RELEASING APERTURE FILTER CONSTRAINTS

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ABSTRACT

Aperture filters are a recently introduced class of non-linear filters used in image processing. In this paper we present a new approach for aperture filter design, improving operator performance with respect to the MSE measure by releasing some of the operator constraints without losing statistical estimation accuracy. With the use of the proposed methods an average of 34% MSE reduction was achieved for deblurring, whereas a standard aperture operator reduced the error by only 10% on the average.

1. INTRODUCTION

In statistical approaches to non-linear filter design the task is to find a filter Ψ, given the training dataset of signal h to be observed and corresponding signal g to be estimated, so that the error measure between $\Psi(h)(t)$ and $g(t)$ is minimized.

The general class of image operators used in non-linear filter design is called window operators (*W-operators*). A window operator is a locally defined function ψ operating on an observed signal $h(t)$ restricted to the finite window W about t, mapping the signal values in the window W to the operator's output value, so that $\Psi(h)(t) = \psi(h(W))$ where $h(W)$ is the observed signal $h(t)$ restricted to the window W. Both signals $h(t)$ and $g(t)$ are assumed to be jointly stationary. Mapping function $\psi(h(W))$ is assumed to be translation invariant with respect to the window placement on signal $h(t)$.

2. APERTURE OPERATORS

A fundamental problem in non-linear filter design is to place an efficient constraint on the filter search space so that it is possible to obtain sufficient training data to achieve good output estimates, but at the same time the constraint should not damage the filter performance.

The domain constraint resulting from window operators does usually not in itself, reduce the search space sufficiently, especially for multidimensional signals. In order to further reduce the class size, a subclass of W-operators called *WK-operators* or aperture operators was introduced in [1]. Aperture operators are not only constrained in their domain by a window, but also restricted to a number of gray levels in signal values. Because of this additional constraint, the operator's class size can be greatly reduced.

For a specified window W the signal $h(t)$ is projected into the aperture K, $K = [-k, k]$ by placing the aperture on a specified value a. Signal values outside the aperture are saturated to the closest aperture boundary.

2.1 Aperture placement

Aperture placement has a strong influence on filter performance. In general the position has to be chosen so as to observe as many signal values as possible, without clipping. Rather than compete with alternative techniques, aperture filtering may build on a good existing estimate of the signal output value i.e. for deblurring, the aperture is usually placed on the center pixel value, and for noise removal it is positioned on the median value.

2.2 Operator configuration

The choice of window size, its shape and the number of aperture levels are the most important design considerations. A large window and aperture assures better filtering quality by capturing more of the input signal characteristics and increasing the output value range. On the other hand, a limited amount of training data requires that the search space be small to maintain good output estimation.

In order to improve filter quality while maintaining good output estimates a number of different techniques were tried [2, 3, 4]. In order to capture more dominant signal characteristic at the cost of losing some detail, scaling can be used. The scaling can be performed in the image domain by mapping several pixel values into one value, thus reducing the resolution; or in gray-scale range [2]. Another technique reduces the search space by using non-rectangular aperture masks [3]. If a signal does not fit into a particular mask without clipping, another mask is tried until the signal is fitted with minimal clipping. All masks together cover a larger region than each one in particular thus the aperture is able to capture more of the signal span with a smaller search space. Another principle can be seen in the pyramidal approach [4]. In this approach a bank of filters is trained, each filter operating with a different level of resolution. During filtering if a pattern was not sufficiently trained with the best quality filter, another one is tried at lower resolution until the output value is found.

All of the above techniques can be combined to achieve better filter performance. A further technique which reduces the impact of some of the operator's constraints on the filtering error is presented in this paper.

2.3 Output mapping

During training for every observed pattern x from input signal $h(t)$ the output value is estimated as the expected value from all corresponding center pixel values from ideal signal $g(t)$, translated by aperture position and clipped to the aperture boundaries. There is a problem of what output value to assign to the unobserved pattern. One way to solve this issue would be to use a Machine Learning framework based on one of the classification algorithms, where estimated output values would become *class labels* for input patterns x. After training the classification algorithm is able to "generalize" and extend the knowledge gathered during training onto unseen input patterns. The authors of earlier aperture operator papers [1] suggest the use of Oblique Decision Trees built with OC1 algorithm [5], which combines deterministic hill-climbing with randomization steps in order to find sub-optimal split in form of a hyperplane.

