16th European Signal Processing Conference (EUSIPCO 2008), Lausanne, Switzerland, August 25-29, 2008, copyright by EURASIP

Robust Adaptive CRLS-GSC Algorithm for DOA Mismatch in Microphone Array

P. Mowlaee Begzade Mahale
Department of Electrical Engineering
Amirkabir University of Technology
Tehran, Iran 15875-4413
P_Mowlaee@ieee.org , kahaei@ee.iust.ac.ir

***Abstract*—In this paper, two new noise reduction algorithms are presented which are robust to the mismatch problem commonly exist between the ideal array look direction and actual speaker's *Direction-Of-Arrival* (DOA). Deriving a set of leakage constraints and incorporating them into the state-of-the-art *Generalized Sidelobe Canceller* (GSC) algorithm leads to two new noise reduction algorithms, namely *Constrained Least Mean-Squares Generalized Side-lobe Canceller* (CLMS-GSC) and *Constrained Recursive Least-Squares Generalized Sidelobe Canceller* (CRLS-GSC). Conducting several simulations, it is demonstrated that the proposed algorithms will flatten the array mainlobe and as a result, can significantly improve noise cancellation of the beamformer in the presence of DOA mismatch. Simulation results indicate that the proposed algorithms successfully attenuate the closely-placed interferers as well as they are robust to small changes in desired source DOA.**

***Index Terms*—Generalized side lobe canceller, DOA mismatch, leakage constraints, robust beamforming.**

I. INTRODUCTION

Adaptive beamformers are able to recover the desired signal while automatically placing deep-pattern nulls along the direction of the interferences, even if its amplitude and DOA are unknown [3],[4],[10]. In the speaker direction finding, it is assumed that the desired speaker location, microphone characteristics and their positions are known a priori, and also reverberation is ignored [2]. In reality, these conditions are rarely fulfilled completely and some distortion is introduced in the desired speech signal. This problem also known as DOA mismatch may lead to significant performance degradation due to the speaker possible mobility at the vicinity of look direction. Among the most popular interference and noise reduction algorithms, the *Generalized Sidelobe Canceller* (GSC) and *Multi-channel Wiener Filtering* (MWF) are more notable [1,2]. Although the latter algorithm seems promising from robustness point of view, it has not yet been considered for implementation due to its higher computational cost. The conventional GSC algorithm, however, is sensitive to the DOA mismatch.

To reduce sensitivity of a given array with respect to the mentioned DOA mismatch, the array mainlobe may be flattened [6]. Two different schemes are reported for narrowband signals in which the derivatives of the beamformer output power with respect to the actual angle of the desired source is set to zero in order to achieve a maximally flat main-lobe for a wider range of DOAs as reported by Tseng in [7]. Another method according to [8] considered two adaptive modules to reduce the speech leakage at the cost of more complication. Such methods are however hard to implement due to the large number of the required constraints to be fulfilled at the same time. In another approach, some phase-independent constraints were used to arrive at a flattened main-lobe beam to alleviate the DOA mismatch for norrowband case [7]. It was observed that the latter algorithm was also computationally burden and also needed much more constraints to generate a maximally flat beam pattern.

In this paper, sensitivity of the GSC algorithm with respect to the speaker DOA mismatch is reduced by flattening the mainlobe. This is performed by deriving and incorporating the 1st and 2nd-order derivatives of the steering vectors into the GSC algorithm. Accordingly, using these constraints, two new adaptive *Constrained Least Mean-Squares Generalized Side-lobe Canceller* (CLMS-GSC) and *Constrained Recursive Least-Squares Generalized Sidelobe Canceller* (CRLS-GSC) are developed. It is demonstrated that the proposed algorithms outperform the GSC in terms of less sensitivity to the DOA mismatch which as a result introduces less distortion in the output speech signal. This as a result leads to a lower speech leakage in look-direction while attenuating closely-placed interferers. The robustness of the proposed algorithm with respect to the DOA mismatch is also comparable to that of the MWF algorithm but with much simpler implementation similar to GSC. Finally, as a conventional approach for beamforming, we will consider the *Linearly Constrained Minimum Variance* (LCMV) reported in [5],[9] as an optimal benchmark for evaluation purposes.

