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JOINT ESTIMATION OF SCAN RATE AND EMITTER LOCATION IN SCAN BASED PASSIVE LOCALIZATION SYSTEMS

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ABSTRACT

Scan based localization is a passive localization method applicable to scanning emitters such as radars with mechanically rotating antennas. The scan based location estimate requires prior knowledge of the emitter scan rate. This paper develops joint estimation techniques for the scan rate and the emitter location using scan time measurements at $N = 4$ separate receivers, doing away with separate estimation of the scan rate. The joint estimators are based on geometric, pseudolinear and maximum likelihood estimation. The latter two are also applicable to the general geolocation problem where $N > 4$. The performance of the proposed estimators is demonstrated by way of computer simulations.

1. INTRODUCTION

Passive emitter localization is an important research problem with civilian and military applications including user location in wireless mobile communication systems, and target location and tracking in electronic warfare systems. Several techniques are available for passive emitter localization utilizing different sensor measurements such as angle of arrival (bearing), Doppler shift, time of arrival, time difference of arrival, and received signal energy. The main limitation of time difference of arrival techniques is the requirement of high-precision time of arrival measurements and highly synchronized clocks at multiple receivers. Furthermore the receivers are required to detect the same signal from significantly different angles, often along sidelobes of a directional signal. The scan-based localization technique dispenses with these limitations and is particularly effective for mechanically scanning radars with narrow azimuth beamwidth [1].

This paper develops improved estimation methods for scan-based emitter localization. The proposed methods are suitable for geolocation of radars performing circular or sector scans [2] and provide a joint estimate of the scan rate and the emitter location in 2D plane. A maximum likelihood estimator and a pseudolinear estimator based grid search method are presented in detail. The latter is particularly useful for initializing an iterative maximum likelihood search algorithm.

The paper is organized as follows. Section 2 provides an overview of the scan based geolocation problem. A geometric solution is presented in Section 3 where the number of receivers is fixed at N=4. In Section 4 the pseudolinear estimation based grid search method is described in detail for large numbers of receivers. Section 5 covers the maximum likelihood estimator. Computer simulations are given in Section 6 and the paper concludes in Section 7.

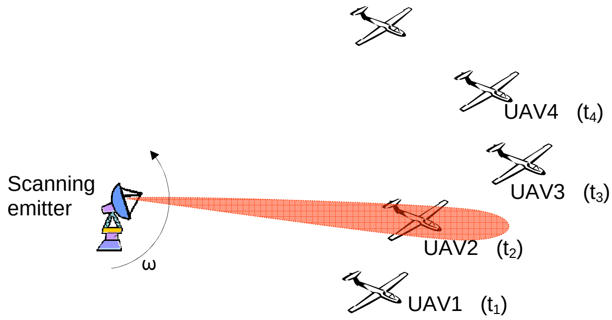

Figure 1: Scan-based geolocation. The times (t_k) correspond to the time instants the emitter beam is intercepted by the UAV receivers.

2. OVERVIEW

Fig. 1 illustrates a typical operational scenario for the scan-based geolocation technique. The figure shows a stationary emitter scanning its main antenna beam at a constant scan rate ω. The main beam sweeps across a number of RF receivers on-board UAVs equipped with GPS receivers. The RF receivers sense the incoming emitter signals (modulated by the antenna beam pattern) and pass the measured data to a processing unit, which estimates the scan intercept times at the beam peaks. The instantaneous UAV positions associated with the beam peaks are also recorded.

The main underlying idea of the scan-based localization technique is to determine the emitter position by exploiting the constraint of antenna uniform rotational motion that the emitter uses to perform its search and track functions. This paper extends the scan-based localization method developed in [1] to include the estimation of the scan rate. This is particularly useful in scenarios where the emitter is only performing a sector scan [2].

