

## CONSTRAINED SPACE-TIME SPREADING FOR MIMO-CDMA SYSTEMS: TENSOR MODELING AND BLIND DETECTION

André L. F. de Almeida <sup>†</sup>, Gérard Favier <sup>†</sup> and João C. M. Mota <sup>‡</sup>

<sup>†</sup>I3S Laboratory, University of Nice-Sophia Antipolis (UNSA)-CNRS.  
2000, route des Lucioles, B.P. 121, 06903, Sophia Antipolis, France.

<sup>‡</sup>Wireless Telecom Research Group, Federal University of Ceará.  
CP 6005, Campus do Pici, 60455-760, Fortaleza, CE - Brazil.  
Phone/Fax: (+33) 492 942 736/ 492 942 896.  
E-mails: {lima,favier}@i3s.unice.fr, mota@gtel.ufc.br

### ABSTRACT

*In this paper, we present a new space-time transmission framework for Multiple Input Multiple Output (MIMO) Code Division Multiple Access (CDMA) systems. A Constrained Space-Time Spreading (CSTS) model is proposed by using a tensor modeling formalism. The key feature of the CSTS model is the presence of two constraint matrices controlling the spatial spreading of the data streams and the reuse factor of the spreading codes across subsets of transmit antennas. The proposed CSTS model allows one to derive several multiple-antenna transmit schemes with different space-time spreading patterns by simply adjusting the structure of these two constraint matrices. Finite sets of CSTS schemes for 2, 3 and 4 transmit antennas are derived, which ensures the blind recovery of the transmitted data streams. Exploiting the constrained tensor model of the received signal, a joint blind detection using the alternating least squares algorithm is used for performance evaluation of several CSTS schemes.*

### 1. INTRODUCTION

The growing research interest in Multiple Input Multiple Output (MIMO) Code Division Multiple Access (CDMA) systems comes together with the expectation that mobile users will be equipped with two or more antennas in the near future. A generalization of classical spatial multiplexing schemes for CDMA systems was proposed in [1]. Transmit diversity schemes for MIMO-CDMA have been proposed in [2, 3]. These methods, commonly called *space-time spreading*, are capable of providing maximum transmit diversity gain without using extra spreading codes and without an increased transmit power. However, space-time spreading methods put more emphasis on diversity than on multiple-access interference.

In a seminal paper [4], the problem of multiuser detection in the context of CDMA systems is linked to Parallel Factor (PARAFAC) modeling. Following this work, some model generalizations were proposed [5, 6, 7, 8, 9] under different assumptions concerning multipath propagation structure (e.g. including frequency-selectivity and/or specular multipath). All these works are limited to single-antenna transmissions.

Recently, tensor decompositions have also been considered for modeling multiple-antenna transmissions [10], [11], [12]. A multi-antenna scheme exploiting the Khatri-Rao product structure of the received signal is proposed in [10]. Despite its diversity-rate flexibility and built-in blind identifiability, this multiple-antenna scheme relies on temporal-only spreading of the data streams (as in a conventional CDMA system). Since there is no spatial spreading of the data streams across the transmit antennas, no transmit spatial diversity is obtained. In [11], a generalized tensor model is proposed for multiple-antenna CDMA systems with blind detection. However, this modeling approach only considers spatial multiplexing, where the number of data streams is restricted to be equal to the number of transmit antennas. The approach of [12]

adds some flexibility at the transmitter by allowing the number of data streams to be different from the number of transmit antennas. Contrarily to [10] and [11], full spatial spreading of each data stream across the transmit antennas is also permitted in [12].

In this work, we present a constrained space-time spreading model for MIMO systems which is based on a tensor modeling approach with constraint matrices. The proposed model allows one to derive several multiple-antenna transmit schemes with different space-time spreading patterns by simply adjusting the structure of two constraint matrices of the tensor signal model. The first constraint matrix controls the coupling of data streams and transmit antennas while the second one controls the coupling of spreading codes and transmit antennas.

As opposed to [12], where each data stream is spread over all the available transmit antennas using necessarily different spreading codes, the proposed model allows the reuse of the same spreading code by each data stream, with the possibility to go from full code reuse to full code multiplexing. Some examples of constrained space-time spreading schemes are presented for illustrating our modeling approach. Multiuser detection based on an alternating least squares algorithm is considered for recovering the transmitted data streams in a complete blind setting, without using training sequences and not even spreading code knowledge. Simulation results are provided for performance evaluation of several constrained space-time spreading schemes using this blind detection.

