PRELIMINARY RESULTS ON NOISE REDUCTION USING STACKGRAMS FOR (LOW-DOSE) X-RAY CT SINOGRAMS

Antti P. Happonen$^{1,2}$ and Matti O. Koskinen$^2$

$^1$Institute of Signal Processing  
Tampere University of Technology  
P.O. Box 553, 33101 Tampere, Finland  
email: antti.happonen(at)elisanet.fi  

$^2$Department of Clinical Physiology  
Medical Imaging Center, Tampere University Hospital  
P.O. Box 2000, 33521 Tampere, Finland

ABSTRACT

In low-dose X-ray computed tomography (CT), it is necessary to reduce the excessive X-ray quantum noise of the acquired data. This is the case especially when the filtered back-projection method is used for image reconstruction from sinogram data. In this study, we compare the conventional radial filtering direction of the sinogram to the recently introduced angular stackgram filtering technique. Our experimental tests were performed on simulated low-dose CT data. We modelled the X-ray quantum noise using a Gaussian distribution with a signal dependent variance for numerical phantoms simulating stop-and-shoot CT acquisitions. The noise model has been reported elsewhere. This experimentally verified model describes calibrated projection data, i.e. the acquired data after CT system calibration (including logarithmic transform, beam hardening, etc.). For our quantitative tests, we employed both linear Gaussian low-pass filters and non-linear L-filters with Gaussian weights. The stackgram technique provides the best resolution-noise trade-off in comparison with the radial sinogram filtering method, and thus a potential filtering approach to low-dose CT sinograms.

1. INTRODUCTION

Minimization of the X-ray radiation exposure to the patients is an important effort in the field of computed tomography (CT). In low-dose CT acquisitions (low mA or short acquisition period), this minimization of the radiation exposure results in severely degraded image quality due to the X-ray quantum noise. Iterative reconstruction algorithms, based on statistical optimization principles, allow quantitative image reconstruction for data like these. In modern CT scanner designs, however, accurate modeling of the noise and signal properties for the iterative image reconstruction algorithms can be challenging [1]. Another and desirable approach is to reduce the quantum noise of the acquired raw or sinogram data prior to linear and fast image reconstruction, such as filtered back-projection (FBP).

Various noise reduction methods have been published for CT sinogram data, in order to improve the FBP-image quality. These methods can be divided roughly into two different approaches. In the first approach, the X-ray quantum noise is modeled prior to full CT system calibration, i.e. the noise is modeled using a compound of Poisson and Gaussian terms, representing the photon counting statistics and the electronic noise, respectively. Such CT sinogram restoration techniques are described e.g. in [2] and [3]. The other approach is to model the noise properties after CT system calibration (including logarithmic transform, beam hardening, etc.), such as in [4][5][6] and [7]. In this paper, we concentrate on this latter approach. In [4] it was experimentally studied that the noise in the acquired sinograms after the full CT system calibration follows a Gaussian distribution with a nonlinear signal-dependent variance. In this study, we have applied this same noise model to simulate calibrated stop-and-shoot CT acquisitions.

Frequently, the (stop-and-shoot) sinogram data are smoothed or filtered only in the radial direction (i.e. along the projections). It is a well-known fact that data filtering along the angular direction (i.e. across the projections) introduces non-uniform or tangential blurring into the reconstructed images. Despite this fact, however, two- and three-dimensional filters [5][6] are routinely employed across the angular projections for noise reduction of the CT data.