However of all classification algorithms known to date, there is no one which can perform best with *all* datasets, and since non-linear image filters are used with many different kinds of imagery, it is impossible to point out just one optimum classification algorithm. Furthermore most classification algorithms are based on the principle of segmenting the search space into continuous regions where the input data is assigned the same label. Therefore in order for this to work the input data belonging to the same class should form one or more clusters in the search space. If the data from one class is too scattered and mixed with data from another class, classification algorithms will give very poor performance.

On the other hand, even a properly trained classifier is unlikely to properly generalize the knowledge over a very large region of the search space. Therefore it is impractical to try filter configurations for which we can only supply data for a small fraction of the search space during training, unless filtered signals have very simple and predictable characteristics.

If we consider only using filter configurations for which we can supply training examples for most part of the search space, the easiest implementation are lookup tables. The disadvantage of this approach is significant memory requirement, and no built-in generalization ability (it is however possible to use k-nearest neighbor classifier for patterns unobserved during training). On the other hand, lookup tables do not introduce any classification error and are very computationally efficient with respect to other classification algorithms, since there is no need for additional training after the statistics for output estimates are gathered and output values are calculated. Yet another advantage of lookup tables is their ability to store output values over a wider range (as opposed to most classification algorithms where output value range, thus the number of labels, needs to be low in order to reduce training cost and achieve good accuracy).

3. APERTURE CONSTRAINTS

The main source of error when using aperture operators, (assuming adequately trained output estimates), arises from the constraints which limit the potential filter performance.

3.1 Windowing constraint.

Corrupted signal $h(t)$ and ideal signal $g(t)$ are assumed to be jointly stationary. This assumption is necessary to proceed with output estimation on the basis of examples of ideal signal $g(t)$. Also the mapping function $\psi(h(W))$ is assumed to be translation invariant with respect to the window placement in the domain of signal $h(t)$, which is required from a statistical point of view to achieve good output estimates. In practice these assumptions could introduce an error in the filter performance, but this is very difficult to avoid. Another source of error is induced by constraining the signal $h(t)$ with a window W. This constraint can cause error if the operator is not able to observe enough of the signal $h(t)$ to efficiently estimate output. This error depends on window size and shape, and in general it would decrease with increasing window size. On the other hand, from the point of view of statistical estimation, the search space of all patterns that can be observed in a window needs to be kept small, which means that the window size also should be small. Optimal filter performance is achieved with a window configuration that balances the error introduced by domain constraint and the output estimation error.

3.2 Aperture output value range constraint.

The idea behind aperture operators is that the filter is only observing a section of the input signal, and is able to filter the subtle signal changes inside this section. According to the originators of the aperture operator concept, the operator's output should also be clipped to the aperture boundaries. In this way the probability mass of the output random variable is more condensed, which in practice means less class labels for Machine Learning output mapping described above. Constraining output value range will obviously introduce error, as for a particular input pattern, even if it fits into the aperture without clipping, output signal does not always fall into the aperture range. Research results presented in this paper suggest that this constraint can seriously restrict the filter performance.

3.3 Output value's translation invariance with respect to the aperture position.

Another assumption in aperture filters is translation invariance in gray-scale of the operator's output value. Every pattern observed through the aperture is assigned only one output value, disregarding the absolute gray-scale level at which the aperture was placed. This assumption increases the number of occurrences for every pattern, however it introduces an error for applications where the relative signal corruption varies with the absolute signal value i.e. blur.

4. RESEARCH RESULTS

4.1 Removing range constraint

The analysis of range constraint impact on filter performance was tested for a deblurring application. The set of 100 grayscale pictures (8 bit) of Glasgow suburbs in 640×480 resolution was blurred with Gausian lowpass filter with 3×3 kernel and standard deviation of 3. From this dataset, 20 pairs were selected for testing purposes. The remaining 80 image pairs were used for training.