This paper is organized in the following manner. Section 2 describes the basic system model and assumptions. In Section 3, we present the constrained space-time spreading model using a tensor formalism. Feasible CSTS structures ensuring the blind recovery of the transmitted data streams are also presented in this section. The tensor received signal model for blind detection is described in Section 4. The blind receiver is presented in Section 5 along with computer simulation results for performance evaluation of some CSTS schemes. The paper is concluded in Section 6.

### 2. GENERAL SYSTEM MODEL AND ASSUMPTIONS

We consider a MIMO system with  $M$  transmit and  $K$  receive antennas.  $R$  denotes the total number of independent transmitted data streams<sup>1</sup> and  $J$  denotes the number of spreading codes available at the transmitter. The block diagram of the considered MIMO-CDMA system is shown in Fig. 1. Each transmitted data stream consists of  $N$  symbols. The wireless channel is assumed to be flat-fading and constant during  $N$  symbol periods. We assume that  $R, J$ , and  $M$  satisfy the following inequality:

$$1 \leq R \leq J \leq M.$$

<sup>1</sup>It is worth mentioning at this point that the  $R$  data streams can belong to different users. We do not distinguish between both interpretations here.

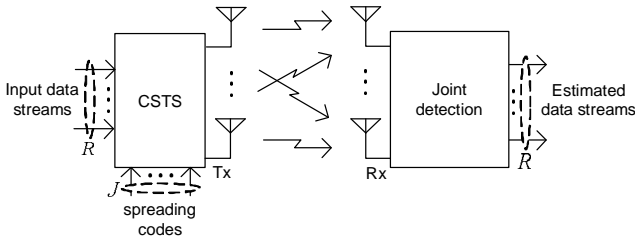


Figure 1: MIMO-CDMA system model

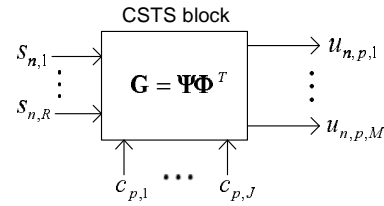


Figure 2: CSTS block-diagram

The multiple-antenna transmission is structured in the following manner. The  $M$  transmit antennas are partitioned into  $R$  smaller antenna subsets of  $M_r$  antennas each so that  $M = M_1 + \dots + M_R$ . At the  $r$ -th antenna subset, space-time spreading is performed to provide transmit diversity to the  $r$ -th data stream using  $J_r$  different spreading codes where  $J = J_1 + \dots + J_R$  and

$$1 \leq J_r \leq M_r, \quad r = 1, \dots, R.$$

We assume that the  $R$  antenna subsets are formed by adjacent antennas. Different antenna subsets transmit different data streams using different spreading codes, i.e., there is no sharing of data streams and spreading codes between any two different antenna subsets. It is to be noted that full transmit diversity and full spatial multiplexing are special cases of this transmission model. We can distinguish the following types of transmit schemes that are covered by our constrained space-time spreading model:

- $R = 1, J = M$ : Full transmit diversity with full code multiplexing;
- $R = M, J = M$ : Full spatial multiplexing (e.g. [1])
- $R = 1, J = 1$ : Full transmit diversity with full code reuse (e.g. [2]);

### 3. CONSTRAINED SPACE-TIME SPREADING

In this section, Constrained Space-Time Spreading (CSTS) is formulated using a tensor modeling formalism. At the transmitter, the serial input data is parallel-to-serial converted into  $R$  data streams of  $N$  symbols each, where

$$s_{n,r} \doteq s((r-1)N + n)$$

denotes the  $n$ -th symbol of the  $r$ -th data stream. Let  $c_{p,j}$  be the  $p$ -th element of the  $j$ -th spreading code and  $h_{k,m}$  be the spatial channel gain between the  $m$ -th transmit antenna and the  $k$ -th receive antenna. Let us define the following matrices

$$\mathbf{S} \in \mathbb{C}^{N \times R}, \quad \mathbf{C} \in \mathbb{C}^{P \times J}, \quad \mathbf{H} \in \mathbb{C}^{K \times M}$$

as the *symbol*, *code* and *channel* matrices, where

$$h_{k,m} \doteq [\mathbf{H}]_{k,m}, \quad s_{n,r} \doteq [\mathbf{S}]_{n,r}, \quad c_{p,j} \doteq [\mathbf{C}]_{p,j}$$

are, respectively, the typical elements of these matrices.