However, note that acoustic echoes can still deteriorate the noise reduction performance of the proposed algorithm in this work. As a result, some works include promoting GSC structure for reverberated environment like the AEC-GSC structure have previously been proposed in [12],[13]. In [14] we have also recently remove the GSC deficiencies in reverberant environment by jointly employing some adaptive *Acoustic Echo Canceller* (AEC), namely *Segment Variable Step-size Proportionate LMS* (SVS-PNLMS) as a pre-processor in front of GSC. However, in this paper we mainly stress on DOA mismatch mentioned earlier. The proposed method can also improve noise reduction performance as demonstrated in [15].

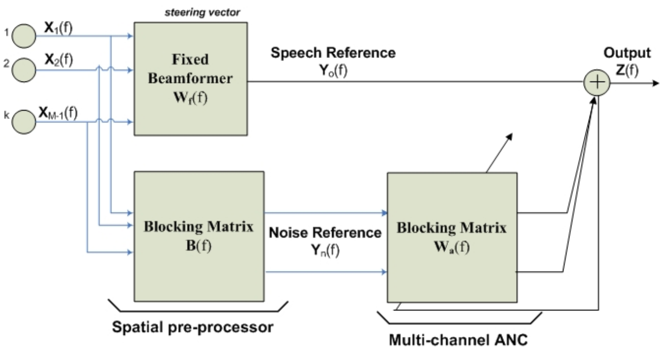

Fig. 1. Block diagram of the GSC algorithm.

In this paper, by incorporating the 1st and 2nd order derivatives of the steering vectors and extending them to the GSC algorithm, the CLMS-GSC and CRLS-GSC algorithms are proposed. In the proposed algorithms, a fewer number of constraints are needed to achieve a maximally flat beampattern response. They are also able to attenuate closely located interferers in addition to extending the acceptable range for look direction DOA mismatch difficulty. It is shown that the proposed algorithms are less sensitive to the speaker location.

The paper is organized as follows. In Section 2, we introduce the signal model used in the rest of the paper. In Section 3 the proposed CLMS-GSC and CRLS-GSC algorithms are introduced. Simulation results are presented in Sections 4. Section 5 concludes.

II. Signal Model

The noise source in general consists of internal and external noise while the former originates from the array sensors and the hardware, the latter is as a result of the background noise and localized interferers including computer fans, cars, and other undesired speakers [1,2]. For the desired speech spectrum $\mathbf{X}^s(f)$ (related to the desired source), the received signal to M microphones is denoted by $\mathbf{X}(f) = [X_1(f), \ldots, X_M(f)]^T$ and defined in matrix form as:

$$\mathbf{X}(f) = \hat{\mathbf{X}}(f) + \mathbf{X}^n(f) \tag{1}$$

$$\hat{\mathbf{X}}(f) = \mathbf{H}(f)\mathbf{X}^s(f) \tag{2}$$

where $\mathbf{X}^n(f)$ shows the additive noise spectrum and $\hat{\mathbf{X}}(f)$ is the received signal at microphone array filtered by an *acoustic transfer function* (ATF) as follows:

$$\mathbf{H}(f) = [\begin{array}{cccc} H_1(f) & H_2(f) \ldots & H_M(f) \end{array}] \tag{3}$$

in which $H_l(f)$ with l=1,...,M models the acoustic transfer function between the desired speech source $\mathbf{X}_s(f)$ and the kth microphone in array.

III. Proposed CRLS-GSC Algorithm

Fig.1 illustrates the block diagram of the GSC algorithm in which $\mathbf{W}_f(f)$ is a fixed beamformer steered to the desired target DOA, **B**(f) is the blocking matrix to extract the interferences, and $\mathbf{W}_a(f)$ denotes the weight vector of a transversal filter. In this algorithm, $\mathbf{W}_f(f)$ adjusts the main lobe towards the desired signal while **B**(f) generates the noise references for the adaptive canceller $\mathbf{W}_a(f)$ by blocking the input signal **X**(f). In the GSC algorithm, **B**(f) is designed based on 0 radians towards the look-direction. Based on Fig. 1, we may write

$$Z(\mathrm{f}) = (\mathbf{W}_f^H(f) - \mathbf{W}_a^H(f)\mathbf{B}^H(f))\mathbf{X}(f) \tag{4}$$