In this experiment 24 aperture filter configurations were tested. For each of the six different window shapes, presented in fig. 1, an aperture filter was designed with 3, 5, 7 and 9 aperture levels. The aperture was positioned on the center pixel value. Each filter was extended by scaling with 8 scaling factors: 1, 4, 8, 12, 16, 20, 24, and 28. If the observed pattern was clipped by the aperture, then it was scaled by a successively higher factor until the signal could be observed through the aperture without clipping (or the maximum scaling factor was reached).

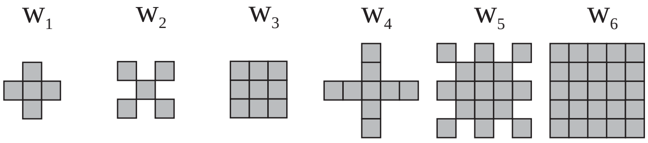

Figure 1: Window shapes of aperture filters used

window shape	aper. levels	number of images in training							
		10	20	30	40	50	60	70	80
initial error (blur)		75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0
W_1	3	73.2	73.1	73.2	73.5	73.4	73.4	73.3	73.3
W_1	5	70.9	70.5	70.5	70.6	70.5	70.4	70.2	70.2
W_1	7	70.9	69.7	69.5	69.6	69.5	69.3	69.2	69.2
W_1	9	70.9	69.6	69.5	69.4	69.3	69.0	69.0	68.9
W_2	3	74.3	74.1	74.2	74.3	74.3	74.3	74.2	74.1
W_2	5	72.3	71.2	71.1	71.1	71.1	71.0	70.8	70.8
W_2	7	71.6	70.2	70.0	69.8	69.7	69.4	69.5	69.5
W_2	9	72.2	70.1	69.8	69.5	69.4	69.2	69.1	69.1
W_3	3	74.3	73.8	73.8	73.8	73.8	73.7	73.6	73.6
W_3	5	74.3	72.4	72.0	72.0	71.8	71.4	71.1	71.0
W_3	7	75.9	73.4	72.9	72.8	72.4	71.6	71.2	70.9
W_3	9	77.8	74.9	74.8	74.9	74.3	73.2	72.7	72.3
W_4	3	72.1	71.4	71.4	71.4	71.4	71.4	71.2	71.1
W_4	5	70.6	68.8	68.5	68.5	68.2	67.7	67.6	67.4
W_4	7	70.5	68.2	67.7	67.4	66.9	66.4	66.1	65.7
W_4	9	71.6	68.9	68.3	68.2	67.8	67.1	66.5	66.0
W_5	3	76.1	74.0	73.9	73.7	73.4	73.0	72.6	72.6
W_5	5	76.8	75.1	75.3	75.6	75.3	74.7	74.3	73.9
W_5	7	76.0	75.0	75.2	75.4	75.3	75.0	74.8	74.3
W_5	9	75.6	75.0	75.0	75.2	75.1	74.9	74.7	74.4
W_6	3	77.8	76.2	76.0	75.9	75.6	75.1	74.7	74.5
W_6	5	76.5	75.7	76.1	76.4	76.2	76.0	75.9	75.6
W_6	7	75.7	75.1	75.2	75.3	75.2	75.1	75.0	74.8
W_6	9	75.3	75.0	75.0	75.0	74.9	74.9	74.8	74.6

Table 1: MSE of filtering results with standard aperture.

Training was performed using a dataset ranging from 10 images ($636 \times 476 \times 10 = 3027360$ sample patterns) to 80 images (24218880 patterns). Every filter was trained in two versions: a standard version with output constrained to the aperture boundaries and scaled appropriately; and an unconstrained version, where the output value was neither clipped nor scaled.

After training the filters were applied on the blurred images in the test set. Filtering results are presented in tables 1 and 2 as a Mean Square Error (MSE) measure between original and filtered images, calculated over the entire test set (20 image pairs) to get an average value.