At the output of the CSTS block, the discrete-time signal sample associated with the  $n$ -th transmitted symbol,  $p$ -th chip and  $m$ -th transmit antenna is represented by:

$$u_{n,p,m} \doteq u_m((n-1)P + p).$$

In this work,  $u_{n,p,m}$  is interpreted as the  $(n, p, m)$ -th element of the third-order tensor  $\mathcal{U} \in \mathbb{C}^{N \times P \times M}$  representing the effective transmitted signal. We propose the following constrained factorization for modeling the CSTS operation:

$$u_{n,p,m} = \sum_{r=1}^R \sum_{j=1}^J g_m(r,j) s_{n,r} c_{p,j}, \quad g_m(r,j) \doteq \Psi_{r,m} \Phi_{j,m}, \quad (1)$$

where  $g_m(r,j)$  is the  $(r,j)$ -th element of  $\mathbf{G}_m \in \mathbb{C}^{R \times J}$ . This matrix defines the coupling between  $R$  data streams and  $J$  spreading codes at the  $m$ -th transmit antenna. Let us define

$$\mathbf{G} = \sum_{m=1}^M \mathbf{G}_m = \Psi \Phi^T \in \mathbb{C}^{R \times J}$$

as a matrix synthesizing the overall CSTS structure.  $\mathbf{G}$  is called here the *coupling matrix*, and is given by the inner product of two *constraint matrices*  $\Psi \in \mathbb{C}^{R \times M}$  and  $\Phi \in \mathbb{C}^{J \times M}$ . Both matrices are composed of canonical vectors controlling the coupling of  $R$  data streams and  $J$  spreading codes at the  $M$  transmit antennas, respectively.  $\Psi$  can be viewed as a *stream-to-antenna selection matrix* and  $\Phi$  as a *code-to-antenna selection matrix*. At this point, we define the basic structure of these constraint matrices:

- The columns of  $\Psi$  and  $\Phi$  are canonical vectors<sup>2</sup> belonging to the canonical bases  $E^{(R)} = \{\mathbf{e}_1^{(R)}, \dots, \mathbf{e}_R^{(R)}\}$ , and  $E^{(J)} = \{\mathbf{e}_1^{(J)}, \dots, \mathbf{e}_J^{(J)}\}$ , respectively;
- $\Psi$  and  $\Phi$  are both full rank matrices,  $\Psi \Psi^T$  and  $\Phi \Phi^T$  being diagonal matrices with equal trace  $M$ .

#### 3.1 Generic CSTS structure

Our final goal is the blind recovery of the transmitted data streams (regardless of the knowledge of the spreading codes). From an identifiability point of view, the coupling involving the columns of  $\mathbf{S}$  and  $\mathbf{C}$  determined by  $\mathbf{G}$ , may induce rotational freedom in subsets of columns of these matrices. A possible choice for  $\mathbf{G}$  that ensures the uniqueness of  $\mathbf{S}$  is given by a ‘‘row-wise’’ block-diagonal matrix, each row containing a non-zero row vector [13]:

$$\mathbf{G} = \begin{bmatrix} \gamma_{1,1} & \dots & \gamma_{1,J_1} & 0 & \dots & 0 & \dots & 0 & \dots & \dots & 0 \\ 0 & \dots & 0 & \gamma_{2,1} & \dots & \gamma_{2,J_2} & 0 & \dots & \dots & \dots & \vdots \\ \vdots & & \vdots & 0 & \dots & 0 & \dots & \dots & \dots & \dots & \vdots \\ \vdots & & \vdots & \vdots & \vdots & \vdots & \dots & \dots & 0 & \dots & 0 \\ 0 & \dots & 0 & 0 & \dots & 0 & \dots & 0 & \gamma_{R,1} & \dots & \gamma_{R,J_R} \end{bmatrix} \quad (2)$$

$R \times (J_1 + J_2 + \dots + J_R)$

where  $\gamma_{r,j_r}$  is the reuse factor of the  $j_r$  spreading code, i.e., the number of times the  $j_r$  spreading code is used in the transmission of the  $r$ -th data stream.  $\gamma_{r,j_r}$  satisfies the following constraint:

$$\sum_{r=1}^R \sum_{j_r=1}^{J_r} \gamma_{r,j_r} = M. \quad (3)$$

Several CSTS schemes can be obtained from this generic CSTS structure for different choices of the  $\gamma_{r,j_r}$ 's.

<sup>2</sup>A canonical vector  $\mathbf{e}_n^{(N)} \in \mathbb{R}^N$  is a unitary vector containing an element equal to 1 in its  $n$ -th position and 0's elsewhere.

