ROBUST SEQUENTIAL INTERFERENCE CANCELLATION FOR SPACE DIVISION MULTIPLE ACCESS COMMUNICATIONS

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ABSTRACT

In this paper[1] *we consider a multiuser detection scheme for space division multiple access communication systems. Sequential interference cancellation (SIC) procedures are subject to performance degradation when the antenna array is only partially calibrated. We propose to incorporate robust beamforming algorithms into the SIC procedure to compensate for the array misalignment. We show by a simulation study that the proposed combination outperforms conventional SIC procedures for various degrees of array misalignment, different SNR values, several array configurations, and two modulation constellations (namely, QPSK and 16-QAM).*

keywords: Robust beamforming, Sequential interference cancellation.

1. INTRODUCTION

Consider a scenario in which several users are communicating with a base-station, equipped with an antenna array. The base-station in this communication scheme, commonly addressed as a *space division multiple access* (SDMA) system, must be capable of receiving the individual sources.

The problem of estimating signals of multiple transmitters using an antenna array is a fundamental problem in array processing. The simplest receivers are the linear receivers where for each signal a different weight vector is used for linearly combining the received signals. The linear receivers suffer from noise amplification causing a severe performance degradation when channel is ill-conditioned.

The second family of receivers is based on decision feedback or, equivalently, sequential cancellation. Among these methods we find the *sequential interference cancellation* (SIC) [1], the *generalized decision feedback equalizer* (GDFE) [2], and the V-BLAST [3]. All these methods assume perfect knowledge of the channel relating the source and received antennas (or, equivalently, the steering vector from the received antenna towards the various transmitting signals). The channel matrix estimate is obtained using a training sequence. However in many cases the initial channel estimate is inaccurate due to lack of sufficient training or due to the presence of strong interference that biases the channel estimate. When the channel estimate is biased we find that sequential cancellation techniques also suffer severe degradation and the nonlinear processing gain diminishes.

A wide-spread solution for designing the array response (i.e. beamforming) is the *minimum variance distortionless response* (MVDR) formulation proposed by Capon [4]. Unfortunately, a perfect estimate of the desired signal direction is assumed to be available in the design process. In the array processing literature there are various robust techniques to overcome steering vectors errors in the beamforming process. One of the first robust methods was proposed by Cox [5]. In this method, the sensitivity of the array to steering errors is decreased by constraining the norm of array weight coefficients. Another commonly used method for improving the robustness of the MVDR is the diagonal loading method [6].

Vorobyov *et al.* [7, 8] propose an approach for increasing the robustness of the Capon beamformer, in presence of arbitrary unknown steering-vector mismatch. Their method is based on the optimization of the worst-case performance using second-order cone (SOC) programming, which can be solved efficiently (in polynomial time). It is also shown that the proposed technique can be interpreted in terms of diagonal loading where the optimal value of the diagonal loading factor is computed based on the known level of uncertainty of the signal steering vector. Rong *et al.* [9] applied this method for improving the performance of *multiple input multiple output* (MIMO) wireless communication systems.

Stoica *et al.* proposed a different extension of the Capon beamformer [10] to the case of uncertain steering vectors. The resulting optimization problem can be solved using the Lagrange multiplier methodology. The Lagrange multiplier is found via Newton-based search. The authors further show that the proposed *robust Capon beamformer* (RCB) is related to the diagonal loading method [11]. The proposed RCB can no longer be expressed in a closed form, but it can be efficiently computed.

Lorenz and Boyd [12] explicitly model the uncertainty in the array manifold via an ellipsoid that gives the possible values of the array for a particular look direction. Their robust weight optimization method can be cast as a second-order cone program that can be solved efficiently using Lagrange multiplier techniques. It is shown that if the ellipsoid reduces to a single point, the method coincides with Capon's method.

Dietrich *et al.* [13] propose a joint channel estimation and decision feedback equalization scheme based on the Bayesian paradigm and show its close relation with robust equalization. Their joint optimization procedure can be viewed as a technique for incorporating the structure of the channel estimation error into the equalizer design.

