MAXIMUM LIKELIHOOD FREQUENCY OFFSET ESTIMATION FOR FREQUENCY SELECTIVE MIMO CHANNELS: AN AVERAGE PERIODOGRAM INTERPRETATION AND ITERATIVE TECHNIQUES.

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ABSTRACT

We provide exact and approximate maximum likelihood solutions and the corresponding Cramer Rao Lower Bounds (CRLBs) for the estimation of channel fading coefficients and frequency offsets (FOs) for a MIMO frequency selective channel. In our problem we considered estimation of parameters of data observed over a number of frames, and assumed FOs during those frames do not vary significantly as compared to the channel fading coefficients that change between frames according to Rayleigh fading profile. We show the resulting estimator for the FO is an average periodogram maximizer. We also provide an approximation to the estimator to reduce the data storage complexity. Since FO estimator requires a large set of data for satisfactory performance, we provided an iterative technique which combines very short pilot symbols with the soft estimate of the transmitted data symbols to iteratively refine the performance of the estimator.

1. INTRODUCTION

Communication through wireless transmission is affected, apart from signal fading, by interference, additive noise and FOs. FO is introduced due to poor synchronisation between the transmitter and receiver local oscillators. The FOs could also be introduced due to Doppler shifts, that are induced due to relative motion between the base station and the mobile station. Various methods have been proposed for the estimation of FOs, [1, 2, 3, 4], and in particular [5, 6, 4, 7, 8] considered the estimation of multiple FOs due to distinct Doppler shifts. In this paper, however, we look at a scenario where signal is observed over a number of frames, and the channel coefficients are assumed to be changing between frames according to a fading process, but the FOs could remain relatively unchanged over a number of frames. This scenario could arise mainly due to poor synchronisation of local oscillators. This might become increasingly an issue, for example in sensor networks, where mass scale production of sensors at low cost might result into imperfect local oscillators. Also a cluster of sensors for transmission and reception could form a virtual MIMO systems experiencing distinct carrier offsets as addressed in this work. We show even if the channel coefficients are time varying between frames, the estimation of FOs, in this case, is maximisation of average periodogram. We also provide an approximation technique to reduce data storage complexity associated with the average periodogram. This technique proves to be very useful specially when available training signal is very short. We looked at this estimation problem for a very general case incorporating multiple antennas at the transmitter and the receiver. We also derived the CRLBs and showed that the variance of the estimator attains this bound. When training signal is very short, it may not be adequate to obtain accurate FOs estimation, mainly due to resolution issue associated with discrete Fourier transforms. We therefore extended the proposed technique to estimate the FOs iteratively by using soft estimate of the transmitted signal and the short pilot sequence. The result shows that the estimation performance is very close to as if the whole transmitted signal is pilot.

2. PROBLEM STATEMENT

Consider a MIMO communication system with n_T transmit and n_R receive antennas, where the signal between any two transmit and receive antennas is assumed to have a multipath channel of length L. The received baseband signal at antenna k can be written as

$$r_k^q(m) = u_k^q(m) + \eta_k^q(m) \tag{1}$$

where

$$u_k^q(m) = \sum_{l=1}^{n_T}\sum_{p=0}^{L-1} h_{kl}^q(p) s_l^q(m-p) e^{j\omega_{kl}m} \tag{2}$$

for $m = n, \ldots, n-N+1$ and $k = 1, \ldots, n_R$. The postscript q denotes the frame/packet number, $q = t,\ t-1, \ldots, t-W+1$. N is the number of symbols received in each frame, W is the total number of frames used for the parameter estimation, $h_{kl}^q(p)$ and ω_{kl} are the channel gain and the FOs, respectively, between the transmit antenna l and the receive antenna k. The channel coefficients are therefore assumed to be changing between frames, but FOs are relatively unchanged. Here, $\{s_l^q(m)\}$ is the q^{th} training signal frame transmitted from the l^{th} transmitter and $\eta_k^q(m)$ is assumed to be zero mean, circularly distributed, spatially uncorrelated, white Gaussian noise with variance σ_η^2. Let

$$\begin{aligned}
\mathbf{s}_{lp}^q &= \left[\begin{array}{ccc} s_l^q(n-p) & \cdots & s_l^q(n-p-N+1) \end{array}\right]^T \\
\mathbf{e}_{kl} &= \left[\begin{array}{ccc} e^{j\omega_{kl}n} & \cdots & e^{j\omega_{kl}(n-N+1)} \end{array}\right]^T \\
\mathbf{h}_{kl}^q &= \left[\begin{array}{ccc} h_{kl}^q(0) & \cdots & h_{kl}^q(L-1) \end{array}\right]^T
\end{aligned}$$