In the GSC algorithm shown in Fig. 1, the array look-direction angle in **B**(f) is ideally defined to be 0 radians. In practice, however, speaker movement with respect to the look direction angle can lead to some DOA mismatch. Since the conventional GSC algorithm is not robust against this type of DOA mismatch [2], it yields, as a result, a leakage of the desired speech signal into the blocking matrix output (noise references denoted by $Y_n(f)$ in Fig.1) which degrades the system performance. To reduce such sensitivity to array look direction, one may re-design the fixed beamformer mainlobe by incorporating a new structure for the array sensors while the new beamformer should still have the same performance of interference and noise cancellation. To guarantee this, a careful design should take the new changes into account. however, This is time consuming, costly, and also should be performed continuously to follow any new variation. The noise reduction transfer function, $\mathbf{T}(f)$ is defined as

$$\mathbf{T}(f) = \mathbf{W}_f(f) - \mathbf{W}_a(f)\mathbf{B}(f) \tag{5}$$

where $\mathbf{W}_a(f)$ as shown in Fig. 1 is the weight vector of an *adaptive noise canceller* (ANC) with a transversal structure. Assuming that the look direction is set to 0 radians, the optimization of the GSC system output *Power Spectral Density* (PSD), $P^z(f)$ can be formulated as

$$P^z(f) = \mathbf{T}^H(f)\mathbf{R}_{xx}\mathbf{T}(f) \tag{6}$$

where $\mathbf{R}_{xx} = E\{\mathbf{X}(f)\mathbf{X}^H(f)\}$. Note that for the rest of our derivations (f) is removed for simplicity. If we assume that the look direction angle is not exactly 0, but close to it, say θ_{exp}, and also both the localized interferers and background noise are uncorrelated with the desired speech signal (neglecting any acoustical echoes in our ATF's in (3)), then the array correlation matrix of the received signal vector $\mathbf{X}(f)$ is directly related to the array correlation of the desired signal which can be expressed as,

$$\mathbf{R}_{xx} = \mathbf{R}_{nn} + \mathbf{R}_{x_s x_s}(\theta_{\mathrm{exp}}) \tag{7}$$

where $\mathbf{R}_{nn}$ is the correlation matrix of noise and $\mathbf{R}_{x_s x_s}$ represents the array correlation matrix of the desired signal as given in (2).

In this paper, we intend to reduce the sensitivity of the GSC algorithm with respect to the DOA mismatch. This, as a result, requires a modification in the blocking matrix $\mathbf{B}(f)$ which can more realistically model the practical scenario. This is carried out by setting the 1st and 2nd derivatives of the array power response to zero in order to achieve a maximally flat response in the look direction. Correspondingly, the use of derivative constraints can arrive at a less computational intensive approach with respect to phase-independent constraints in [7]. To proceed, by writing the Taylor series expansion at the vicinity of 0 radians for array correlation matrix of the desired signal, we have,

$$\mathbf{R}_{ss}(\theta_{exp}) = \mathbf{R}_{ss}(0)+\frac{\partial \mathbf{R}_{ss}}{\partial \theta_{exp}}|_{\theta_{exp}=0}\theta_{\mathrm{exp}}+\frac{\partial^2 \mathbf{R}_{ss}}{\partial \theta_{exp}^2}|_{\theta_{exp}=0}\frac{\theta_{\mathrm{exp}}^2}{2!}+\ldots \tag{8}$$

To ease the mathematical tract, we also assume that the blocking matrix, $\mathbf{B}(f)$ is able to perfectly block the signal received from 0 radians, since we have excluded the reverberation and acoustic echoes from the ATFs' in (3) in our scenario in this work. Hence we have,

$$\mathbf{B}(f)\mathbf{R}_{ss}(0) = \mathbf{0} \tag{9}$$

In the following subsection, we derive some derivative constraints which are found useful while flattening the mainlobe beam for GSC structure. In simulation results, it will be demonstrated that the proposed CRLS-GSC algorithm can successfully reduce the array sensitivity to DOA mismatch problem and, as a result, improve the over-all interference cancellation of the algorithm.