In our contribution we propose a new sequential interference cancellation (RSIC) method, for which the MMSE estimator is replaced by an improved array weight vector. The array is designed following the procedure given in [11]. Nevertheless, the proposed me[illegible]st

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beamformer design to overcome channel estimation errors. The resulting algorithm combines the benefits of the SIC procedure with the improved robustness of the beamformer.

The structure of this paper is as follows. In Sec. 2 the problem of multiuser detection by an antenna array is formulated. The robust Capon beamformer [11] concepts are reviewed in Sec. 3. In Sec. 4 a method for combining SIC with the RCB is devised. In Sec. 5 we provide a simulation study of the proposed method and demonstrate its superiority over standard MMSE based SIC for QPSK and 16-QAM signalling.

2. PROBLEM FORMULATION

Consider K signals $\mathbf{s}(m) = [s_0(m), s_1(m), \ldots, s_{K-1}(m)]^T$, from directions $\theta_0, \theta_1, \ldots, \theta_{K-1}$, impinging on array of L sensors with array manifold

$$\mathbf{h}^T(\theta) = \begin{bmatrix} 1 & e^{-j\pi\sin(\theta)} & \ldots & e^{-j\pi(L-1)\sin(\theta)} \end{bmatrix}$$

where a far-field regime is assumed. The received signal can be described by

$$\mathbf{x}(m) = H\mathbf{s}(m) + \mathbf{n}(m),\; m = 1, 2, \ldots, M \quad (1)$$

where M is the number of snapshots, $\mathbf{n}(m) = [n_0(m), n_1(m), \ldots, n_{L-1}(m)]^T$ is a Gaussian, spatially and temporally white, receiver noise vector, and H is the array response towards the directions of the signals given by:

$$H = [\mathbf{h}(\theta_0), \mathbf{h}(\theta_1), \ldots, \mathbf{h}(\theta_{K-1})]. \quad (2)$$

We further assume that the signals $s_k(m)$, $k = 0, 1, \ldots, K-1$ are uncorrelated and chosen from a finite alphabet.

Given the observations in Eq. (1), the corresponding correlation matrix is given by,

$$R = E\{\mathbf{x}(m)\mathbf{x}^*(m)\} = H\Sigma H^* + \sigma_N^2 I \quad (3)$$

where $(\bullet)^*$ denotes the Hermitian conjugate, $\Sigma = \mathrm{diag}\{\sigma_0^2, \sigma_1^2, \ldots, \sigma_{K-1}^2\}$ is a diagonal matrix comprised of the various signals power, σ_N^2 is the receiver noise power, and I is the $L \times L$ identity matrix.

Let $\bar{H}$ be an initial estimate of the channel matrix H. Let $\psi = \mathrm{vec}(H - \bar{H})$ be a vector concatenation of the matrix columns. Assume that the covariance of the estimate

$$\mathbf{C}_{\bar{H}} = E[\psi\psi^*] \quad (4)$$

is known in advance. The goal of the multi-channel decoder is to recover the signal vectors $\mathbf{s}(1), \mathbf{s}(2), \ldots, \mathbf{s}(M)$ with the lowest possible bit error rate.

There are several alternatives for detecting the multiple signals. Sequential interference cancellation uses the estimated channel matrix to sequentially estimate the signals using *minimum mean squared error* (MMSE) or *zero forcing* (ZF) beamformer, and then decode the estimated signals. The decoded signals are then remodulated and subtracted from the received data. The process continues until all signals are estimated. The main drawback of this method is its dependance on the channel estimate accuracy. When the estimate is imperfect the beamformer based on $\bar{H}$ can cause partial cancellation of the desired signal and hence a performance degradation.