where $(\cdot)^T$ denotes the transpose operator, $\mathbf{e}_{kl}\ \varepsilon\ C^{N\times 1}$ contains FOs between the receive antenna k and the transmit antenna l. $\mathbf{h}_{kl}^q\ \varepsilon\ C^{L\times 1}$ is the vector of channel gains between the receive antenna k and the transmit antenna l for the frame q. Let $\mathbf{S}_{lp}^q$ denotes the N×N diagonal matrix formed from the signal vector $\mathbf{s}_{lp}^q$. The training signals from different antennas are assumed to be orthogonal to each other. Further, suppose that $\mathbf{S}_{kl}^q = \left[\begin{array}{ccc} \mathbf{S}_{l0}^q\mathbf{e}_{kl} & \cdots & \mathbf{S}_{l(L-1)}^q\mathbf{e}_{kl} \end{array}\right]$, $\mathbf{S}_k^q = \left[\mathbf{S}_{k1}^q \cdots \mathbf{S}_{kn_T}^q\right]$, and $\mathbf{h}_k^q = \left[\mathbf{h}_{k1}^{q\ T} \cdots \mathbf{h}_{kn_T}^{q\ T}\right]^T$.

Since the Gaussian noise is assumed to be spatially uncorrelated, we could estimate the FOs, and channel gains for different receiver antennas independently. Therefore without loss of generality, we explain the proposed algorithm for the k^{th} receive antenna only. We write the signal received during the frame number q in a vector form as follows:

$$\mathbf{r}_k^q = \left[\begin{array}{ccc} r_k^q(n) & \cdots & r_k^q(n-N+1) \end{array}\right]^T = \mathbf{S}_k^q\mathbf{h}_k^q + \eta_k^q$$

Now we append the received signal vector of the kth receiver antenna over the last W frames of observations as $\mathbf{r}_k = [\mathbf{r}_k^{(t-W+1)^T} \cdots \mathbf{r}_k^t T]^T$. The received signal vector $\mathbf{r}_k\ \varepsilon\ C^{WN\times 1}$ could be related to the transmitted signal and the channel parameters as $\mathbf{r}_k = \mathbf{S}_k\mathbf{h}_k + \eta_k$. Where $\mathbf{S}_k$ =diag$[\mathbf{S}_k^{(t-W+1)}\mathbf{S}_k^{(t-W)}...\mathbf{S}_k^t]$

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is a block diagonal matrix with $\mathbf{S}_k \; \varepsilon \; C^{WN \times Ln_T W}$ and $\mathbf{h}_k = [\mathbf{h}_k^{t-W+1^T} \cdots \mathbf{h}_k^{t\,T}]^T \; \varepsilon \; C^{Ln_T W}$ is a vector of all channel fading coefficients. Here we are interested in estimating various channel gains, $\mathbf{h}_{kl}^q$ and FOs w_{kl}. Let $\mathbf{w}_k = [w_{k1}, \cdots, w_{kn_T}]^T$. Then the parameter vector $\boldsymbol{\theta}_k$ to be estimated can be written as $\boldsymbol{\theta}_k = [\,\mathbf{h}_k{}^T \mathbf{w}_k{}^T]^T$. The log-likelihood function could be written as

$$p(\mathbf{r}_k|\,\theta) = \frac{1}{(\pi)^N |\mathbf{C}_{\eta_k}|} \, e^{-(\mathbf{r}_k - \mathbf{S}_k\mathbf{h}_k)^H \mathbf{C}_{\eta_k}^{-1} (\mathbf{r}_k - \mathbf{S}_k \mathbf{h}_k)}, \tag{3}$$

where $(\cdot)^H$ denotes the conjugate transpose and $\mathbf{C}_{\eta_k}$ is the noise covariance matrix, which according to the assumption is a diagonal matrix with σ_η^2 as its elements. Taking the natural logarithm and ignoring the constant terms, (3) can be written as

$$\ln p(\mathbf{r}_k|\,\theta) = -\frac{1}{\sigma_\eta^2} \, (\mathbf{r}_k - \mathbf{S}_k \mathbf{h}_k)^H \, (\mathbf{r}_k - \mathbf{S}_k \mathbf{h}_k). \tag{4}$$

Maximising (4) with respect to $\boldsymbol{\theta}$ is equivalent to minimising the following cost function:

$$J = (\mathbf{r}_k - \mathbf{S}_k \mathbf{h}_k)^H (\mathbf{r}_k - \mathbf{S}_k \mathbf{h}_k) \tag{5}$$