A. Employing derivative constraints

Comparing with the algorithm presented in [7], the proposed method is easier to implement with less derivative constraints. Substituting (8) in (7) and then in (6) and setting the 1st and 2nd derivatives of the resulting function with respect to θ_{exp} to zero and incorporating (9), we obtain the constraints as derived in Appendix. These constraints generate a maximally flat response in the look direction angle, and thus, solve the DOA mismatch problem. If we summarize all the constraints in one matrix, say, the optimization problem in (A-2), as well as employing (7) and (8) in (6) and incorporating (9) for the system output power results in constraints $\mathbf{C}_0$ in matrix form on adaptive canceller coefficients, as below,

$$\mathbf{C}_0^T\mathbf{W}_a = \mathbf{0} \tag{10}$$

where

$$\mathbf{C}_0 = [\mathbf{C}_{1,1} \;\; \mathbf{C}_{2,1} \;\; \mathbf{U}] \tag{11}$$

$$\mathbf{C}_{1,1} = \mathbf{W}_f^H\frac{\partial \mathbf{R}_{ss}(\theta_{exp})}{\partial \theta_{exp}}|_{\theta_{exp}=0}\mathbf{B}^H \tag{12}$$

$$\mathbf{C}_{2,1} = \mathbf{W}_f^H\frac{\partial^2 \mathbf{R}_{ss}(\theta_{exp})}{\partial \theta_{exp}^2}|_{\theta_{exp}=0}\mathbf{B}^H \tag{13}$$

and $\mathbf{U}$ is an orthogonal matrix corresponding to the respective eigenvectors of $\mathbf{R}_{ss}(\theta_{exp})$. These constraints are computationally less complicated compared to the phase-independent derivative constraint algorithm reported in [7].

B. Adaptive Framework

By incorporating the constraints in a matrix, namely $\mathbf{C}_0$, the CLMS-GSC and CRLS-GSC algorithms are respectively introduced to update the adaptive filter weight vector $\mathbf{W}_a(f)$. The CLMS-GSC algorithm is obtained based on the adaptive LMS algorithm as,

$$\mathbf{e}^{(n)}(f) = \mathbf{X}^{(n)}(f)\mathbf{W}_f(f) - \mathbf{X}^{H^{(n)}}(f)\mathbf{B}(f)\mathbf{W}_a^H(f) \tag{14}$$

$$\mathbf{W}_a^{(n+1)}(f) = \mathbf{W}_a^{(n)}(f) + \mu\mathbf{P}^{\dagger}\mathbf{B}^H(f)\mathbf{X}^{(n)}(f)\mathbf{e}^{(n)}(f) \tag{15}$$

where

$$\mathbf{P}^{\dagger} = \mathbf{I} - \mathbf{C}_0(\mathbf{C}_0^H\mathbf{C}_0)^{-1}\mathbf{C}_0^H \tag{16}$$

where n denotes the iteration number, μ is the step-size parameter to adjust weight vector changes at each iteration, and $\mathbf{P}^{\dagger}$ is the projection matrix. Similarly, we derive the CRLS-GSC algorithm based on the *Recursive Least Square* (RLS) algorithm [9] by incorporating (16) as

$$\mathbf{P}^{\dagger(n+1)} = \frac{1}{\lambda}\mathbf{P}^{\dagger(n)}(1-\frac{\mathbf{B}^H(f)\mathbf{X}^n(f)\mathbf{X}^{(n)H}(f)\mathbf{B}^H(f)\mathbf{P}^{\dagger}}{\lambda+\mathbf{X}^{(n)H}(f)\mathbf{B}(f)\mathbf{P}^{\dagger}\mathbf{B}(f)\mathbf{X}^n(f)}) \tag{17}$$

$$\mathbf{W}_a^{(n+1)}(f) = \mathbf{W}_a^{(n)}(f) + \mathbf{P}^{\dagger}\mathbf{B}^H(f)\mathbf{X}^n(f)e^{*(n)}(f) \tag{18}$$

where λ is the forgetting factor. We use (14),(15) and (17) as our iterative procedure to update the weight vectors for this RLS algorithm. Based on the well-known properties of the LMS and RLS algorithm [9], the CLMS-GSC algorithm is simpler with a lower convergence rate while the CRLS-GSC algorithm is more complex but faster. The latter property leads to a better performance in interference cancellation as demonstrated in simulation results in the following section.

IV. SIMULATION RESULTS

The performance of the proposed algorithms is compared to that of the phase-independent constraints method as well as the GSC and fixed beamformer algorithm [10]. In all the simulations, a five-element *Uniform Linear Array* (ULA) microphone array with inter-spacing of d=1.5 cm is considered. Four interferers are located at $[-55^o, -45^o, -35^o, 45^o]$ where the 1st three ones are very close to each other. The interferers are spectrally colored noise (modeled as computer fan noise) with in the frequency range [0,4kHz]. The desired speech signal is embedded in additive WGN with 0 dB SNR and 5 kHz bandwidth, and the powers of interferers are 10 dB. Speech signals are uttered by a male speaker for different sentences from a persian database. The simulation parameters are as follows. The sampling frequency is set to f_s=8 kHz in all experiments. The desired speech signal is located at one meter far from the array in the look-direction angle $\theta = 0^o$, by which the far field assumption is fulfilled [11]. Also, the DOA is measured only from -90 to 90 degrees. The results are presented after the convergence based on averaging of 100 independent trials, each one with 300 iterations. The set-up configuration for one interferer is shown in Fig.2.