In all tested configurations filters with unconstrained output performed better than standard aperture filters for any number of training samples. The only exception was the filter with window shape W_3 (see fig. 2) and 9 aperture levels, which performed slightly worse when trained with the smallest training sets. However for those training sets both the standard and unconstrained filters were seriously undertrained.

It can be seen that the Mean Squared Error (MSE) as a function of the number of training samples decreases monotonically as expected, with the exception of a point around 12500000 samples, where for most filters there is a very noticeable error increase. This is due to a few images included into the training set from this point, which have non representative statistical properties, hence worsening output estimates. This performance deterioration is much less noticeable with the standard aperture filter with constrained output, which suggests that unconstrained filters are more susceptible for improper training.

window shape	aper. levels	number of images in training							
		10	20	30	40	50	60	70	80
initial error (blur)		75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0
W_1	3	67.8	67.2	67.5	67.9	68.1	67.8	67.6	67.4
W_1	5	65.3	64.0	64.2	64.6	64.6	64.2	63.9	63.8
W_1	7	66.0	63.6	63.5	63.9	63.8	63.3	63.0	62.9
W_1	9	67.4	64.2	63.9	64.2	63.8	63.3	62.9	62.7
W_2	3	66.4	65.9	66.1	66.5	66.6	66.3	66.2	66.1
W_2	5	64.1	62.8	62.9	63.2	63.2	62.8	62.6	62.5
W_2	7	65.4	62.9	62.6	62.9	62.7	62.1	61.9	61.7
W_2	9	67.6	63.7	63.2	63.4	63.0	62.4	62.0	61.8
W_3	3	66.4	65.3	65.3	65.7	65.7	65.3	65.1	65.0
W_3	5	69.3	65.5	65.2	65.4	64.9	64.0	63.5	63.2
W_3	7	74.2	69.1	68.6	68.9	68.0	66.5	65.6	65.1
W_3	9	78.2	72.6	72.6	73.4	72.4	70.5	69.2	68.4
W_4	3	57.4	56.4	56.2	56.3	56.3	56.1	56.0	55.9
W_4	5	56.9	53.5	52.7	52.6	52.1	51.4	51.0	50.7
W_4	7	60.2	55.7	54.7	54.5	53.7	52.5	51.7	51.3
W_4	9	63.5	58.7	57.8	57.8	56.8	55.2	54.2	53.5
W_5	3	64.9	61.2	60.7	61.0	60.4	59.5	58.9	58.5
W_5	5	68.8	65.3	65.0	65.6	64.9	63.6	62.6	61.7
W_5	7	71.4	68.9	68.6	68.8	68.4	67.5	66.9	66.1
W_5	9	72.5	70.9	70.6	70.7	70.3	69.7	69.4	68.6
W_6	3	69.9	65.9	65.7	66.2	65.5	64.3	63.3	62.7
W_6	5	70.8	68.8	68.8	69.2	68.8	68.1	67.5	66.7
W_6	7	72.6	71.3	71.0	70.9	70.5	70.1	69.9	69.3
W_6	9	73.5	72.7	72.3	72.3	72.0	71.7	71.5	71.1

Table 2: MSE of filtering results with aperture with unconstrained output

The best unconstrained filter found was with window W_4 and 5 aperture levels. This filter outperformed other tested filters for all training sets. A comparison of best unconstrained filters for each window is in fig. 4. It can be seen that the window shape plays a very important role in filter performance, not only its size. Filters with windows W_3 and W_4 with 5 aperture levels have the same search space size. The only difference in those configurations is different window shape, and the filter with cross-shaped window performs much better than the square-shaped.

In conclusion, aperture filters with unconstrained output considerably outperform standard aperture filters. In this experiment it was shown that in practice the ideal output value does not always fit inside the aperture boundaries, and clipping it to the aperture boundaries damages the filter performance, even with the use of scaling. Output value clipping is beneficial for the implementation of classification algorithms like Decision Trees, where reducing the number of classes is crucial for finding effective operator representation, both in terms of classification accuracy and computational cost. For more constrained configurations, it is however much more cost effective to implement filter representation in the form of simple lookup tables. In such implementations constraining output value reduces the memory requirement, but it increases the filter error significantly.