3. ROBUST CAPON BEAMFORMING

In this section we summarize the RCB recently proposed in [10, 11]. In the *standard Capon Beamformer* (SCB) we seek for the best beamformer $\mathbf{w}_k^{\mathrm{SCB}}$, $k = 0, 1, \ldots, K-1$ in the following sense:

$$\min_{\mathbf{w}_k^{\mathrm{SCB}}} \mathbf{w}^* R\mathbf{w} \text{ subject to } \mathbf{w}^*\mathbf{h}(\theta_k) = 1. \quad (5)$$

The solution to this criterion is given by

$$\mathbf{w}_k^{\mathrm{SCB}} = \frac{R^{-1}\mathbf{h}(\theta_k)}{\mathbf{h}^*(\theta_k)R^{-1}\mathbf{h}(\theta_k)}. \quad (6)$$

However, the exact steering vector $\mathbf{h}(\theta_k)$ is usually unknown and only a rough estimate thereof, $\bar{\mathbf{h}}(\bar{\theta}_k)$, exists, where $\bar{\theta}_k$ is an initial estimate of θ_k. Moreover, it is assumed that the true steering vector is within an uncertainty ellipsoid around $\bar{\mathbf{h}}(\bar{\theta}_k)$. In our contribution we use the covariance matrix in Eq. (4) as the indicated ellipsoid. For the simplicity of the exposition we assume that $\mathbf{C}_{\bar{H}} = \varepsilon I$, where I is the $(LK) \times (LK)$ identity matrix. Hence, using an alternative interpretation of the Capon beamformer [14], we can formulate the minimizer in the following manner:

$$\min_{\mathbf{h}(\theta_k)} \mathbf{h}^*(\theta_k)R^{-1}\mathbf{h}(\theta_k) \text{ subject to } \|\mathbf{h}(\theta_k) - \bar{\mathbf{h}}(\bar{\theta}_k)\|^2 = \varepsilon. \quad (7)$$

The steering vector estimate is then given by,

$$\hat{\mathbf{h}}(\hat{\theta}_k) = \bar{\mathbf{h}}(\bar{\theta}_k) - (I + \lambda R)^{-1}\bar{\mathbf{h}}(\bar{\theta}_k) \quad (8)$$

where λ is the Lagrange multiplier obtained by solving

$$g(\lambda) \triangleq \|(I + \lambda R)^{-1}\bar{\mathbf{h}}(\bar{\theta}_k)\|^2 = \varepsilon. \quad (9)$$

Eq. (9) is easily solved by applying the following stages:

1. Obtain the eigenvalue decomposition of the covariance matrix, $R = \mathbf{U}\mathbf{\Gamma}\mathbf{U}^*$, where $\gamma_0 \geq \gamma_1 \geq \ldots \geq \gamma_{L-1}$ are the elements of the diagonal matrix $\mathbf{\Gamma}$.
2. Use this diagonalization to obtain $\mathbf{z} = \mathbf{U}^*\bar{\mathbf{h}}(\bar{\theta}_k)$.
3. Defining $\mathbf{z}^T = \begin{bmatrix} z_0 & z_1 & \ldots & z_{L-1} \end{bmatrix}$, Eq. (9) can be rewritten as:

$$g(\lambda) = \sum_{l=0}^{L-1} \frac{|z_l|^2}{(1 + \lambda\gamma_l)^2} = \varepsilon. \quad (10)$$

Eq. (10) can be solved by common search methods[2].

Using the estimated steering vector, $\hat{\mathbf{h}}(\hat{\theta}_k)$, an estimate of the desired signal power is obtained by

$$\hat{\sigma}_k^2 = \frac{1}{\hat{\mathbf{h}}^*(\hat{\theta}_k)R^{-1}\hat{\mathbf{h}}(\hat{\theta}_k)}. \quad (11)$$

Note, that an inherent gain ambiguity problem exists. If the channel matrix is comprised of steering vectors, this ambiguity can be easily resolved by using the following normalization:

$$\hat{\hat{\sigma}}_k^2 = \hat{\sigma}_k^2 \frac{\|\hat{\mathbf{h}}(\hat{\theta}_k)\|^2}{M}. \quad (12)$$

[2] We used Matlab© *fsolve* rou

The robust Capon beamformer can now be calculated by using the estimated steering vector

$$\mathbf{w}_k^{\mathrm{RCB}} = \frac{R^{-1}\hat{\mathbf{h}}(\hat{\theta}_k)}{\hat{\mathbf{h}}^*(\hat{\theta}_k)R^{-1}\hat{\mathbf{h}}(\hat{\theta}_k)}. \quad (13)$$