3. GEOMETRIC SOLUTION

It is possible to determine jointly the emitter location and scan rate using four receivers by exploiting the trigonometric relationships in the localization geometry. Fig. 2 shows a radar scanning across four receivers. This emitter/receiver geometry gives rise to three adjacent triangles. As shown in

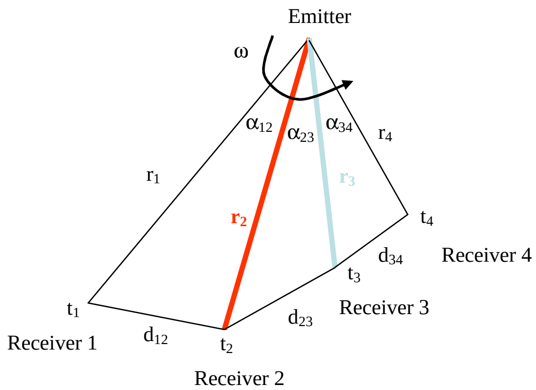

Figure 2: Joint estimation of the emitter position and scan rate using four separate receivers.

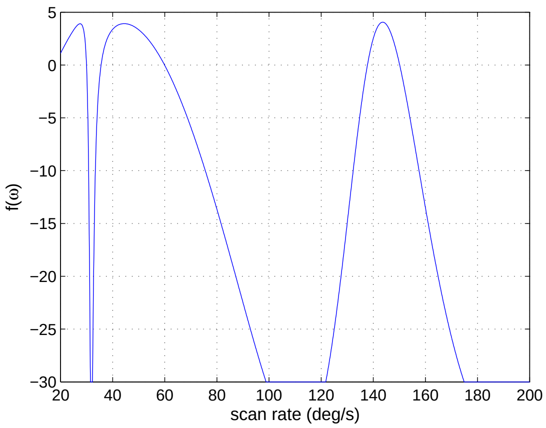

Figure 3: A sample $f(\omega)$.

Appendix A at the end of the paper, r_2 and r_3 are given by

$$r_2(\omega) = \frac{|\sin(\beta_2 + \omega(t_3 - t_1))|}{\sqrt{\mu_{12}^2 + \mu_{23}^2 + 2\mu_{12}\mu_{23}\cos(\beta_2 + \omega(t_3 - t_1))}}$$
$$r_3(\omega) = \frac{|\sin(\beta_3 + \omega(t_4 - t_2))|}{\sqrt{\mu_{23}^2 + \mu_{34}^2 + 2\mu_{23}\mu_{34}\cos(\beta_3 + \omega(t_4 - t_2))}} \tag{1}$$

with $\mu_{ij} = (\sin\omega(t_j - t_i))/d_{ij}$, β_i the baseline deviation angle at the ith receiver, and d_{ij} the distance between receivers i and j.

The only unknown in (1) is the scan rate ω. If we consider the central triangle in Fig. 2, then the scan rate must also satisfy the cosine rule:

$$f(\omega) = r_2^2(\omega) + r_3^2(\omega) - 2r_2(\omega)r_3(\omega)\cos\omega(t_3 - t_2) - d_{23}^2 = 0. \tag{2}$$

Fig. 3 gives an illustration of $f(\omega)$ for a true scan rate 100 deg/s and no time scan measurement errors. Clearly there are a finite number of zero-crossings, which include the zero-crossing associated with the true scan rate. In practice one may apply prior knowledge of the expected scan rate range to further limit its possible values. The scan rate is usually within the range of 12–240 deg/s corresponding to a scan cycle of 1.5–30 s. Scan rates outside this range are very rare [2].

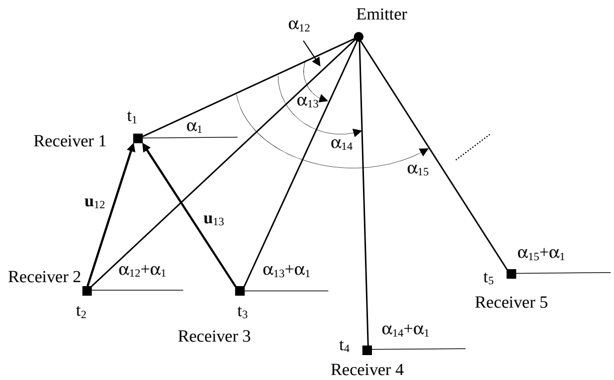

Figure 4: AOA formulation.

