle \right] \right).$$

(14)

Provided $N/M \leq 1$, choosing $g_0[n], w_0[n] \in \mathbb{R}$ such that they constitute a Gabor frame and a corresponding dual frame,

$$\epsilon_{\text{MSSE}}(g_0, w_0) = 1 + \frac{M}{N} \left( \langle g_0, w_0 \rangle - 2\langle g_0, w_0 \rangle \right).$$

(15)

It follows from $\langle g_0, w_0 \rangle = N/M \epsilon_{\text{MSSE}}(g_0, w_0) = 0$.

**Increasing density of lattice points in time-frequency plane:**

For constant $S_c[m]$ and $\phi[\omega]$ the MSSE tends to zeros as $N \rightarrow 1$ and $M \rightarrow \infty$. To see this, define $g_0[n] = M^{-1} \delta_n$ for $n \in \{Z \cap [-M/2, M/2]\}$ and $g_0[n] = 0$ otherwise, and $w_0[n] = \delta_n$. Since $\langle w_{k,\ell}, w_0 \rangle = 0 \forall k \neq 0$, and $\langle g_0, w_\ell \rangle = 0 \forall \ell \neq 0$,

$$\epsilon_{\text{MSSE}}(g_0, w_0) =$$

$$1 + \frac{M}{N} \left( \langle g_0, g_{k,\ell} \rangle \langle w_{k,\ell}, w_0 \rangle - 2\langle g_0, S_c \rangle \phi \langle w_0 \rangle \right).$$

(16)

Note that $\langle g_0, g_{k,\ell} \rangle \langle w_{k,\ell}, w_0 \rangle = M^{-1}$. Additionally, due to the increasing concentration of $g_0[n]$ around $\omega = 0$ in frequency, $\langle g_0, S_c \rangle \phi \langle w_0 \rangle \rightarrow M^{-1}$. Hence, $\epsilon_{\text{MSSE}}(g_0, w_0) \rightarrow 0$ as $N = 1$ and $M \rightarrow \infty$.

### 5. NUMERICAL RESULTS

Let us now compute optimized prototype pairs on the basis of (13). We assume a delay power profile with an exponential decay, i.e.,

$$S_c[m] = (1 - \exp(-1/\tau_d)) \exp(-m/\tau_d), \quad m \in \{0, 1, 2, \ldots\},$$

(17)

with $\tau_d$ representing the root mean-squared (RMS) delay spread in samples. As for the Doppler power profile, a Laplacian function with a two-sided exponential decay is assumed, translating to the time correlation function

$$\phi[\omega] = \frac{1}{1 + 2\pi^2 \nu_d^2 |\omega|^2}, \quad n \in \mathbb{Z}$$

(18)

with the RMS Doppler spread $\nu_d$ relative to the sampling rate.

Analytical solutions for our optimization problem

$$\hat{g}, \hat{w} = \arg \min_{\langle g_0, w_0 \rangle} \epsilon_{\text{MSSE}}(g_0, w_0) \quad \text{given} \quad \tau_d, \nu_d, N, M$$

(19)
are not available. Hence, we resort to a numerical method. The prototype functions \( g_0[n] \) and \( w_0[n] \) are constrained to be real-valued with the support \( \mathbb{Z} \cap (-N_s/2,N_s/2) \) where \( N_s > N \). Starting from initial 
\[
g_0[n] = w_0[n] = \begin{cases} N^{-1/2}, & n \in (\mathbb{Z} \cap (-N_s/2,N_s/2)) \\ 0, & \text{otherwise}, \end{cases}
\]
the two prototypes are optimized alternatingly by a simple gradient method. An optimization of \( g_0[n] \) involves the computation of the gradient of \( \text{MSSE}(g_0,w_0) \) w.r.t. \( g_0 \) in the sample space \( \mathbb{R}^{N_s} \), and the choice of a step size in the direction determined by the gradient. After the update of \( g_0[n] \) an optimization of \( w_0[n] \) follows and so on, until a local minimum of \( \text{MSSE}(g_0,w_0) \) is found. Since the objective function (13) is non-convex, there is no guarantee that the procedure arrives at the global minimum.

Figs. 2-4 display the obtained pairs \((\hat{g},\hat{w})\) for \( N = 24, M = 32, N_s = 64 \), and different RMS delay and Doppler spreads as given in the figure captions. In the first scenario with a relatively small RMS delay spread equivalent to 1.6 samples the resulting \( \hat{g} \) and \( \hat{w} \) are not very different from two functions defining rectangular windows with widths equal to \( N \) and \( M \) samples, respectively, which would constitute a frame and a dual frame and thus provide perfect reconstruction in the flat fading case. For larger delay and Doppler spreads more degenerated shapes result. At a sampling rate of 3.84 Msps, for instance, in accordance with UMTS signals, RMS delay spreads of \( \tau_d = 1.6 \) and \( \tau_d = 3.2 \) samples correspond to approximately 417 ns and 833 ns, respectively, which are realistic values for suburban environments. A Doppler spread of \( \nu_D = 0.004 \) translates to 15.36 kHz, which is a very large value.