4.2 Filter output invariance with respect to the aperture position level

Another common assumption made with aperture filters is that of translation invariance as applied to gray-scale. In order to reduce this constraint without losing estimation quality a simple approach was tested. The full gray-scale range (256 levels) was divided into 10 sections. Every pattern observed during training was assigned 10 separate output values corresponding to the gray-scale section, where the aperture was placed. If the number of examples in the training set for a

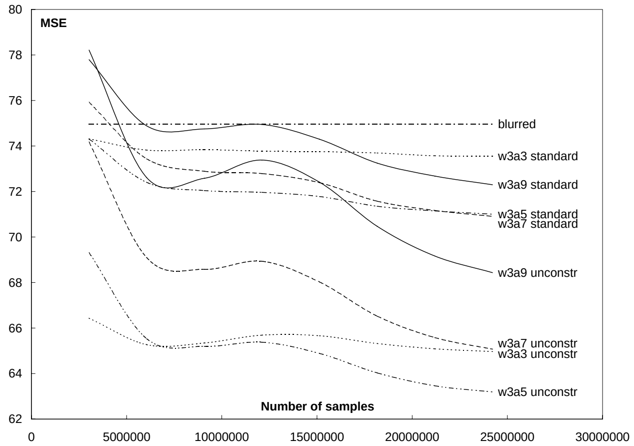

Figure 2: Comparison of standard and unconstrained output apertures with window W_3.

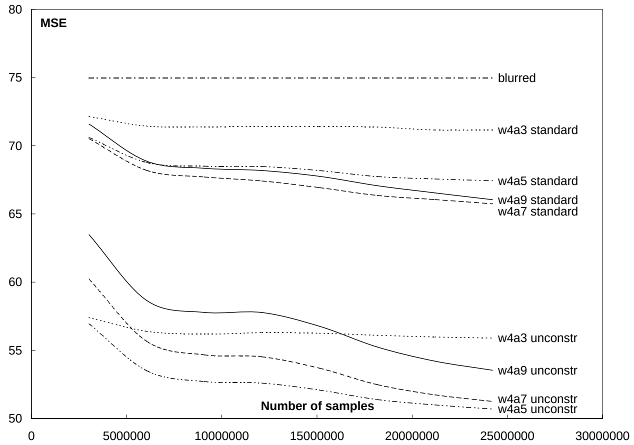

Figure 3: Comparison of standard and unconstrained output apertures with window W_4.

given section was lower than 10, the global estimate was used (calculated over all sections). This way the operator's output value was dependent on the aperture position.

In order to test this approach the same dataset was used as described above. In this experiment 16 filter configurations were tested with windows W_1 to W_4 (see fig. 1) and with 3, 5, 7 and 9 aperture levels. As with previous experiment the same scaling strategy and factors were used.

Filtering results in form of MSE measures calculated over the entire test set are presented in table 3. It can be seen that this approach was beneficial for small apertures (fig. 5). The best filter configuration (window W_4 with 5 aperture levels) improvement with respect to the aperture with unconstrained output was still noticeable (fig. 6), especially with large training sets (around 2% reduction in MSE). With this filter configuration 34% of MSE reduction was achieved as compared to 10% with the standard aperture filter of the same configuration. In fig. 8 a fragment (400×300) of one of the test images filtered with this configuration is presented. The image was cropped to improve detail reproduction in this paper. It can be seen that aperture with multiple output values slightly improves filtering of small details (light reflections

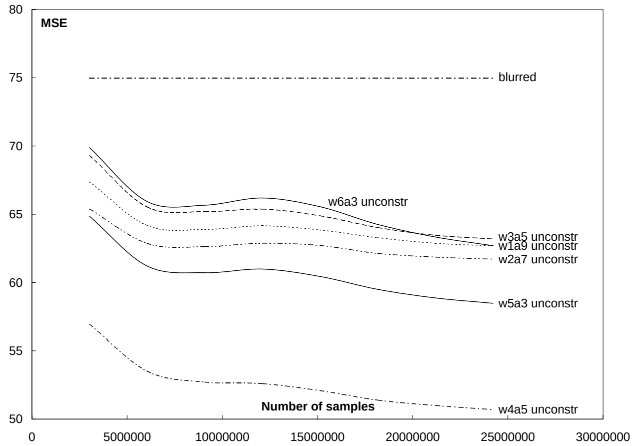

Figure 4: Comparison of best unconstrained output apertures for each window.