For each scan rate solution, the radar position, $[x,y]^T$, in the 2D plane can be determined using

$$(x - x_2)^2 + (y - y_2)^2 = r_2^2 \tag{3a}$$
$$(x - x_3)^2 + (y - y_3)^2 = r_3^2 \tag{3b}$$

where $[x_2, y_2]^T$ and $[x_3, y_3]^T$ are the known locations of receivers 2 and 3, and r_2 and r_3 are calculated using (1).

4. PSEUDOLINEAR ESTIMATION BASED GRID SEARCH

Fig. 4 shows several receivers intercepting the emitter main beam as it scans across them at different times t_k with k ranging from 1 to N. The angles subtended at the emitter can be expressed as $\alpha_{1k} = \omega(t_k - t_1)$, where ω is the emitter scan rate to be estimated. As is clear from the figure, each receiver presents an angle of arrival given by $\alpha_i = \alpha_1 + \alpha_{1i}$ where α_1 is the (unknown) AOA angle associated with the first receiver. If the scan rate is known, then the problem of localizing the emitter is equivalent to the angle of arrival (AOA) problem but with an extra unknown, the offset angle α_1. When the scan rate is not known, a total of two extra unknowns, viz. ω and α_1, need to be estimated in conjunction with the emitter position $[x,y]^T$.

We introduce a cost function inspired by pseudolinear estimation for AOA localization [3]

$$E = \sum_{k=1}^{N} (r_k \sin\varepsilon_k)^2 \tag{4}$$

where $\varepsilon_k = \alpha_k - \beta_k(x,y)$ and $\beta_k(x,y)$ is the true bearing angle at receiver k. For AOA-only problems, this cost function results in a linear estimator (pseudolinear estimator or PLE) [3], [4], [5], [6]. In our scan based problem, however, the two additional unknowns introduce nonlinearities, which makes the minimization problem intractable analytically.

To take advantage of the AOA pseudolinear estimator, we remove the nonlinear effects of the scan rate and angle offset by implementing a 2D grid search. For each hypothesized pair of scan rate and offset angle in the grid, the bearing angle at each receiver is computed using $\alpha_k = \alpha_1 + \alpha_{1k} = \alpha_1 + \omega(t_k - t_1)$ where t_k $(1 < k \le N)$ is the measured scan intercept time. Minimizing E given the bearing values α_k becomes a straightforward least squares problem as shown next.

Expand the right-hand side of (4) as

$$E(\omega,\alpha_1)=\sum_{k=2}^{N}\left(r_{kx}\sin\alpha_k-r_{ky}\cos\alpha_k\right)^2 \tag{5}$$

where r_{kx} and r_{ky} are the coordinates of the vector r_k joining the emitter to the kth receiver. Each vector r_k may alternatively be expressed in terms of r_1 as $r_k=r_1+u_{1k}$. Knowing the positions of the receivers, the vectors u_{1k} in Fig. 4 can be computed. The error E then becomes

$$E(\omega,\alpha_1)=\sum_{k=2}^{N}\left[(r_{1x}+u_{kx})s_k-(r_{1y}+u_{ky})c_k\right]^2 \tag{6a}$$

$$=\sum_{k=2}^{N}(r_{1x}s_k-r_{1y}c_k-z_k)^2 \tag{6b}$$

where $s_k \triangleq \sin(\omega(t_k-t_1)+\alpha_1)$, $c_k \triangleq \cos(\omega(t_k-t_1)+\alpha_1)$ and $z_k=u_{ky}c_k-u_{kx}s_k$. If we let

$$A=\begin{bmatrix}s_2 & -c_2\\ \vdots & \vdots\\ s_N & -c_N\end{bmatrix}\quad r_1=\begin{bmatrix}r_{1x}\\ r_{1y}\end{bmatrix}\quad z=\begin{bmatrix}z_2\\ \vdots\\ z_N\end{bmatrix}$$

then the least square error estimator is obtained as

$$E(\omega,\alpha_1)=\|A\hat{r}_1-z\|^2,\quad \hat{r}_1=\left(A^TA\right)^{-1}A^Tz. \tag{7}$$

In a grid search framework the separation between grid points is governed by $\varepsilon/M\sqrt{2}$ [7] where ε is the required accuracy for determining the global minimum and M is the Lipschitz constant of the cost function E satisfying

$$|E(y_1)-E(y_2)|\le M\|y_1-y_2\| \tag{8}$$

for all y_1 and y_2 within the search region. The Lipschitz constant M can be estimated by fitting largest directional derivatives of E to a reverse Weibull distribution as shown in [8]. The location parameter of the reverse Weibull distribution then gives an estimate of M.