In Fig. 1 the directivity pattern of the resulting robust Capon beamformer (RCB) is depicted together with directivity pattern of the standard Capon beamformer (SCB). In

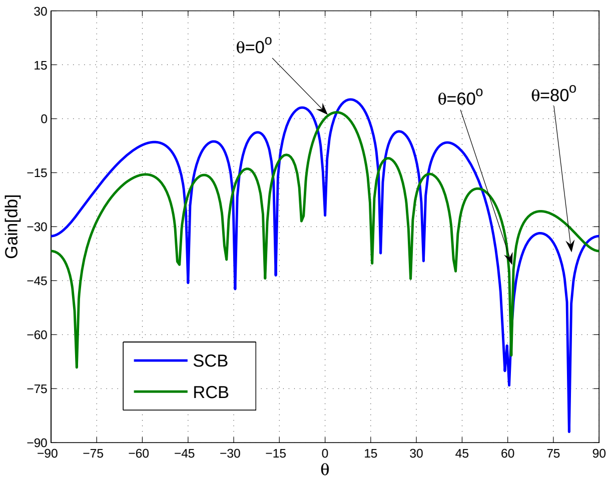

Figure 1: Spatial response of the standard (SCB) and robust (RCB) Capon beamformers. No. of sensors 10. No. of signals 3. Signals' angles of arrival $\theta_k = \{0^o, 60^o, 80^o\}$. Signals' power $\sigma_k^2 = \{10, 100, 100\}$. Noise power $\sigma_N^2 = 1$. Array look direction $\theta_0 = 0^o$

the figure, the performance for $K = 3$ sources, and $L = 10$ sensors is demonstrated. The sources impinge on the array from the directions $\theta = \{0^o, 60^o, 80^o\}$, and their power is $\sigma_k^2 = \{10, 100, 100\}$, respectively. Noise power is $\sigma_N^2 = 1$. A steering error of $\Delta\theta = 3^o$ is assumed for all directions. The array is designed to look at $\theta = 0^o$ direction. It can be verified from the figure that the SCB does not maintain the desired direction, while the RCB does. Both methods suppress the interferences' directions, although the SCB suppression level is somewhat better.

As a final remark, we emphasize that the criterion proposed in Eq. (7) is not sensitive to a phase-only multiplicative gain, i.e. $e^{j\phi}\mathbf{w}_k^{\mathrm{RCB}}$ is also a valid solution for the criterion for any ϕ. This phase ambiguity can cause a severe degradation in the obtained performance. We will elaborate on this issue in the Section 5.

4. COMBINING ROBUST CAPON WITH SEQUENTIAL INTERFERENCE CANCELLATION

In this section we show how the robust Capon beamformer can be combined with sequential interference cancellation to obtain robust multiuser detection. The following steps constitute our proposed method:

1. Assuming that the channels are under-spread, i.e. the *direction of arrival* (DoA) is time-invariant during the snapshot, the data covariance matrix $R = E\{\mathbf{x}(m)\mathbf{x}^*(m)\}$ is estimated using,

$$\hat{R} = \frac{1}{M}\sum_{m=1}^{M}\mathbf{x}(m)\mathbf{x}^*(m). \quad (14)$$

2. Let $\bar{H}$ be the available estimate of the channel and $\mathbf{C}_{\bar{H}}$ be the covariance of the estimate. The *Cramér-Rao lower bound* (CRLB) of the DoA estimation might be utilized for determining $\mathbf{C}_{\bar{H}}$.
3. Denote by i_k, $k = 0, 1, \ldots, K-1$ the source with best SINR given an array weight vector $\mathbf{w}_{i_k}$, where the SINR is determined by

$$\mathrm{SINR}(\mathbf{w}_{i_k}) = 10\log_{10}\frac{\left|\mathbf{w}_{i_k}^*\hat{\mathbf{h}}(\hat{\theta}_{i_k})\right|^2}{\mathbf{w}_{i_k}^* R_n^{i_k}\mathbf{w}_{i_k}}.$$