Minimising with respect to $\mathbf{h}_k$ yields $\hat{\mathbf{h}}_k = (\mathbf{S}_k^H \mathbf{S}_k)^{-1} \mathbf{S}_k^H \mathbf{r}_k$. Substituting this into the cost function J we obtain

$$J(w_{kl}) = \mathbf{r}_k^H \mathbf{r}_k - \mathbf{r}_k^H \mathbf{S}_k (\mathbf{S}_k^H \mathbf{S}_k)^{-1} \mathbf{S}_k^H \mathbf{r}_k \tag{6}$$

Thus, to minimise the cost function we maximise the term shown below:

$$\mathbf{w}_k = \arg\max_{\mathbf{w}_k} \; \mathbf{r}_k^H \mathbf{S}_k (\mathbf{S}_k^H \mathbf{S}_k)^{-1} \mathbf{S}_k^H \mathbf{r}_k \tag{7}$$

We note that $\mathbf{S}_k^H \mathbf{S}_k$ will be dominated by the large diagonal terms, with almost negligible contribution from the off-diagonal terms [7]. Thus, $\mathbf{S}_k^H \mathbf{S}_k \approx \sum_{n=0}^{N-1} |x_l(n)|^2 \, \mathbf{I} \stackrel{\triangle}{=} \kappa \mathbf{I}$, where κ is constant over the frame considered, enabling us to approximate the minimum of (6) as the maximum of

$$J'(\omega_{kl}) = \mathbf{r}_k^H \mathbf{S}_k \mathbf{S}_k^H \mathbf{r}_k$$

$$= \sum_{q=t-W+1}^{t} \sum_{l=1}^{n_T} \sum_{p=0}^{L-1} \left| \sum_{n=0}^{N-1} r_k^*(n) s_l^q(n-p) e^{j\omega_{kl} n} \right|^2 \tag{8}$$

Hence the estimator for the FO is the average periodogram maximiser. For this, we need to save the Fourier transforms of the last W frames. This might result into consumption of significant memory at the receiver. Hence we propose an approximation to the above periodogram maximiser using recursive smoothing operation. Writing f_t as the periodogram obtained using the frame t, i.e

$$f_t \;=\; \sum_{l=1}^{n_T} \sum_{p=0}^{L-1} \left| \sum_{n=0}^{N-1} r_k^*(n) s_l^t(n-p) e^{j\omega_{kl} n} \right|^2. \tag{9}$$

We write the average periodogram required in (8) as

$$F_t = (1-\alpha)\, F_{t-1} + \alpha\, f_t \tag{10}$$

where α is chosen approximately as $1/W$. In this case we need to store only the approximate sum of the periodogram of the previous frame. In the simulation we will compare estimation performance of the exact method with the approximation.

3. ITERATIVE MMSE MIMO EQUALIZER DESIGN

In most wireless system, the available short pilot symbols are inadequate to estimate FOs. For example, a GSM burst consists of only 26 pilot symbols and 116 data symbols. We therefore use an iterative technique (turbo equalization) to generate soft estimates of the transmitted data symbols and treat them as pilot symbols to estimate channel and FOs iteratively as shown in Fig 1. We first explain the iterative equalizer for a particular frame, hence without loss of generality avoid the frame index q. Also we denote the transmitted data signal as $\mathbf{s}(n)$ and the corresponding received signal as $\mathbf{r}(n)$. For an equalizer of temporal length M, the received signal vector of dimension $n_R M \times 1$ is written as [5] [6]

$$\mathbf{r}(n) = \mathbf{H}_c(n)\mathbf{s}(n) + \eta(n) \tag{11}$$

where $\eta(n)$ is additive white Gaussian noise vector and $\mathbf{H}_c(n)$ is the $n_R M \times n_T(L+M-1)$ channel convolution matrix [6],

$$\begin{aligned}
\mathbf{r}(n) &= \left[\; \mathbf{r}_1^T(n) \quad \ldots \quad \mathbf{r}_{n_R}^T(n) \;\right]^T \\
\mathbf{r}_k(n) &= \left[\; r_k(n) \quad \ldots \quad r_k(n-M+1) \;\right]^T, \\
\mathbf{s}(n) &= \left[\; \mathbf{s}_1^T(n) \quad \ldots \quad \mathbf{s}_{n_T}^T(n) \;\right]^T, \\
\mathbf{s}_l(n) &= \left[\; s_l(n) \quad \ldots \quad s_l(n-M-L+2) \;\right]^T \\
\mathbf{H_c}(n) &= \begin{bmatrix} \mathbf{C}^n & 0 & \ldots & 0 \\ 0 & \mathbf{C}^{(n-1)} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \ldots & 0 & \mathbf{C}^{(n-M+1)} \end{bmatrix}, \\
\mathbf{C}^n &= [\mathbf{h}_1^n \, \mathbf{h}_2^n \ldots \mathbf{h}_{n_R}^n]^T, \\
\mathbf{h}_k^n &= [\mathbf{h}_k^n(0)^T \; \mathbf{h}_k^n(1)^T \; \mathbf{h}_k^n(L-1)^T]^T, \\
\mathbf{h}_k^n(p) &= [h_{k1}(p) e^{jw_{k1} n} \ldots h_{kn_T}(p) e^{jw_{kn_T} n}]^T,
\end{aligned}$$