### 3.2 Examples of CSTS schemes

For a fixed number  $M$  of transmit antennas, a finite set, or codebook, of CSTS schemes exists, each element of this set being given by a different combination of the two constraint matrices  $\Psi$  and  $\Phi$  satisfying the generic CSTS structure (2). Let  $S_M(\Psi, \Phi)$  denote the feasible set of CSTS schemes for a fixed  $M$ , each element of this set being given by the matrix pair  $(\Psi, \Phi)$ . A criterion for deriving a feasible set  $S_M(\Psi, \Phi)$  of CSTS schemes ensuring the blind recovery of the data streams is proposed in [13] using concepts of partition theory. In this work, we restrict ourselves to the presentation of feasible CSTS schemes, where the constraint matrices  $\Psi$  and  $\Phi$  are derived from a common set of *generating arrays*, as will be clear from the following examples.

•  $M = 2$ : The 2 generating arrays are:

$$\mathbf{u}_2 = [1 \ 1], \quad \mathbf{U}_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The set  $S_2(\Psi, \Phi)$  of feasible CSTS schemes for  $M = 2$  can be derived from these generating arrays by:

$$S_2(\Psi, \Phi) = \left\{ \underbrace{(\mathbf{u}_2, \mathbf{u}_2)}_{J=1}, \underbrace{(\mathbf{u}_2, \mathbf{U}_{11})}_{J=2}, \underbrace{(\mathbf{U}_{11}, \mathbf{U}_{11})}_{R=J=2} \right\}$$

Note that 3 CSTS schemes are possible. The first one, where  $\Psi = \Phi = \mathbf{u}_2$ , indicates that a single data stream is transmitted over both transmit antennas using the same spreading code ( $R = J = 1$ ). This is a full transmit diversity scheme with full code reuse. In the second scheme we have  $\Psi = \mathbf{u}_2$  and  $\Phi = \mathbf{U}_{11}$ , meaning that the single data stream is now associated with 2 different spreading codes at the first and second transmit antennas. In this case, transmit diversity with full code multiplexing takes place. Finally, the third scheme  $\Psi = \Phi = \mathbf{U}_{11}$  indicates that two different data streams are transmitted from both antennas using different spreading codes. This full spatial multiplexing scheme with full code multiplexing (i.e. different codes at different antennas), as in a standard spatial multiplexing CDMA system [1].

•  $M = 3$ : In this case, we have 3 generating arrays:

$$\mathbf{u}_3 = [1 \ 1 \ 1], \quad \mathbf{U}_{21} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{U}_{111} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The feasible set  $S_3(\Psi, \Phi)$  is given by:

$$S_3(\Psi, \Phi) = \left\{ \underbrace{(\mathbf{u}_3, \mathbf{u}_3)}_{J=1}, \underbrace{(\mathbf{u}_3, \mathbf{U}_{21})}_{J=2}, \underbrace{(\mathbf{u}_3, \mathbf{U}_{111})}_{J=3}, \right. \\ \left. \underbrace{(\mathbf{U}_{21}, \mathbf{U}_{21})}_{J=2}, \underbrace{(\mathbf{U}_{21}, \mathbf{U}_{111})}_{J=3}, \underbrace{(\mathbf{U}_{111}, \mathbf{U}_{111})}_{R=J=3} \right\},$$

yielding a total of 6 possible CSTS schemes. The first scheme  $\Psi = \Phi = \mathbf{u}_3$  is a 3-antenna transmit diversity with full code reuse. The last one  $\Psi = \Phi = \mathbf{U}_{111}$  is a spatial multiplexing system using different spreading codes at each transmit antenna. In between both extremes, we have 4 intermediary schemes which are different combinations of transmit diversity and spatial multiplexing with different code reuse patterns. For instance, the second and third schemes transmit a single data stream using 2 or 3 spreading codes, respectively. Similarly, the fourth and fifth schemes transmit 2 data streams using 2 and 3 spreading codes, respectively.

•  $M = 4$ : Now, we have a total of 5 generating arrays:

$$\mathbf{u}_4 = [1 \ 1 \ 1 \ 1], \quad \mathbf{U}_{31} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{U}_{22} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \quad \mathbf{U}_{211} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{U}_{1111} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and the feasible set  $S_4(\Psi, \Phi)$  is composed of 12 different CSTS schemes satisfying (2):