$R_n^{i_k}$ is the interference covariance matrix when all source signals (except for signal i_k) and the noise signals are considered as interference.
Calculate the optimal weight vector $\mathbf{w}_{i_k}^{\mathrm{RCB}}$ using

$$\mathbf{w}_{i_k}^{\mathrm{RCB}} = \frac{\hat{R}^{-1}\hat{\mathbf{h}}(\hat{\theta}_{i_k})}{\hat{\mathbf{h}}(\hat{\theta}_{i_k})^*\hat{R}^{-1}\hat{\mathbf{h}}(\hat{\theta}_{i_k})}. \quad (15)$$

where the array manifold vector $\hat{\mathbf{h}}(\hat{\theta}_{i_k})$ is estimated using the RCB algorithm.
4. For the entire snapshot $m = 1, 2, \ldots, M$ the source i_k with best SINR is estimated by

$$\hat{s}_{i_k}(m) = F\left((\mathbf{w}_{i_k}^{\mathrm{RCB}})^*\mathbf{x}^{(k)}(m)\right) \quad (16)$$

where F is a decision function mapping the soft estimates into symbols using ML decoding.
5. For the entire snapshot $m = 1, 2, \ldots, M$ remodulate $\hat{s}_{i_k}(m)$ and subtract it from the data

$$\mathbf{x}^{(k+1)}(m) = \mathbf{x}^{(k)}(m) - \hat{\mathbf{h}}^*(\hat{\theta}_{i_k})\hat{s}_{i_k}(m) \quad (17)$$
$$k = 0, 1, \ldots, K-2$$

where $\mathbf{x}^{(k=0)}(m) = \mathbf{x}(m)$.
6. Go back to step 3 until all sources have been extracted.

5. SIMULATION RESULTS

In this section a multi-user communication system is tested and its performance improvement is demonstrated. In the first set of experiments we use (uncoded) QPSK modulation. The series length is set to 2^{12}. Three methods were compared: the SIC method with known steering vector using MMSE beamforming, the SIC method with the erroneous steering vector using MMSE beamforming, and the proposed robust-SIC method (denoted as R-SIC in the sequel).

As mentioned in Section 2 the phase ambiguity is inherent to the RCB method. We chose to resolve this ambiguity by using a short training sequence, although other common methods can be applied as well.

We first test BER curves as a function of SNR (the interference level is fixed) for *two* users impinging on the array from $\theta = \{0^o, 40^o\}$ with po[illegible]

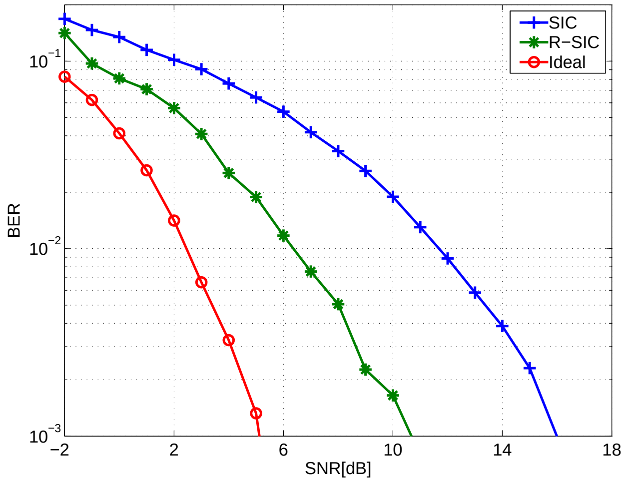

Figure 2: BER vs. SNR curves for SIC with known steering, SIC with erroneous steering and R-SIC. No. of sensors 6. No. of signals 2. Signals' directions $\theta_k = \{0^o, 40^o\}$. Signals' power $\sigma_k^2 = \{0,0\}$dB. Steering error $\Delta\theta = 4^o$.

steering error is $\Delta\theta = 4^o$. The number of sensors in this test was set to 6. The results are depicted in Fig. 2. A 4dB gain in BER=0.05 is encountered.