The effect of the leakage constraints on the mainlobe flattening is studied by investigating the polar beampattern

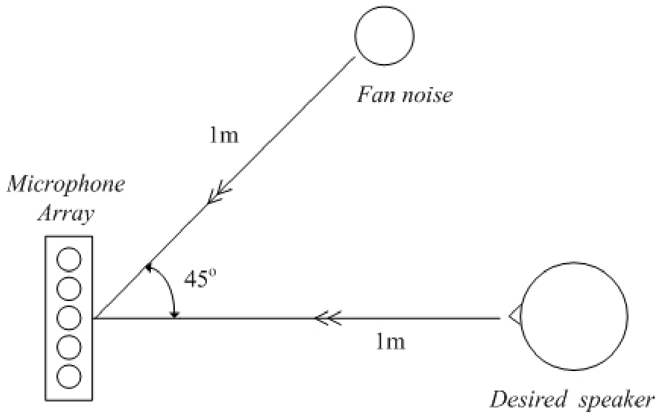

Fig. 2. The set-up configuration for a 5-element *Unform Linear Array* (ULA) microphone array with an interferer at 45^o.

TABLE I
ESTIMATED DOAS FOR THE INTERFERERS AT

Algorithm	CPU (sec)	Estimated DOA
LCMV	187	[-76 to -36.8],[46]
CLMS-GSC	212	[-76.3 to 38.7],[46]
CRLS-GSC	216	[-55 to -36.9],[45]

gain diagrams in Fig.3. The beamformer spatial response is shown in the top panel wherein the fixed beamformer only steers toward the look direction with a very narrow mainlobe and no sensitivity to interferers. The so called *Linearly Constraint Minimum Variance* (LCMV) algorithm introduces deeper attenuations at the nulls for closely located interferers with lower sidelobes. The phase-independent approach also generates a flat mainlobe at the cost of more computations. Instead, the proposed CLMS-GSC and CRLS-GSC algorithms flatten the mainlobe toward the look direction angle with almost a similar attenuation. The middle panel in Fig.3 illustrates the corresponding polar beam patterns. As the CRLS-GSC algorithm performs beam steering with a flattened mainlobe toward the speaker direction, thus, less sensitivity to the speaker movement is experienced.

To compare the accuracy of the angles estimated by the LCMV, CLMS-GSC, and CRLS-GSC algorithms, Table 1 shows the results corresponding to the beam pattern gains given in Fig.3. Although LCMV is faster in terms of CPU time, the range of estimated angles is not good enough. However, the CRLS-GSC is more accurate for closely located interferers at $[-55^o, -45^o, -35^o]$ at a cost of slightly more computations. This algorithm also outperforms the CLMS-GSC algorithm due to less mean-square error of the adaptive RLS with respect to the LMS algorithm.

V. CONCLUSION

In this paper, a set of leakage constraints was derived for the conventional GSC algorithm leading to the new CLMS-GSC and CRLS-GSC algorithms. Using simulation results, it was shown that the speech leakage problem addressed in the GSC algorithm is considerably alleviated and also the