$$S_4(\Psi, \Phi) = \left\{ \underbrace{(\mathbf{u}_4, \mathbf{u}_4)}_{J=1}, \underbrace{(\mathbf{u}_4, \mathbf{U}_{31})}_{J=2}, \underbrace{(\mathbf{u}_4, \mathbf{U}_{22})}_{J=2}, \underbrace{(\mathbf{u}_4, \mathbf{U}_{211})}_{J=3}, \underbrace{(\mathbf{u}_4, \mathbf{U}_{1111})}_{J=4}, \right. \\ \left. \underbrace{(\mathbf{U}_{31}, \mathbf{U}_{31})}_{J=2}, \underbrace{(\mathbf{U}_{22}, \mathbf{U}_{22})}_{J=2}, \underbrace{(\mathbf{U}_{31}, \mathbf{U}_{211})}_{J=3}, \underbrace{(\mathbf{U}_{22}, \mathbf{U}_{211})}_{J=3}, \right. \\ \left. \underbrace{(\mathbf{U}_{211}, \mathbf{U}_{211})}_{J=3}, \underbrace{(\mathbf{U}_{211}, \mathbf{U}_{1111})}_{J=4}, \underbrace{(\mathbf{U}_{1111}, \mathbf{U}_{1111})}_{R=J=4} \right\}.$$

We remark that  $(\mathbf{U}_{31}, \mathbf{U}_{31})$  and  $(\mathbf{U}_{22}, \mathbf{U}_{22})$  are two schemes having the same number of transmitted data streams and spreading codes but differing in the way the transmit antennas and spreading codes are shared between the data streams. Let us take now the two schemes  $(\mathbf{U}_{31}, \mathbf{U}_{211})$  and  $(\mathbf{U}_{22}, \mathbf{U}_{211})$ . Both transmit 2 data streams over 4 transmit antennas using 3 spreading codes.

Let us also look at scheme  $(\mathbf{U}_{31}, \mathbf{U}_{211})$ . It transmits the first data stream over 3 transmit antennas using 2 different spreading codes (one is reused at 2 transmit antennas) and the second data stream is transmitted by a single antenna using a single spreading code. On the other hand, the scheme  $(\mathbf{U}_{22}, \mathbf{U}_{211})$  transmits both data streams by assigning 2 transmit antennas to each one. The first data stream is associated with a single spreading code while the second one uses 2 different spreading codes.

All the component CSTS schemes belonging to the sets  $S_2(\Psi, \Phi)$ ,  $S_3(\Psi, \Phi)$  and  $S_4(\Psi, \Phi)$  satisfy the generic structure (2) which is required for the blind recovery of the transmitted data streams. It is worth noting that *not all* the pairwise combinations of generating arrays is a feasible CSTS scheme. For instance, the scheme  $(\Psi, \Phi) = (\mathbf{U}_{22}, \mathbf{U}_{31})$  is not feasible for blind symbol recovery. In this scheme, the *same* spreading code is reused by the two *different* data streams, inducing a coupling between different data streams. It can be shown [13] that this coupling leaves rotational freedom within the antenna subset associated with the second data stream, thus affecting uniqueness of  $\mathbf{S}$  (only the first data stream can be uniquely recovered). Therefore, care must be taken when deriving the CSTS schemes from the generating arrays.

## 4. RECEIVED SIGNAL MODEL

At the receiver, the discrete-time chip-rate sampled baseband version of the received signal at the  $n$ -th symbol period,  $p$ -th chip, and  $k$ -th receive antenna (in absence of noise) is defined as:

$$x_{n,p,k} \doteq x_k((n-1)P + p),$$

where  $x_{n,p,k}$  is the  $(n, p, k)$ -th element of a received signal tensor  $\mathcal{X} \in \mathbb{C}^{N \times P \times K}$ . The MIMO signal model is then given in tensor notation by:

$$x_{n,p,k} = \sum_{m=1}^M u_{n,p,m} h_{k,m}. \quad (4)$$

The structure of the CSTS model defined in (1) yields the following noise-free received signal model:

$$x_{n,p,k} = \sum_{m=1}^M \sum_{r=1}^R \sum_{j=1}^J g_m(r,j) s_{n,r} c_{p,j} h_{k,m}. \quad (5)$$

It is worth noting that (5) is known in the specialized literature as a Tucker3 model [14],  $g_m(r,j)$  being the  $(r,j,m)$ -th element of the core tensor  $\mathcal{G}$  of dimension  $R \times J \times M$ . To be specific, (5) is a constrained Tucker2 model because  $\mathcal{G}$  has only 1's and 0's elements, since we have defined  $g_m(r,j) = \psi_{r,m} \phi_{j,m}$ . Instead of using the Tucker2 notation, we will adopt here a "constrained PARAFAC" notation for representing (5) in matrix form, since it allows us to explicit the constraint matrices when working with the model.