We set the temporal equalizer length same as the channel length, i.e., $M = L$, and decompose (11) to explicitly show user l symbol with delay (L-1) as

$$\mathbf{r}(n) = \mathbf{H}_c(n)\tilde{\mathbf{s}}(n) + \mathbf{H}_c(n)\mathbf{c}_{lL} s_l(n-L+1) + \eta(n) \tag{12}$$

where operator $\mathbf{c}_{lL}$ is a coordinate vector such that $\mathbf{H}_c(n)\mathbf{c}_{lL}$ will choose the L^{th} column of $\mathbf{H}_c(n)$ corresponding to the l^{th} transmitter antenna, and vector $\tilde{\mathbf{s}}(n)$ includes all the elements of $\mathbf{s}(n)$ except $s_l(n-L+1)$, i.e., $\tilde{\mathbf{s}}(n) = \left[\; \tilde{\mathbf{s}}_1^T(n) \quad \ldots \quad \tilde{\mathbf{s}}_{n_T}^T(n) \;\right]^T$, where $\tilde{\mathbf{s}}_i(n) = [s_i(n) \ldots s_i(n-L+2) \;\; 0 \;\; s_i(n-L) \ldots s_i(n-2L+2)]^T$ if $i = l$, and $\tilde{\mathbf{s}}_i(n) = [s_i(n) \ldots s_i(n-2L+2)]^T$ when $i \neq l$. According to (12), the FOs can be removed by

$$\begin{aligned}
\tilde{\mathbf{r}}(n) &= (\mathbf{u}(n) - \mathbf{H}_c(n)\bar{\mathbf{s}}(n)) \odot \begin{bmatrix} e^{-j\mathbf{w}_{l1}} \\ \vdots \\ e^{-j\mathbf{w}_{ln_R}} \end{bmatrix} \\
&= \mathbf{D}\left(\mathbf{H}_c(n)(\mathbf{s}(n) - \bar{\mathbf{s}}(n)) + \eta(n)\right)
\end{aligned} \tag{13}$$

where $\odot$ denotes the Schur-Hadamard product, $\mathbf{w}_{lk} = [w_{lk}(n) \ldots w_{lk}(n-L+1)]^T$, $\bar{\mathbf{s}}(n)$ is the mean of $\tilde{\mathbf{s}}(n)$ obtained from the extrinsic information passed from the BCJR based MAP decoder and $\mathbf{D}$ = diag($\boldsymbol{\theta}$), $\boldsymbol{\theta} = [e^{-j\mathbf{w}_{l1}} \ldots e^{-j\mathbf{w}_{ln_R}}]^T$. Assuming transmitted symbols are temporally uncorrelated, we could write $E\{(\mathbf{s}(n)-\bar{\mathbf{s}}(n))(\mathbf{s}(n)-\bar{\mathbf{s}}(n))^T\}$ as a diagonal matrix $diag(\mathbf{v}_l(n))$, where $\mathbf{v}_l(n)$ provides the variance of the symbols transmitted from antenna l as will be explained later in (16). Hence $E\{\tilde{\mathbf{r}}(n)\tilde{\mathbf{r}}(n)^H\} = \mathbf{D}(\mathbf{H}_c(n)\mathrm{diag}(\mathbf{v}_l(n))\mathbf{H}_c(n)^H + \sigma_\eta^2 \mathbf{I})\mathbf{D}^H$ and $E\{s_l(n-L+1)\tilde{\mathbf{r}}(n)\}$ = $E\{s_l(n-L+1)\mathbf{D}(\mathbf{H}_c(n)(\mathbf{s}(n)-\bar{\mathbf{s}}(n)) + \eta(n))\}$ = $\mathbf{D}\mathbf{H_c}(n)\mathbf{c}_{lL}$. $E\{s_l(n-L+1)\bar{\mathbf{s}}(n)\}$ is $\mathbf{0}$ because $\bar{\mathbf{s}}(n)$ does not contain

$s_l(n-L+1)$ and all other symbols in $\bar{\mathbf{s}}(n)$ are uncorrelated with $s_l(n-L+1)$. Hence the MMSE equalizer for the lth user is written as,

$$\phi_l(n) = \left(\mathbf{D}(\mathbf{H}_c(n)\mathrm{diag}(\mathbf{v}_l(n))\mathbf{H}_c(n)^H + \sigma_\eta^2\mathbf{I})\mathbf{D}^H\right)^{-1}\mathbf{D}\mathbf{H}_c(n)\mathbf{c}_{lL} \quad (14)$$