To reduce estimation errors arising from the use of a grid, one can use a single Gauss-Newton (Taylor series) iteration with the grid search estimate as the initial guess. This will lead to a refined estimate. If we write the measurement equation as

$$\tau=g(x)+n$$

where τ is the scan time measurement vector and n is the scan time measurement error vector with covariance Σ, the singe Gauss-Newton iteration is given by

$$\hat{x}=x_0+\left(G^T\Sigma^{-1}G\right)^{-1}G^T\Sigma^{-1}(\tau-g(x_0))$$

where $x_0=[x_0,y_0,\omega_0]^T$ is the computed initial guess and G is the Jacobian of g evaluated at x_0.

5. MAXIMUM LIKELIHOOD ESTIMATOR

The first step in maximum likelihood estimation is to determine the conditional joint probability density function of scan time measurements, or the likelihood function, as a function of the scan rate ω and the emitter location $p=[x,y]^T$. If we let $t_1,\ t_2,\ t_3,\dots,\ t_N$, denote the measured scan intercept times then the scan times (i.e. intercept time differences) can be formed as $t_{1i}=t_i-t_1$. The measured scan time vector is given by

$$\tau=[t_{12},t_{13},t_{14},\dots,t_{1N}]^T$$

and is related to the scan time vector mean

$$t(\omega,p)=\frac{1}{\omega}v(p),\quad v(p)=\begin{bmatrix}\cos^{-1}\frac{(p_1-p)^T(p_2-p)}{\|p_1-p\|\|p_2-p\|}\\ \cos^{-1}\frac{(p_1-p)^T(p_3-p)}{\|p_1-p\|\|p_3-p\|}\\ \vdots\\ \cos^{-1}\frac{(p_1-p)^T(p_N-p)}{\|p_1-p\|\|p_N-p\|}\end{bmatrix} \tag{9}$$

by $\tau=t(\omega,p)+n$ where $n=[n_{12}\ n_{13}\ \dots\ n_{1N}]^T$ is the time scan error vector. The scan time error covariance matrix is defined as

$$\Sigma=E\{n^Tn\}=\sigma_t^2Q \tag{10}$$

where the $(N-1)\times(N-1)$ matrix Q has all its off-diagonal elements set to 1 and its diagonal elements set to 2.

The conditional joint pdf of τ is given by

$$f(\tau|\omega,p)=\frac{1}{(2\pi)^{(N-1)/2}|\Sigma|^{1/2}}\times\exp\left\{-\frac{1}{2}(\tau-t(\omega,p))^T\Sigma^{-1}(\tau-t(\omega,p))\right\}. \tag{11}$$

Maximizing the log-likelihood function over $[\omega,p]$ results in

$$[\hat{\omega}_{\mathrm{ML}},\hat{p}_{\mathrm{ML}}]=\arg\min_{\omega,\ p}J_{\mathrm{ML}}(\omega,p) \tag{12}$$

where $\hat{\omega}_{\mathrm{ML}}$ and $\hat{p}_{\mathrm{ML}}$ are the ML estimates of the scan rate and emitter position, respectively, and $J_{\mathrm{ML}}(\omega,p)$ is the ML cost function:

$$J_{\mathrm{ML}}(\omega,p)=e^T(\omega,p)\Sigma^{-1}e(\omega,p),\quad e(\omega,p)=\tau-t(\omega,p). \tag{13}$$

The ML optimization problem does not have a closed-form solution. Iterative numerical search techniques, such as the Gauss-Newton (GN) algorithm, the Nelder-Mead simplex method [9] and the method of scoring, can be used to compute the ML estimate. All these search algorithms require an appropriate initial solution to avoid divergence. The geometric solution derived in Section 3 can be used to generate such an initial guess.