The received signal tensor can be organized as a set of matrices  $\{\mathbf{X}_{..k}\} \in \mathbb{C}^{N \times P}$ ,  $k = 1, \dots, K$ , each one containing  $N$  symbols  $\times P$  chips of the received signal associated with the  $k$ -th receive antenna. It can be shown that  $\mathbf{X}_{..k}$  admits the following "constrained PARAFAC" factorization [9]:

$$\mathbf{X}_{..k} = \mathbf{S} \Psi D_k(\mathbf{H}) \Phi^T \mathbf{C}^T, \quad k = 1, \dots, K, \quad (6)$$

where  $D_k(\mathbf{H})$  is a diagonal matrix holding the  $k$ -th row of  $\mathbf{H}$  on the main diagonal. We can also define two other matrix sets  $\mathbf{X}_{n..} \in \mathbb{C}^{P \times K}$  collecting the received signal samples over  $P$  chips and  $K$  receive antennas associated with the  $n$ -th transmitted symbol; and  $\mathbf{X}_{.p.} \in \mathbb{C}^{K \times N}$  collecting the received signal samples over  $N$  symbol periods and  $K$  receive antennas associated with the  $p$ -th chip of the spreading code. These matrices can be respectively factored as

$$\mathbf{X}_{n..} = \mathbf{C} \Phi D_n(\mathbf{S} \Psi) \mathbf{H}^T, \quad \mathbf{X}_{.p.} = \mathbf{H} D_p(\mathbf{C} \Phi) \Psi^T \mathbf{S}^T, \quad (7)$$

$n = 1, \dots, N$ ,  $p = 1, \dots, P$ . The received signal models (6) and (7) are three different (but equivalent) writings of the received signal tensor  $\mathcal{X} \in \mathbb{C}^{N \times P \times K}$ .

Let us define the three matrices  $\mathbf{X}_1 = [\mathbf{X}_{..1}^T \dots \mathbf{X}_{..K}^T]^T \in \mathbb{C}^{KN \times P}$ ,  $\mathbf{X}_2 = [\mathbf{X}_{.1.}^T \dots \mathbf{X}_{.P.}^T]^T \in \mathbb{C}^{PK \times N}$ , and  $\mathbf{X}_3 = [\mathbf{X}_{1..}^T \dots \mathbf{X}_{N..}^T]^T \in \mathbb{C}^{NP \times K}$  concatenating the third-mode, second-mode and first-mode slices of the received signal tensor, respectively, so that  $[\mathbf{X}_1]_{(k-1)N+n,p} = [\mathbf{X}_2]_{(p-1)K+k,n} = [\mathbf{X}_3]_{(n-1)P+p,k} = x_{n,p,k}$ . It can be shown [9] that these matrices admit the following constrained factorization:

$$\begin{aligned} \mathbf{X}_1 &= (\mathbf{H} \diamond \mathbf{S} \Psi) (\mathbf{C} \Phi)^T \\ \mathbf{X}_2 &= (\mathbf{C} \Phi \diamond \mathbf{H}) (\mathbf{S} \Psi)^T \\ \mathbf{X}_3 &= (\mathbf{S} \Psi \diamond \mathbf{C} \Phi) \mathbf{H}^T, \end{aligned} \quad (8)$$

where  $\diamond$  is the Khatri-Rao (column-wise Kronecker) product.

## 5. SIMULATION RESULTS

The Bit-Error-Rate (BER) performance of some CSTS schemes for MIMO-CDMA systems is evaluated by means of computer simulations. After detailing the simulation assumptions, the blind receiver is described and the simulation results are shown.

### 5.1 Simulation assumptions

The average BER versus Signal-to-Noise Ratio (SNR) curves are obtained from 5000 Monte Carlo runs. Unless otherwise stated, the BER results represent the average performance of the  $R$  data streams. At each run, the additive noise power is generated according to the sample SNR value given by  $\text{SNR} = 10 \log_{10} (\|\mathbf{X}_1\|_F^2 / \|\mathbf{V}_1\|_F^2)$ . The spatial channel gains are redrawn from an i.i.d. complex-valued Gaussian generator. The transmitted symbols are redrawn from a pseudo-random Quaternary Phase Shift Keying (QPSK) sequence. When considering orthogonal spreading codes at the receiver, Hadamard( $P$ ) codes are used. For simulating the presence of inter-chip interference due to multipath propagation, we consider pseudo-random codes as the "effective" codes, which are redrawn from an i.i.d. complex-valued Gaussian generator at each run. We assume that "guard chips" are used to avoid inter-symbol interference, following the approach of [4]. In this case,  $P$  denotes the number of ISI-free chips/symbol.