The lth user symbol is estimated as $\hat{s}_l(n-L+1) = \phi_l(n)^H\tilde{\mathbf{r}}(n)$. The estimated symbols at the equalizer output are de-interleaved and sent to the decoder. We assumed the encoder and the decoder are based on data from four bursts as explained below. The soft estimates of the transmitted signal $\bar{s}_n$ at the decoder output is interleaved and treated as a pilot signal to improve the estimation performance. The data $\mathbf{s}_d(n)$ in the packet is encoded and interleaved to form four bursts. A pilot sequence of length 26 is inserted to each burst and transmitted through a frequency selective channel as shown in Fig. 1. At the receiver, the channel corresponding to each burst is estimated and equalized separately, but the data symbols from all four bursts are collected, de-interleaved and decoded. The initial channel estimate is obtained using the pilot sequence contained in the middle of a burst. This estimate is used to design an MMSE equalizer and to obtain an initial estimate of the transmitted data. In subsequent iterations, the received signal vector $\mathbf{r}(n)$ in (11) will be passed to the iterative MMSE equalizer in (14) together with prior information from the decoder so that the contribution of all other users except the user of interest can be removed from the received signal as in (13) for each user l. The MMSE equalizer output $\hat{s}_l(n)$ is used to obtain the difference between the posteriori and a priori log-likelihood ratio (LLR) as follows [9, 10]

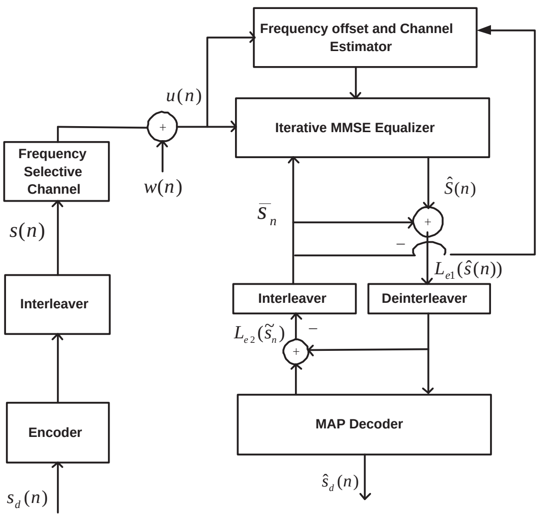

Figure 1: The block diagram describing the transmitter as well as the iterative channel estimation, equalization and decoding at the receiver.

$$\begin{aligned} L_{e1}[\hat{s}_l(n)] &= \ln\frac{p\{s_l(n)=+1|_{\hat{s}_l(n)}\}}{p\{s_l(n)=-1|_{\hat{s}_l(n)}\}} - \ln\frac{p\{s_l(n)=+1\}}{p\{s_l(n)=-1\}} \\ &= \ln\frac{p\{\hat{s}_l(n)|_{s_l(n)=+1}\}}{p\{\hat{s}_l(n)|_{s_l(n)=-1}\}} = \frac{4\mathrm{Re}\{\hat{s}_l(n)\}}{1-\phi_l(n)^H\mathbf{H}_c(n)\mathbf{c}_{lL}} \end{aligned}$$

where we assumed Gaussian probability density function (PDF) for $p\{\hat{s}_l(n)|_{s_l(n)=b}\}$ as follows

$$p[\hat{s}_l(n)] = \frac{1}{\sqrt{2\pi}\sigma_s}e^{-\frac{(\hat{s}_l(n)-m_b)^2}{2\sigma_{\hat{s}_l}^2}} \quad (15)$$

For the above PDF, the conditional mean and the variance are obtained from the knowledge of the channel and the equalizer as follows [10]

$$\begin{aligned} m_b &= E[\hat{s}_l(n)|_{s_l(n)=b}] = \phi_l(n)^H\mathbf{D}\mathbf{H}_c(n)\mathbf{c}_{lL}b \\ \sigma^2_{s_l|s_l(n)=b} &= \mathrm{Cov}\{\hat{s}_l(n), \hat{s}_l(n)|_{s_l(n)=b}\} \\ &= E\{\hat{s}_l(n)\hat{s}_l(n)^H\} - m_b m_b^H \\ &= \phi_l(n)^H\mathbf{D}\mathbf{H}_c(n)\mathbf{c}_{lL}(1-\phi_l(n)^H\mathbf{H}_c(n)\mathbf{c}_{lL}) \end{aligned}$$