An alternative to divergence-prone iterative search algorithms is to employ grid search as pursued in the previous section. The grid search for ML estimation proceeds as follows. For each emitter position in a 2D grid we calculate the ML cost function:

$$J_{\mathrm{ML}}=\left(\tau-\frac{1}{\omega}v(p)\right)^T\Sigma^{-1}\left(\tau-\frac{1}{\omega}v(p)\right). \tag{14}$$

To find an ML estimate of ω from τ and given emitter position on the 2D grid, we minimize J_{ML} or equivalently $I_{\mathrm{ML}}=\sigma_t^2J_{\mathrm{ML}}$:

$$\frac{\partial I_{\mathrm{ML}}}{\partial\omega}=\frac{2}{\omega^2}v^T(p)Q^{-1}\left(\tau-\frac{1}{\omega}v(p)\right)\Bigg|_{\omega=\hat{\omega}}=0 \tag{15}$$

which yields

$$\hat{\omega} = \frac{v^T(p)Q^{-1}v(p)}{v^T(p)Q^{-1}\tau}. \tag{16}$$

Substituting $\hat{\omega}$ back into (14) in order to eliminate the scan rate parameter gives

$$I_{\mathrm{ML}}|_{\omega=\hat{\omega}} = \left(\tau - \frac{1}{\hat{\omega}}v(p)\right)^T Q^{-1}\left(\tau - \frac{1}{\hat{\omega}}v(p)\right) \tag{17a}$$

$$= \tau^T Q^{-1}\tau - \frac{(v^T(p)Q^{-1}\tau)^2}{v^T(p)Q^{-1}v(p)} \tag{17b}$$

which is essentially a profile likelihood cost function.

Due to the special form of the symmetric positive definite matrix Q, the inverse of Q can be written as

$$Q^{-1} = \frac{1}{N}\begin{bmatrix} N-1 & -1 & \cdots & -1 \\ -1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & -1 \\ -1 & \cdots & -1 & N-1 \end{bmatrix} = I - \frac{1}{N} \tag{18}$$

where I is the identity matrix and 1 is the matrix whose elements are all 1. After replacing Q^{-1} by $I - 1/N$ in (17b) and carrying out some vector/matrix algebra, we obtain

$$I_{\mathrm{ML}}|_{\omega=\hat{\omega}} = \|\tau\|^2 - \frac{\tau_s^2}{N} - \frac{\left(\tau^T v(p) - \frac{\tau_s v_s}{N}\right)^2}{\|v(p)\|^2 - \frac{v_s^2}{N}} \tag{19}$$

where τ_s and v_s are the sums of the vector elements of τ and v, respectively.

Noting that the first two terms on the righthand-side of (19) are independent of the emitter position, the cost function for grid search can be rewritten as

$$E_{\mathrm{ML}}(p) = -\frac{\left(\tau^T v(p) - \frac{\tau_s v_s}{N}\right)^2}{\|v(p)\|^2 - \frac{v_s^2}{N}} \tag{20}$$

which has a significantly reduced computational complexity of $\mathcal{O}(N)$ compared with $\mathcal{O}(N^2)$ for (17).

6. SIMULATION STUDIES

This section presents Monte Carlo computer simulations carried out for the modified pseudo-linear and maximum likelihood estimators. The simulation scenario consists of 8 receivers and one emitter located at $[5, 20]^T$. The actual emitter/receiver geometry is shown in Fig. 5 where the true scan rate was set to 60 deg/s (6 s per scan cycle).

To speed up processing, the grid search is carried out in two steps. The first search step makes use of a coarse grid and its purpose is to identify a significantly smaller subregion where a finer grid search is initiated to determine the state associated with the global minimum of the cost function. In the actual implementation of the PLE and MLE, the 2D grid is subdivided in an *ad hoc* manner into 16 subregions where coarse search is initiated. A small selection of subregions (two subregions in this work) where the cost function is lowest is recorded. These selected subregions are then subjected to a finer grid search and the emitter state associated with the lowest cost value is noted. Because of errors introduced by the application of a grid, a single Gauss-Newton iteration is applied to the grid computed state and used to estimate the final scan rate and emitter position.