Recently, Happonen et al. have introduced a so-called stackgram approach to tomographic data processing utilizing the angular information of the projection data in a novel way [8]. They have studied sinogram data filtering using this promising stackgram representation for emission tomography (ET) data [9][10][11], in which the noise and resolution properties differ substantially from the calibrated CT data. In this study, we performed a similar comparison as Happonen et al. in their previous stackgram studies, but for simulated CT sinograms (instead of ET data). Our experimental investigation was carried out by using both shift-invariant Gaussian filters and nonlinear L-filters. By employing these filters, we evaluated resolution-noise tradeoffs of the conventional radial sinogram filtering technique in comparison with the novel stackgram approach.
2. METHODS

2.1 Stackgram Decomposition

Let the function \( g(l, \theta) \) denote a sinogram, i.e., a Radon transform of an object function \( f(x,y) \). The stackgram reorganizes the sinogram into a three-dimensional (3D) domain, in which the signals consisting of values along the sinusoids of the sinogram are decomposed into separate signals. The stackgram \( h(x,y,\theta) \) is defined as in [8][12].

\[
 h(x,y,\theta) = Sg(l, \theta) = g(x \cos \theta + y \sin \theta, \theta).
\]  

(1)

In our study, the one-dimensional (1D) signals that we processed for noise reduction are referred to as locus-signals, and defined as \( h_{\text{locus}}(x,y) \) for each \((x,y)\) coordinate.

A generalized inverse of the stack-operator can be written as

\[
 g(l,\theta) = S^{-1} h(x,y,\theta) = \int \int w(x,y;l,\theta) h(x,y,\theta) \delta(x \cos \theta + y \sin \theta - l) \, dx \, dy,
\]  

(2)

where \( w \) is a weight function and \( \delta \) denotes the Dirac delta function.

In this study, the stackgram or a stack of back-projected projections (Figure 3) was implemented using two-dimensional (2D) data rotations for back- and re-projections (Eqs. 1 and 2). We implemented the rotations with the three-pass rotation algorithm employing sinc-interpolation. This provides a reversible sinogram-to-stackgram transformation, as described in [8].

2.2 Simulated Low-Dose CT Data

To examine the performances of both the stackgram filtering approach and the conventional radial filtering technique, we simulated noisy projections of a numerical ellipse phantom. A sinogram of 512 parallel beam bins by 512 angles acquired over 180 degrees was computed. We simulated calibrated low-dose X-ray CT acquisitions with this sinogram.

Calibrated projection data of a low-dose CT follow approximately a Gaussian distribution with signal dependent variance. This can be described by the analytical formula [4][5] as

\[
 \sigma_i^2 = f_i \times \exp(\lambda_i / S),
\]  

(3)

where \( \lambda_i \) is the mean and \( \sigma_i^2 \) is the variance of the projection data at a detector bin \( i \), and \( S \) is a scaling parameter and \( f_i \) is a parameter adaptive to the different detector bins. For simplicity, we regarded the parameter \( f_i \) as constant. By adding a signal dependent Gaussian noise according to (3), we generated sinogram data of ten noise realizations. An FBP reconstruction of the data is shown in Figure 4. The exposure level of the image corresponds to \( 1.0 \times 10^6 \) approximately.

2.3 Filters

In our study, we employed two different types of 1D filters: conventional Gaussian low-pass filters and nonlinear L-filters with Gaussian weights. The Gaussian low-pass filters were implemented as finite-impulse-response filters. Each output point of an L-filter is obtained as a weighted sum of ordered data values inside a sliding window of length \( N \) [13].

For the filter weights, we used samples from Gaussian distributions (\( \mu=0 \)) with the standard deviations (\( \sigma \)) of 0.25, 0.5, 0.75, …, 2.5. These specifications result in ten different low-pass filters and ten different L-filters. The corresponding lengths (\( N \)) of the filters were as 3, 5, …, 21 samples.

The ten simulated noisy sinograms were filtered employing the 20 filters in the following ways:
Figure 3 - Gaussian filters: resolution-noise trade-off curves of the compared methods at the center (a) and left (b) circular inserts in the ellipse phantom (see Fig. 2). The points of the curves represent the different filter kernels. The chosen CR level depicts the kernels used for visual assessments of the two methods. Radial filtering provides the best trade-off, according to these plots.

(1) along the radial $l$-direction of the sinogram, and (2) along the $\theta$-direction in the stackgram domain, resulting in filtered sinograms.