The log-likelihood ratios of all user symbols will be determined in a similar way. Then the LLR of four consecutive bursts are collected, de-interleaved and decoded using MAP algorithm [11]. The MAP decoder would then provide the extrinsic information $L_{e2}(\tilde{s}_n)$ of the uncoded symbols. The mean of the symbol (soft estimate) $\bar{s}_n$ to be used in (13) is then found from this extrinsic information as $\bar{s}_n = \tanh(L_{e2}(\tilde{s}_n))$ [10]. The soft estimates of the transmitted signal are also treated as a pilot signal to determine the multiple FOs and to refine the channel estimates in an iterative fashion. Finally the variance required for the MMSE equalizer in (14) is also computed as follows [10],

$$\mathbf{v}_l(n) = 1 - \bar{s}_l(n)^2 \quad (16)$$

where $\bar{s}_l(n)$ denotes the soft estimates of the transmitted symbol from the decoder output.

4. SIMULATION

A scenario involving two transmitting and three receiving antennas, with 5 multi-paths between each pair, is considered. Moreover assuming quasi-stationary channel, the channel parameter $h_{kl}^q(p)$ remains the same throughout the frame but changes between frames, according to a complex Gaussian distribution, whereas the FO, w_{kl} is assumed to be constant over a number of frames. Binary Phase Shift Keying (BPSK) signals of length 200 are chosen as training data. The FOs are chosen as $0.003, 0.001, 0.002$ *and* $0.004\ 0.005\ 0.006$- one for each pair formed between the two transmitting and three receiving antennas. In the simulation, parameters are estimated using 10 frames and compared with corresponding CRLBs which is derived in the Appendix. The Fig. 2 and Fig. 3 depict the variance of the estimates of FOs(for the first two antenna pair) and the channel gains(for the first four paths) respectively. The simulation results show that the proposed estimator attains the CRLBs.

In the second simulation, we repeated the first simulation but for various number of frames. Fig. 4 depicts the variance of the FO estimator and we observe the performance improves with increasing number of frames.

In the third simulation, we compared FO estimation performance using W = 10 windows with approximation technique using $\alpha = 0.1$. For this simulation we considered a training sequence of 142 symbols. We also compared the performance with an iterative scheme using 26 pilot symbols and 116 data symbols as in GSM in Fig.5. Again we used 10 frames(bursts) for the estimation of FOs. We observe the estimator performance for 142 training symbols and the 26 training symbols (using iterative approach) is very close to each other, confirming the efficiency of the iterative scheme in the data re-use for training. The approximation method using $\alpha = 0.1$ seems to outperform the exact 10 window method, but this is not surprising because $\alpha = 0.1$ will infact consider more than 10 frames with exponentially decaying weight as follows:

$$\begin{aligned} F_n &= (1-\alpha)^W F_{n-W} + \alpha\sum_{i=0}^{W-1}(1-\alpha)^i f_{W-i} \\ &\approx (1-\alpha)^W F_{n-W} + \alpha\sum_{i=0}^{W-1} f_{n-i}\ , \ \ for\ small\ \alpha \end{aligned} \quad (17)$$

For the iterative equalization simulation, we considered a half rate convolutional coder and a MAP decoder [10]. The polynomials for the coder has been chosen as in GPRS CS1-CS3, (i.e, G0 = $1+D^3+D^4$ and G1 = $1+D+D^3+D^4$), [12] and [13]. The data bits corresponding to four consecutive bursts have been interleaved using a random interleaver, coded and modulated according to BPSK. Then pilot symbols of length 26 have been inserted in each burst of length 142 and transmitted. At the receiver, each burst is separately equalized as explained in the previous section, and the equalizer outputs of four consecutive bursts are collected, deinterleaved and decoded using MAP algorithm. The soft estimates of the uncoded bits are interleaved again and fedback to the iterative equalizers. We computed the uncoded BER performance for four iteration, and compared the result to the BER curve of an MMSE equalizer that was designed ignoring the effect of FOs in Fig.6. The proposed scheme significantly outperforms the conventional equalizer.

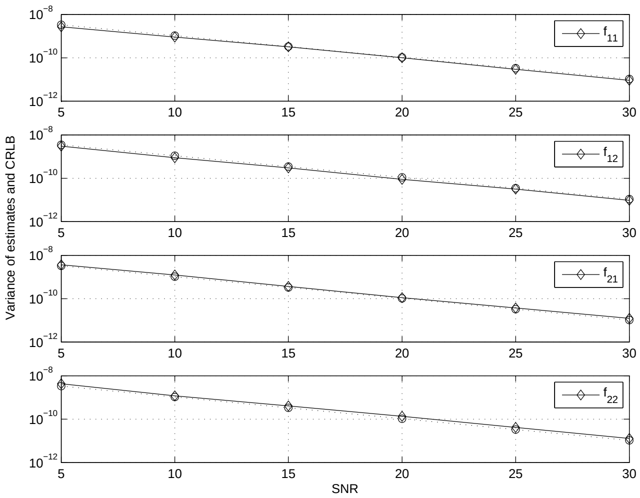

Figure 2: Comparison of the variance of the estimation error of FOs (diamond) with the corresponding CRLB (circle). Here, f_{kl} is the FO between the first receive antenna and the transmit antenna k.