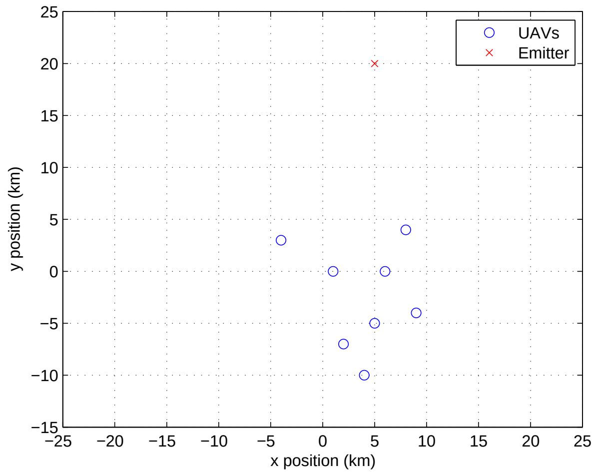

Figure 5: Simulated localization geometry.

A set of 30,000 Monte Carlo runs were used for each estimator to determine the bias and the standard deviation. The bias for all three parameters (i.e. emitter scan rate and emitter location) turned out to be considerably small and hence no bias plots are presented. As can be seen from Fig. 6, the estimated emitter scan rate standard deviation meets the Cramer-Rao lower bound for both PLE and MLE. However, Fig. 6 shows superior emitter positioning performance for the MLE as scan time errors increase due to the inherently better estimation performance of MLE than PLE.

7. CONCLUSION

This paper presented algorithms for joint estimation of scan rate and location of a scanning emitter. A geometric solution exploiting the trigonometric properties of the geolocation problem was developed. Joint estimation algorithms based on the PLE and MLE techniques were proposed. The former utilizes an equivalent AOA formulation of the scan-based localization problem. To alleviate convergence issues with iterative search techniques, grid search algorithms were developed. The estimate obtained from grid search was used to initialize a single-iteration GN algorithm. Computational complexity associated with grid search was significantly reduced by exploiting the dependence between the parameters to be estimated and by employing coarse and fine grid search methods. The MLE was shown to yield a better positioning performance than the PLE for large scan time errors.

Appendix A

The emitter position is either in the receiver sector or the opposite sector as shown in the left and right configurations of Fig. 7. An expression for the distance of the emitter from the central receiver can readily be derived by the application of trigonometric properties, as shown next.