2.4 Evaluation Methods

The filtered sinograms were reconstructed with the FBP algorithm (ramp-filter). Contrast recoveries (CRs) and standard deviations (STDs) were quantified on the FBP-images for both filtering techniques and for each filter.

The CR is defined as $\frac{(H-B)}{B}$, where $H$ and $B$ represent average intensities (in Hounsfield unit (HU)) of a high intensity region (H) and a low intensity region (B). For CR quantification, we applied two different locations in the images: the left and center circular inserts (at the “bones” in Figure 2). For both locations, the region $H$ was a circular region with a radius of 20 pixels covering the bone, whereas the thickness of a ring-shaped region $B$ was 11 pixels round the bone. In this way, the CR corresponds to a resolution measure.

The STD values were measured from the FBP-images using a large region between the left and center inserts (or bones) at the center of the images.

All the quantified and shown values were averaged over ten data values resulting from the ten noise realizations.

3. RESULTS

STD versus CR plots are shown in Figure 3 and Figure 4 for the low-pass filters and the L-filters, respectively. Considering both figures, stackgram filtering with the L-filters provides the best resolution-noise tradeoff at the appropriate resolution (or CR) levels.
In order to obtain a fair visual assessment of the filtering methods, the chosen or matched resolution levels are shown in the figures as well. These levels provide matched filters for the radial and stackgram filtering approaches. The widths ($\sigma$) of the matched Gaussian filter kernels are as 1.0 (the standard Gaussian filter) and 1.25 for radial and stackgram filtering, respectively (Figure 3). The widths ($\sigma$) for the matched L-filter weights happen to be the same (Figure 4), (note that we do not try to match the linear and nonlinear filters). FBP-images for these four matched filters are show in Figure 5 and Figure 6.

### 4. DISCUSSION

The resolution-noise tradeoffs (Figure 3 and Figure 4) as well as the visual quality of the images (Figure 5 and Figure 6) seem to be congruent with those of reported on conventional ET data [8]-[11]. The non-linear L-filters seem to be suitable for the stackgram approach. There is, however, one notable difference in the trade-off curves, as compared to the previously reported performance of the stackgram filtering. That is, stackgram domain filtering leads to a non-uniform resolution (or CR) in the reconstructed images. We concluded this by observing the differences in the distances between the curves representing the radial sinogram and angular stackgram filtering methods at the different locations (i.e. (a) vs. (b) in Figure 3). After stackgram filtering, the difference in the resolution or contrast in the images at the different locations, however, seems to be quite small, and it cannot be observed visually (Figure 5 and Figure 6). Note that shift-invariant (low-pass) filters employed along the radial direction of the sinogram always introduce a uniform resolution to the image. We believe that an explanation for the non-uniform CR of the stackgram-filtered images lies on the huge resolution of the CT data (which is substantially higher as compared to ET). Our implementation of the discrete stackgrams (i.e. three-pass rotation with sinc-interpolation) [8] may introduce these non-uniformities. This would require further investigation.

In practice, huge matrix sizes of the X-ray CT sinograms make the stackgram approach impractical for noise reduction of the data, since the corresponding 3D stackgrams require an enormous space of computer memory. Still, because our results seem to be promising, a useful approach could be to “imitate” the stackgram filtering technique in the sinogram domain, such as in [14].

Our study for noise reduction of low-dose CT data is preliminary. The noise model for the calibrated CT sinograms (Eq. 3) should be incorporated into stackgram (based) filter design more accurately. Moreover, since the CT noise characteristic is local [7], an adaptive approach to the stackgram filter design should be studied to find a proper balance between the streak artifact suppression and the spatial resolution preservation.

In this study, we simulated conventional 2D stop-and-shoot CT acquisitions. Deeper mathematical considerations of the stackgram approach need to be explored for CT scanning designs, since modern CT scanners utilize a helical or spiral geometry in 3D X-ray acquisitions.

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