We proceed by increasing the number of sources to 3 with the directions set to $\theta = \{0^o, 60^o, 80^o\}$ and power levels to $\sigma^2 = \{0,0,0\}$dB. The steering error is set $\Delta\theta = 2.5^o$ and the No. of sensors is raised to 10. The results depicted in Fig. 3 demonstrates the advantage of the proposed method over the SIC method even for the more complicated task.

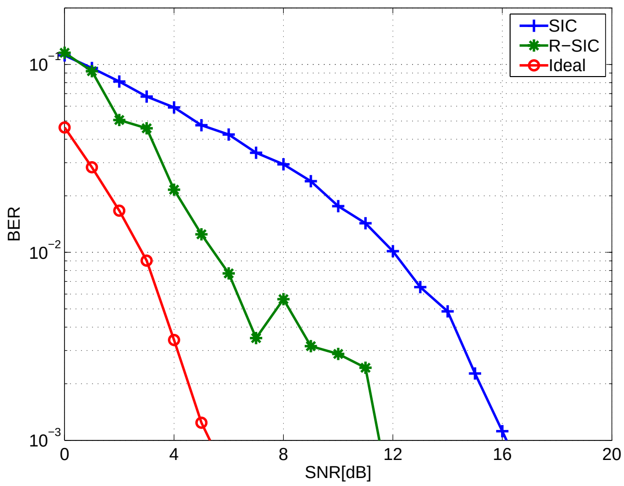

Figure 3: BER vs. SNR curves for SIC with known steering, SIC with erroneous steering and R-SIC. No. of sensors 10. No. of signals 3. Signals' directions $\theta_k = \{0^o, 60^o, 80^o\}$. Signals' power $\sigma_k^2 = \{0,0,0\}$dB. Steering error $\Delta\theta = 2.5^o$.

In Fig. 4 the two sources case is examined again. Now, the steering error is set to $\Delta\theta = 3^o$. The obtained performance for several values of sensors number is depicted in the graph. We compare the SIC method (with steering error) with the proposed method. It can be shown that the R-SIC

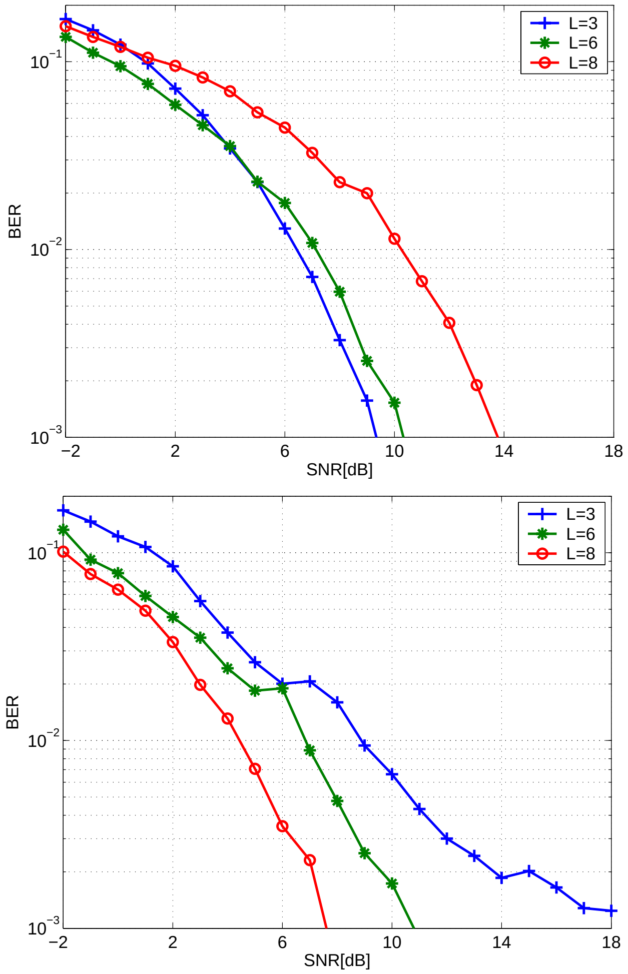

Figure 4: BER vs. SNR curves for SIC with erroneous steering (top) and R-SIC (bottom) for various number of sensors. No. of signals 2. Signals' directions $\theta_k = \{0^o, 40^o\}$. Signals' power $\sigma_k^2 = \{0,0\}$dB. Steering error $\Delta\theta = 3^o$.

performance exhibit a consistent behavior with the number of sensors, while the SIC does not. The reason for that is the ability of the RCB, embedded in the R-SIC method, to steer itself towards the desired signals. As the SIC method is using a wrong steering vector, the obtained performance highly depends on the directivity pattern of the array.