window shape	aper. levels	number of images in training							
		10	20	30	40	50	60	70	80
initial error (blur)		75.0	75.0	75.0	75.0	75.0	75.0	75.0	75.0
W_1	3	64.6	63.2	63.2	63.9	63.7	63.1	62.8	62.7
W_1	5	64.9	62.5	62.3	62.8	62.5	61.6	61.2	61.0
W_1	7	66.1	62.9	62.5	63.0	62.6	61.6	61.2	60.9
W_1	9	67.8	63.9	63.4	63.8	63.2	62.1	61.6	61.2
W_2	3	63.9	62.4	62.4	62.9	62.8	62.3	62.0	61.9
W_2	5	64.6	62.1	61.8	62.2	61.9	61.0	60.6	60.4
W_2	7	66.4	63.1	62.6	62.8	62.4	61.4	60.9	60.6
W_2	9	68.6	64.3	63.7	63.9	63.3	62.3	61.7	61.3
W_3	3	64.9	62.5	62.3	62.8	62.6	61.9	61.5	61.3
W_3	5	69.7	65.3	64.8	65.0	64.4	63.2	62.5	62.0
W_3	7	74.8	69.4	68.8	69.2	68.2	66.6	65.5	64.9
W_3	9	78.8	72.9	72.9	73.7	72.7	70.7	69.8	68.9
W_4	3	55.9	53.7	53.2	53.5	53.4	52.9	52.6	52.5
W_4	5	57.2	53.1	52.1	52.0	51.2	50.2	49.7	49.3
W_4	7	60.5	55.6	54.5	54.2	53.3	52.0	51.1	50.6
W_4	9	63.8	58.8	57.9	57.8	57.1	55.4	54.3	53.5

Table 3: MSE of filtering results with aperture with multiple output values

are more apparent on the front right wheel of the car).

With larger apertures the improvement was minimal (see fig. 7) and in a few cases the filter performance was actually degraded. Possibly for larger search spaces the numbers empirically chosen for this approach (10 output values and 10 minimal number of examples described above) were not optimal. However for smaller apertures the improvement achieved was over 6% for window W_1 and 3 aperture levels, which increases the performance by a factor of 2.

5. CONCLUSIONS AND FURTHER WORK

It was shown in this paper that releasing some of the aperture filter constraints can improve filtering performance by a factor of up to 3. Output value constraint removal was beneficial for this particular dataset for all of the tested filter configurations, and it gave a massive performance improvement. Estimating separate output values for several gray-scale ranges was shown to be beneficial in most cases, however more detailed research is required to address the reduced performance with large apertures.

Further work will include combining the presented methods with other techniques described to further improve the results.

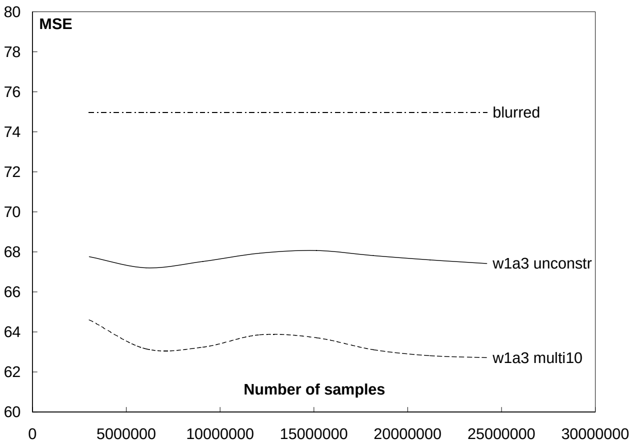

Figure 5: Comparison of multiple and single unconstrained output apertures for window W_1 and 3 aperture levels.