As both the signal power at the channel input and the mean gain of the channel are normalized to unity, the MSSE \( \text{MSSE}(\hat{g},\hat{w}) \) corresponds to the signal-to-error ratio, where error stands for the deviation of the channel output estimated using the model as compared to the actual channel output. Tab. 1 provides the obtained MSSEs in decibels for a number of WSSUS channels. Note that the delay and Doppler spreads in the first three entries lead to the prototypes shown in Figs. 2-4.

Besides of the channel characteristics, the attainable MSSE depends on the lattice constants \( M \) and \( N \) as has been seen in Sect. 4. Given the over-sampling rate \( M/N \), which generally needs to be larger than 1, there may be constraints on the choice of the two constants arising from the signal format or preferable FFT-sizes. Without such restrictions, \( M \) and \( N \) are properly chosen if the ratios \( \tau_d^{-1}/M^{-1} \) (i.e., coherence bandwidth over spacing of elementary functions in frequency) and \( \nu_D^{-1}/N \) (i.e., coherence time over spacing of elementary functions in time) are of the same order of magnitude.

6. RECONFIGURABLE BASEBAND RECEIVER

Fig. 5 shows a possible architecture of a reconfigurable baseband receiver for general linearly modulated signals, incorporating the proposed channel model. Tunable pilot and waveform generators reproduce transmitter-side versions of signal parts compliant with the designated air interface. The pilot signal generator yields the signal parts serving for the channel estimation. The waveform generator provides elementary waveforms corresponding to the unmodulated pulse shapes, their spread versions in the case of DS-CDMA, or the truncated harmonic waveforms providing the basis for synthesizing OFDM signals.

Three analysis filter banks (AFBs) yield time-frequency representations of the received signal \( r[n] \) and the reproduced signal parts. Estimates for the channel coefficients \( \langle h_{k,\ell} \rangle_{(k,\ell) \in \Lambda} \) are in the following computed from the output of the upper two AFBs in the figure, representing the received signal as a bandpass-reconstructed version of the original signal.

<table>
<thead>
<tr>
<th>( \tau_d )</th>
<th>( \nu_D )</th>
<th>( \text{MSSE}(\hat{g},\hat{w}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>0</td>
<td>-32.1 dB</td>
</tr>
<tr>
<td>3.2</td>
<td>0</td>
<td>-18.6 dB</td>
</tr>
<tr>
<td>3.2</td>
<td>0.004</td>
<td>-15.3 dB</td>
</tr>
<tr>
<td>4.8</td>
<td>0.006</td>
<td>-11.1 dB</td>
</tr>
<tr>
<td>6.4</td>
<td>0.008</td>
<td>-8.5 dB</td>
</tr>
<tr>
<td>8.0</td>
<td>0.01</td>
<td>-6.5 dB</td>
</tr>
</tbody>
</table>

Table 1: Signal-to-error ratios due to the channel modeling, utilizing numerically optimized prototypes, \( N = 24, M = 32 \).
received and the pilot signals, in a similar fashion as in OFDM receivers. The discussed channel model is employed for replicating the mapping of the elementary waveforms by the channel. This involves the third AFB and the ”single-tap mapper”. The synthesis part of the channel model is omitted since the correlations between the received signal and the receiver-side versions of the elementary waveforms are computed directly in the time-frequency domain, implying proper configuration of the AFBs. The coefficients resulting from the correlations facilitate the decoding by standard methods.

The constants $N$ and $M$ for the AFBs are chosen in compliance with the signal format. As for the prototypes there may be further criteria besides of minimizing the MSSE due to the approximate channel model, like high time-frequency concentration in order to limit processing complexity in the correlator. For OFDM signals there is also the possibility of choosing prototypes with rectangular shapes and thus emulating a conventional OFDM receiver taking advantage of the orthogonality preservation thanks to the cyclic signal extensions.

7. CONCLUSIONS

A model for the signal transformation in dispersive channels has been discussed which is adaptable to different signal formats and thus an auspicious solution for flexible radios. The single-tap nature of the model facilitates complexity-limited and scalable signal processing similar to OFDM receivers. The efficiency comes at the cost of a model error which, however, is tolerable unless channel dispersions are severe in both time and frequency. The affinity with generalized multicarrier schemes featuring highly flexible time-bandwidth occupation makes the model also interesting in the context of cognitive radios.

REFERENCES