### 5.2 Blind receiver

As the blind receiver, we make use of the alternating least squares (ALS) algorithm [4, 14]. Given the received signal tensor  $\mathcal{X} \in \mathbb{C}^{N \times P \times K}$ , this algorithm consists in alternated least squares updates of  $\hat{\mathbf{S}}$ ,  $\hat{\mathbf{C}}$  and  $\hat{\mathbf{H}}$  based on the constrained model (8).  $\hat{\mathbf{S}}_{(0)}$  and  $\hat{\mathbf{H}}_{(0)}$  are randomly initialized before starting the algorithm. At the  $t$ -th iteration, the three least square updates are:

$$\begin{aligned} \hat{\mathbf{C}}_{(t)}^T &= \left[ (\hat{\mathbf{H}}_{(t-1)} \diamond \hat{\mathbf{S}}_{(t-1)} \Psi) \Phi^T \right]^\dagger \tilde{\mathbf{X}}_1 \\ \hat{\mathbf{S}}_{(t)}^T &= \left[ (\hat{\mathbf{C}}_{(t)} \Phi \diamond \hat{\mathbf{H}}_{(t-1)}) \Psi^T \right]^\dagger \tilde{\mathbf{X}}_2, \\ \hat{\mathbf{H}}_{(t)}^T &= \left[ (\hat{\mathbf{S}}_{(t)} \Psi \diamond \hat{\mathbf{C}}_{(t)} \Phi) \right]^\dagger \tilde{\mathbf{X}}_3, \end{aligned} \quad (9)$$

where  $\tilde{\mathbf{X}}_1 = \mathbf{X}_1 + \mathbf{V}_1$  is the noisy version of  $\mathbf{X}_1$ ,  $\mathbf{V} \in \mathbb{C}^{KN \times P}$  being an additive white gaussian noise matrix. The convergence of the algorithm at the  $i$ -th iteration is declared when the error between the true received signal tensor and its reconstructed version from the estimated matrices does not change significantly between iterations  $t$  and  $t+1$ . In this work we assume that  $\mathbf{C}$  is known at the receiver so that the first estimation step in (9) is skipped. The estimation of  $\mathbf{S}$  is affected by an inherent scaling factor, i.e.,  $\hat{\mathbf{S}} = \mathbf{S} \cdot \text{Diag}(\delta_1 \dots \delta_R)$ , where  $\delta_1, \dots, \delta_R$  are the scaling factors. These scaling factors are eliminated by assuming that the first transmitted symbol of each data stream is equal to one [4].

### 5.3 BER performance

We first compare the performance of different CSTS schemes. We consider the schemes  $(\mathbf{u}_2, \mathbf{U}_{11})$  and  $(\mathbf{U}_{11}, \mathbf{U}_{11})$  for  $M = 2$ , and the schemes  $(\mathbf{U}_{21}, \mathbf{U}_{21})$  and  $(\mathbf{U}_{111}, \mathbf{U}_{111})$  for  $M = 3$ . Note that  $(\mathbf{U}_{11}, \mathbf{U}_{11})$  and  $(\mathbf{U}_{111}, \mathbf{U}_{111})$  are full spatial multiplexing schemes,  $(\mathbf{u}_2, \mathbf{U}_{11})$  is a full transmit diversity scheme, while  $(\mathbf{U}_{21}, \mathbf{U}_{21})$  is a combined transmit diversity and spatial multiplexing scheme. The receiver works with  $K = 2$  antennas and a data block of  $N = 10$  symbols. Figure 3 shows the performance of these schemes with blind detection. Considering  $M = 2$ , we can see that  $(\mathbf{u}_2, \mathbf{U}_{11})$  outperforms  $(\mathbf{U}_{11}, \mathbf{U}_{11})$  as expected, but this comes with a reduction in spectral efficiency by a factor of two. Also for  $M = 3$ ,  $(\mathbf{U}_{21}, \mathbf{U}_{21})$  offers a better performance than  $(\mathbf{U}_{11}, \mathbf{U}_{11})$ .

Figure 4 shows the performance of two CSTS schemes  $(\mathbf{U}_{21}, \mathbf{U}_{21})$  and  $(\mathbf{U}_{21}, \mathbf{U}_{111})$  valid for  $M = 3$ . We consider both orthogonal and random spreading codes. Note that both schemes have the same spatial spreading pattern with  $R = 2$ . The difference is on the code reuse pattern ( $J = 2$  or 3). Here, we assume  $K = 2$ ,  $N = 10$  and  $P = J$ . First we note that a performance degradation occurs when the spreading codes are not orthogonal (e.g. due to multipath propagation). This causes a loss in the diversity gain, as shown in Fig. 4. Note also that the scheme with  $\Phi = \mathbf{U}_{111}$  ( $J = 3$ ) offers a better performance than the scheme with  $\Phi = \mathbf{U}_{21}$  ( $J = 2$ ). Such an improvement is due to the use of more spreading codes.