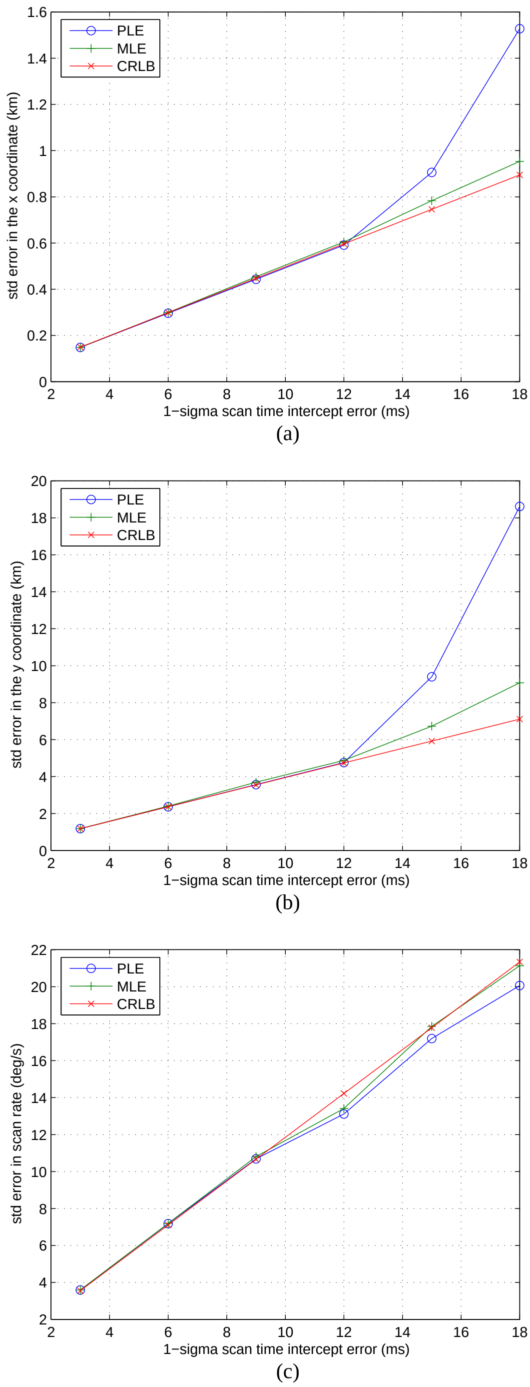

Figure 6: Standard deviation for location estimates (a) and (b), and scan rate (c).

First let $\alpha_{12} = \omega(t_2 - t_1)$, $\alpha_{23} = \omega(t_3 - t_2)$ be the subtended angles at the emitter. From the sine rule we have

$$\mu_{12} = \frac{\sin\alpha_{12}}{d_{12}} = \frac{\sin\theta_{12}}{r_2} \quad \text{and} \quad \mu_{23} = \frac{\sin\alpha_{23}}{d_{23}} = \frac{\sin\theta_{23}}{r_2}.$$

The angles θ_{12} and θ_{23} therefore satisfy $\sin\theta_{12} = \mu_{12}r_2$ and $\sin\theta_{12} = \mu_{12}r_2$ with μ_{12} and μ_{23} known.

The total angular sum, associated with the left and right triangles in Fig. 7, is $\alpha_{12} + \alpha_{23} + \theta_{12} + \theta_{23} + \beta_2 = 2\pi$ where

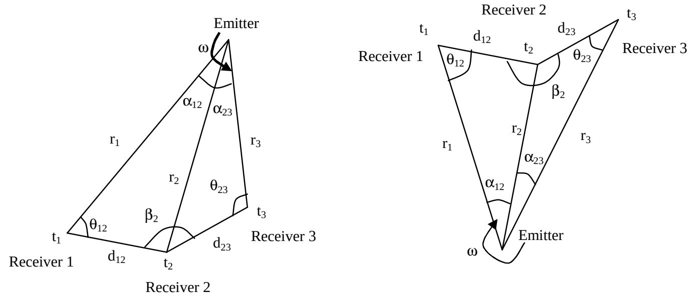

Figure 7: Possible emitter/receiver geometric configurations

$0 < \beta_2 \leq \pi$ in the left configuration of Fig. 7 and $\pi < \beta_2 < 2\pi$ in the right one. Hence $\lambda = \cos(\theta_{12} + \theta_{23}) = \cos(\alpha_{12} + \alpha_{23} + \beta_2)$ is known and we can write $\cos\theta_{12}\cos\theta_{23} = \lambda + \sin\theta_{12}\sin\theta_{23}$. Squaring both sides gives $1 - \lambda^2 = \sin\theta_{12}^2 + \sin\theta_{23}^2 + 2\lambda\sin\theta_{12}\sin\theta_{23}$.

Substituting $\sin\theta_{12}$ and $\sin\theta_{23}$ with $\mu_{12}r_2$ and $\mu_{23}r_2$ leads to

$$r_2 = \sqrt{\frac{1 - \lambda^2}{\mu_{12}^2 + \mu_{23}^2 + 2\mu_{12}\mu_{23}\cos(\beta_2 + \alpha_{12} + \alpha_{23})}}.$$

Defining $\chi_2 = \beta_2 + \alpha_{12} + \alpha_{23}$, we obtain

$$r_2 = \frac{|\sin\chi_2|}{\sqrt{\mu_{12}^2 + \mu_{23}^2 + 2\mu_{12}\mu_{23}\cos\chi_2}}. \tag{21}$$

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