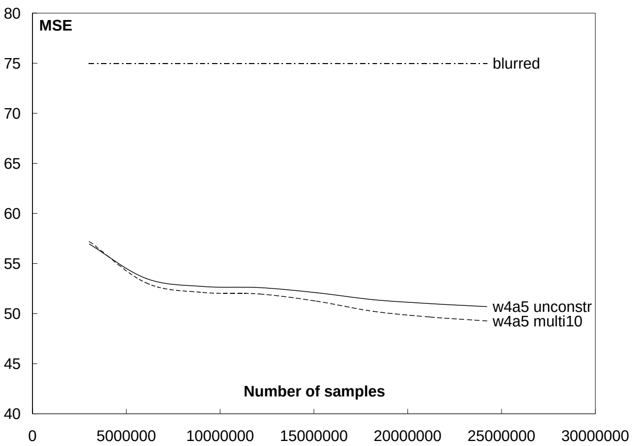

Figure 6: Comparison of multiple and single unconstrained output apertures for window W_4 and 5 aperture levels.

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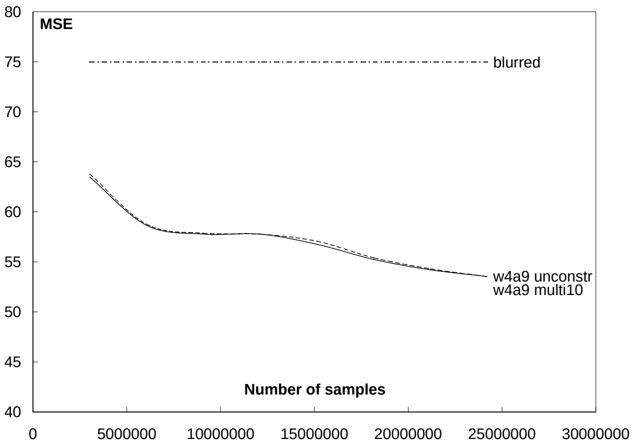

Figure 7: Comparison of multiple and single unconstrained output apertures for window W_4 and 9 aperture levels.

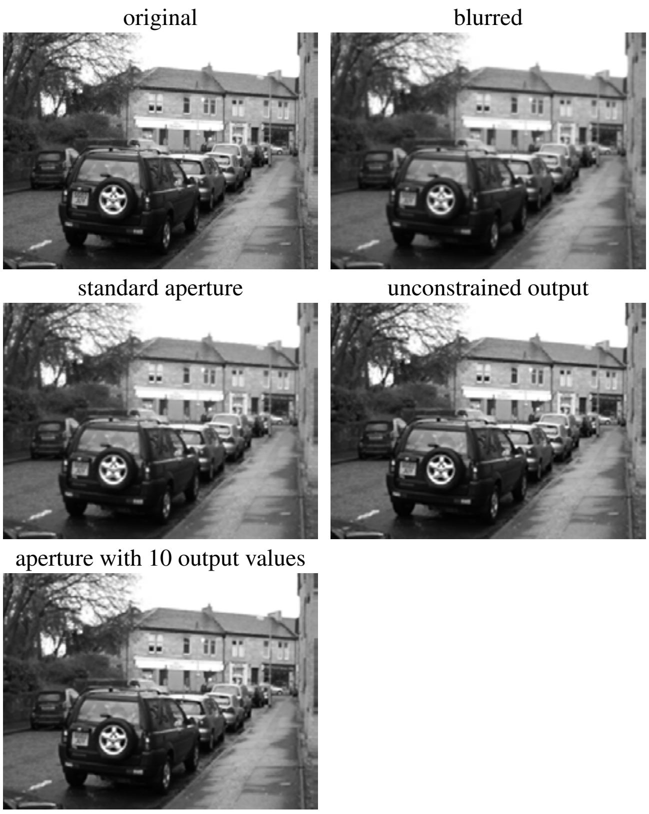

Figure 8: Example image from the test set filtered with aperture filters with window W_4 and 5 aperture levels.

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