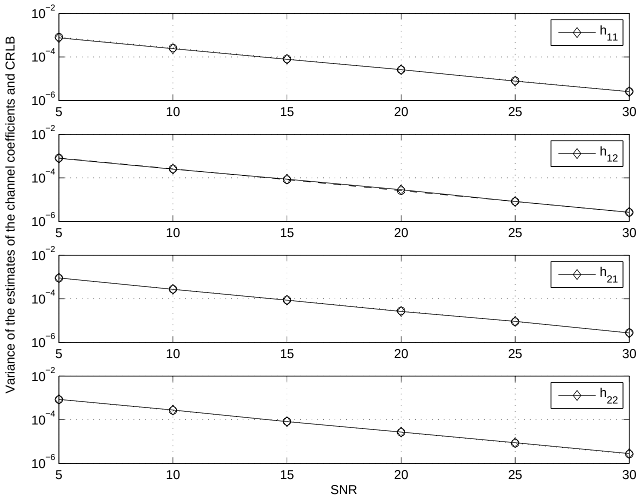

Figure 3: Comparison of the variance of the channel estimates (diamond) with the corresponding CRLB (circle). Here, h_{kl} is the channel coefficients between the first receive antenna and the transmit antenna k.

5. CONCLUSION

We studied estimation of FOs in a MIMO frequency selective fading channel. We show even if the MIMO channel coefficients change according to a fading profile, the estimation of the FOs observed over a number of frames is the average periodogram maximiser, under the assumption of relatively stationary FOs. We also provide an approximation to the maximum likelihood estimator. Since pilot symbols are generally inadequate for FO estimation, we provided an iterative solution so that soft estimate of the transmitted symbols are treated as pilot. We also demonstrated the performance of the iterative scheme is very close to as if all the transmitted symbols are pilot signal.

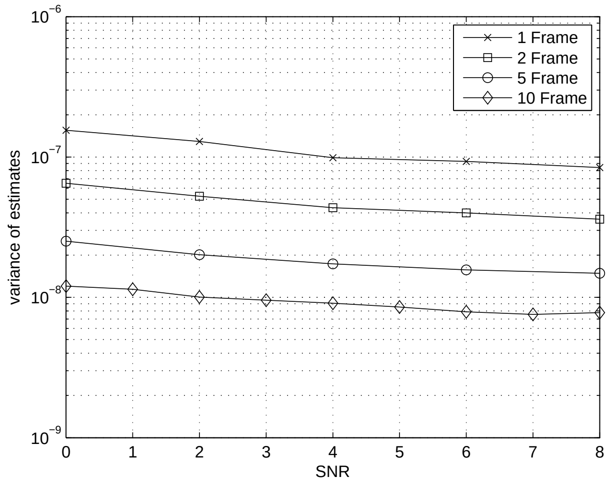

Figure 4: Comparison of the variance of the FO estimates using different number of frames.Only the FO from the 1st transmitter antenna to the 1st receiver antenna is shown

Appendix: DERIVATION OF CRLBs

This section is devoted for the derivation of the CRLB for the problem at hand. Recalling (1), stacking all the received samples from time n to $(n-N+1)$, from all antennas, (1) can be written in vector form as

$$\mathbf{r} = \mathbf{u} + \eta, \tag{18}$$

where $\mathbf{r} = \left[\ \mathbf{r}(n)^T \ \cdots \ \mathbf{r}(n-N+1)^T \ \right]$, and $\mathbf{r}(n) = \left[\ r_1(n) \ \cdots \ r_{n_R}(n)\ \right]^T$ with $\mathbf{u}$ and η formed similarly. Denote the unknown desired vector parameters, $\varphi \triangleq \left[\ \varphi_1^T \ \ \varphi_2^T \ \ \ldots \ \ \varphi_{n_R}^T\ \right]^T$, where $\varphi_k \triangleq \left[\ Re(\mathbf{h}_k)^T \ \ Im(\mathbf{h}_k)^T \ \ \omega_k^T\ \right]^T$. Since the noise sequence $\eta_k(n)$ is spatially uncorrelated and Gaussian, the Fisher information matrix (FIM) for the estimation of φ can be found using Slepian-Bangs formula (see, e.g, [14], [15]).