It is also interesting to test the sensitivity of the methods to steering errors. It is easy to verify from Fig. 5 that while the SIC method is sensitive to steering errors, the R-SIC method is indeed much more robust.

Finally, we demonstrate in Fig. 6 that the obtained enhancement of the R-SIC method is even more evident in higher constellations such as 16-QAM, where the conventional method rendered useless in steering error as low as $\Delta\theta = 2.5^o$.

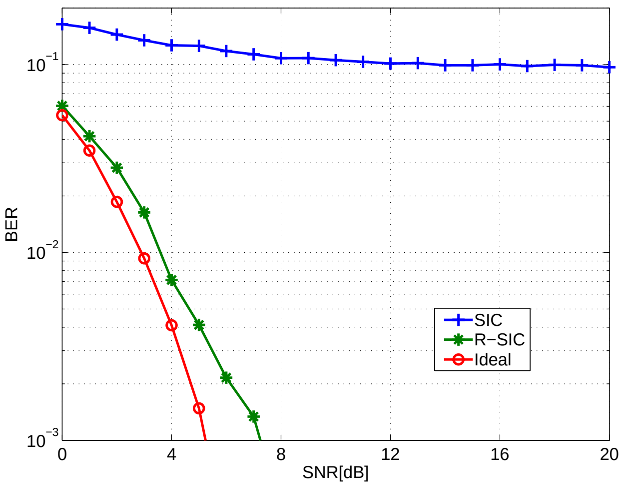

Figure 6: BER vs. SNR curves for SIC with known steering, SIC with erroneous steering and R-SIC. Constellation 16-QAM. No. of sensors 6. No. of signals 2. Signals' directions $\theta_k = \{0^o, 40^o\}$. Signals' power $\sigma_k^2 = \{0, 0\}$dB. Steering error $\Delta\theta = 2.5^o$.

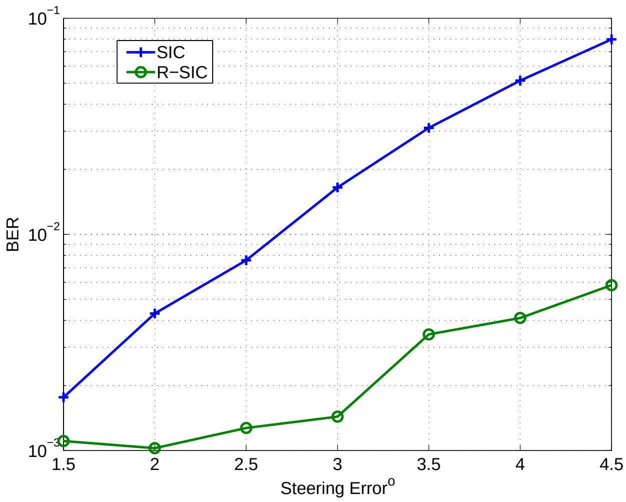

Figure 5: BER vs. steering error curves for SIC with erroneous steering and R-SIC. No. of sensors 6. No. of signals 2. Signals' directions $\theta_k = \{0^o, 40^o\}$. Signals' power $\sigma_k^2 = \{0, 0\}$dB. SNR 6dB.

6. CONCLUSIONS

We considered the incorporation of the recently proposed robust Capon beamformer into a multiuser detection scheme in partially calibrated antenna array scenarios. We show by simulation study that the proposed combination outperforms standard sequential interference cancellation methods. This conclusion is applicable for various test scenarios, including several steering errors, several number of sources and sensors, and two constellations (QPSK and 16-QAM).

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