Now, we consider a CSTS scheme for  $M = 5$  transmit antennas using  $(\Psi, \Phi) = (\mathbf{U}_{221}, \mathbf{U}_{1121})$ . In this scheme,  $R = 3$  data streams are transmitted using  $J = 4$  spreading codes. The first data stream is transmitted over  $M_1 = 2$  antennas using  $L_1 = 2$  spreading codes. The second data stream is also transmitted over  $M_2 = 2$  antennas, but using only a single spreading code. The third data stream is transmitted by a single antenna using a single spreading code. We focus on the individual performance of each data stream in order to investigate the influence of the number of used transmit antennas and codes on their performance. We consider  $K = 3$ ,  $N = 10$  and  $P = 4$ . Figure 5 shows the performance of each data stream. A performance gain of the third data stream over the second one is obtained considering both orthogonal and random spreading codes. Note that the third data stream uses different spreading codes while the second data stream reuses the same spreading code. This result confirms that using different codes for transmit diversity generally provides a higher diversity gain than reusing the same code [1].

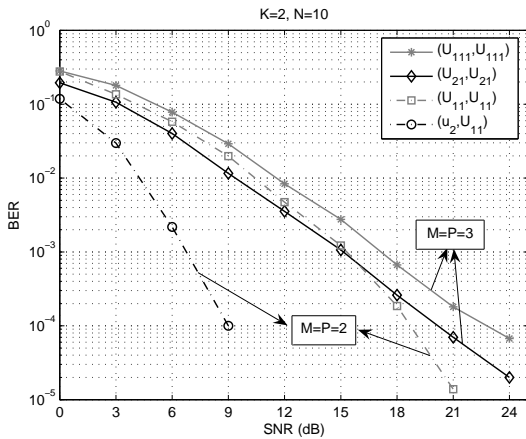


Figure 3: Performance of some CSTS schemes for different choices of the constraint matrices  $\Psi$  and  $\Phi$ .

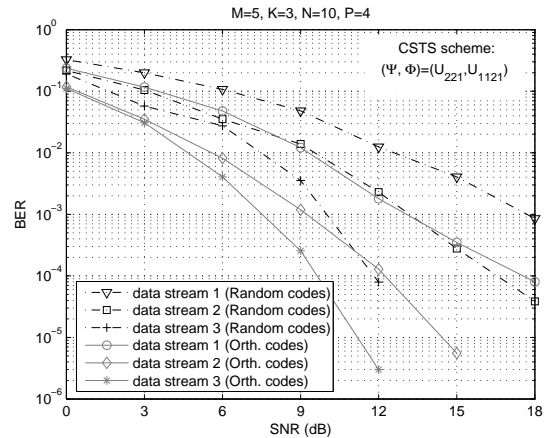


Figure 5: Individual data-stream performance for  $\Psi = U_{221}$  and  $\Phi = U_{1121}$ , using both orthogonal and random spreading codes.

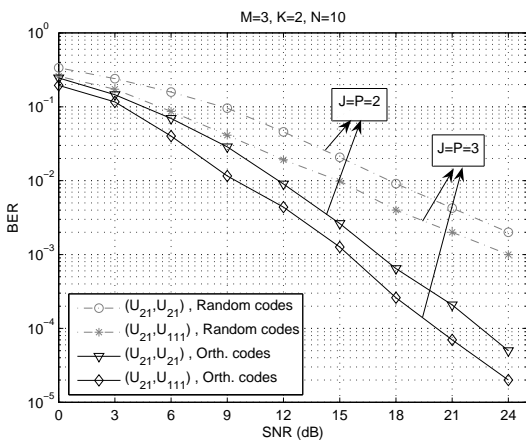


Figure 4: Performance of two CSTS schemes using orthogonal and random spreading codes.

## 6. CONCLUSION

In this paper, we have presented a constrained space-time spreading (CSTS) framework for MIMO-CDMA systems relying on a tensor modeling of the space-time spreading process. We have shown that the two constraint matrices  $\Psi$  and  $\Phi$  characterizing the CSTS model can be viewed as stream-to-antenna and code-to-antenna selection matrices, respectively. The proposed CSTS model covers several classes of multiple-antenna CDMA schemes, spatial multiplexing with full code multiplexing to transmit diversity with full code reuse. By focusing on the blind joint detection of the data streams, we have presented sets of CSTS schemes for  $M = 2, 3$  and 4 transmit antennas for different choices of the constraint matrices. Bit-error-rate performance of some CSTS schemes has been evaluated using a blind alternating least squares receiver.

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