$$\mathbf{F}(k,l) = \frac{2}{\sigma_\eta^2} Re \sum_{q=0}^{W-1} \sum_{n=0}^{N-1} \left(\frac{\partial \mathbf{u}^{qH}(n)}{\partial \varphi_k^q} \frac{\partial \mathbf{u}^q(n)}{\partial \varphi_l{}^{qT}} \right), \tag{19}$$

where

$$\frac{\partial \mathbf{u}^H}{\partial \varphi_k} = \begin{bmatrix} \frac{\partial \mathbf{u}^H}{\partial Re(h_k)} \\ \frac{\partial \mathbf{u}^H}{\partial Im(h_k)} \\ \frac{\partial \mathbf{u}^H}{\partial \omega_k} \end{bmatrix}$$

$$\frac{\partial \mathbf{u}}{\partial \varphi_l^T} = \begin{bmatrix} \frac{\partial \mathbf{u}}{\partial Re(h_l{}^T)} & \frac{\partial \mathbf{u}}{\partial Im(h_l{}^T)} & \frac{\partial \mathbf{u}}{\partial \omega_l{}^T} \end{bmatrix}$$

which the elements of the $\mathbf{F}(k,l)$ can be found using the following differentials

$$\frac{\partial u_k^q(n)}{\partial Reh_{kl}^{\tilde{q}}(p)} = \begin{cases} e^{j\omega_{kl}n} s_l^q(n-p) & if \quad q=\tilde{q} \\ 0 & if \quad q \neq \tilde{q} \end{cases}$$

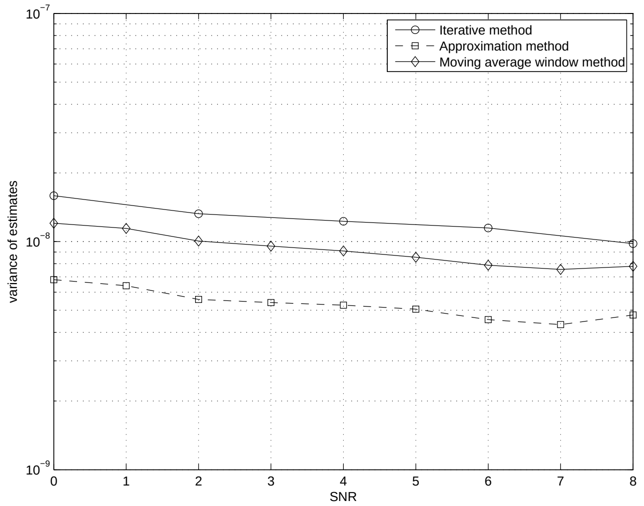

Figure 5: Comparison of the variance of the FO estimates using a)iterative method using 26 training symbols and 116 data symbols, b)exact method but using 142 training symbols, c)approximation method using142 training symbols.Only the FO from the 1st transmitter antenna to the 1st receiver antenna is shown

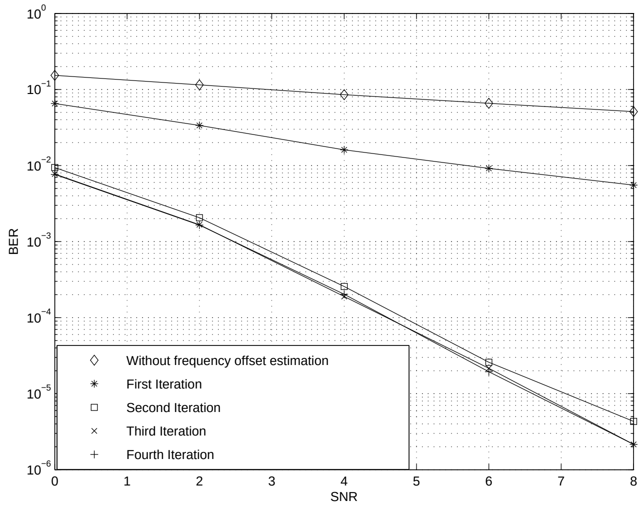

Figure 6: The BER performance of the proposed iterative equalizer and MMSE equalizer which ignores the effect of FOs. FOs have been estimated using 10 frames.

$$\frac{\partial u_k^q(n)}{\partial Imh_{kl}^q(p)} = \begin{cases} je^{j\omega_{kl}n} s_l^q(n-p) & if \quad q = \tilde{q} \\ 0 & if \quad q \neq \tilde{q} \end{cases}$$

$$\frac{\partial u_k^q(n)}{\partial \omega_{kl}} = \sum_{p=0}^{L-1} jnh_{kl}^q(p) e^{j\omega_{kl}n} s_l^q(n-p)$$

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