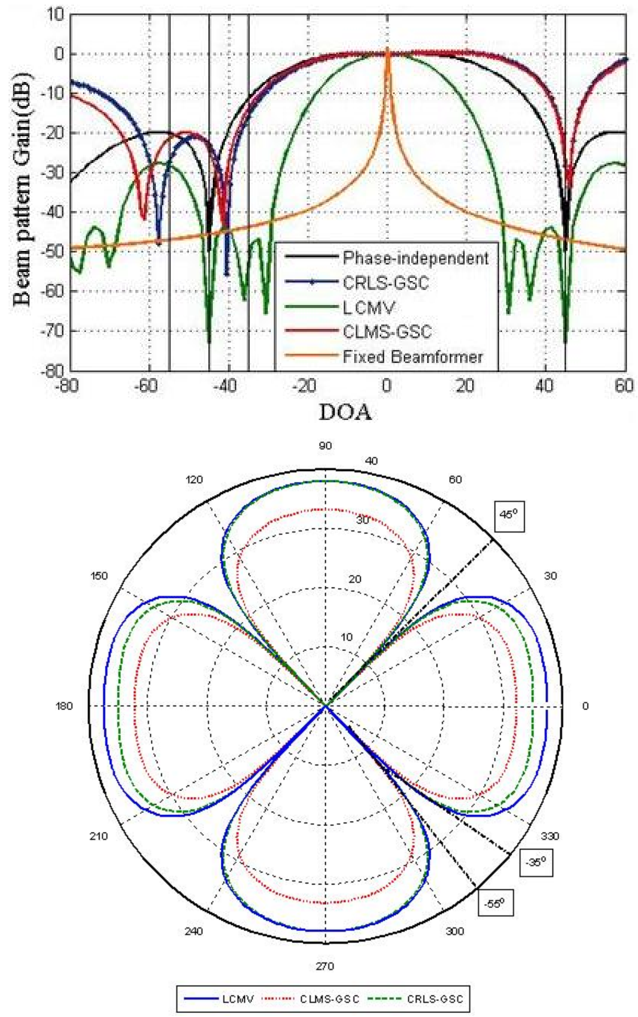

Fig. 3. Top panel: Comparison of spatial response and beampattern of the phase-independent, LCMV, and fixed beamformer to the CRLS-GSC and CLMS-GSC algorithms in dB for interferers located at $[-55^o, -45^o, -35^o, 45^o]$, middle: polar beampattern and bottom: beampattern gain results for CRLS-GSC, CLMS-GSC and LCMV for look directions at 0^o.

closely placed interferences are more attenuated. Simulation results also revealed that the DOA mismatch problem is highly resolved with a lower computational cost comapred with other algorithms.

REFERENCES

[1] I. Thng, A. Cantoni, and Y. H. Leung,*Constraints for maximally flat optimum broadband antenna arrays*, IEEE Trans. on Signal Processing, 43(6):1334-1347, Jun. 1995.

[2] A. Spriet, M. Moonen, J. Wouters, *Robustness analysis of multi-channel Wiener filtering and generalized Sidelobe cancellation for multi-microphone noise reduction in hearing aid applications*, IEEE Trans. on Speech and Audio Processing, 2004.

[3] L. J. Griffiths and C. W. Jim, *An Alternative Approach to Linearly Constrained Adaptive Beamforming*, IEEE Trans. Antennas and Propag, vol. 30, pp. 27-34, Jan. 1982.

[4] S. P. Applebaum and D. J. Chapman, *Adaptive arrays with main beam constraints*, IEEE Trans. Antenna., Propagate. Vol. Ap-24, pp. 650-662, Sept. 1976.

[5] J. Li, P. Stoica, *Robust Adaptive Beamforming*, New Jersey, John Wiley and Sons, 2006.

[6] S. Zhang and I. Thng, *Robust pre-steering derivative constraints for broadband antenna arrays*, IEEE Trans. on Signal Processing, 50(1):1-10, Jan. 2002.

[7] C. Y. Tseng, *Minimum Variance Beamforming with Phase-Independent Derivative Constraints*, IEEE Trans. Antenna., Propagate. Vol. 40,pp. 285-294, March. 1992.

[8] O. Hoshuyama, A. Sugiyama, and A. Hirano, *A robust adaptive beamformer for microphone arrays with a blocking matrix using constrained adaptive filters*, IEEE Trans. Signal Processing, vol. 47, pp. 2677-2684, Oct. 1999.

[9] S. Haykin, *Adaptive Filter Theory*, Prentice Hall, New Jersey, 3rd edition, 1996.

[10] O. L. Frost, *An algorithm for linearly constrained adaptive array processing*, Proc. IEEE, vol. 60, pp. 926-935, Aug. 1972.

[11] M. Brandstein and D. Ward, *Microphone arrays-signal processing techniques and applications*, New York, Springer- Verlag, 2001.

[12] W. Herbordtt, S. Nakamura and W. Kellermann, *Joint optimization of LCMV beamforming and acoustic echo cancellation for automatic speech recognition*, IEEE Int. Conf. on Acoustics Speech Signal Processing, pp.77-80, Mar. 2005.

[13] W. Herbordt and W. Kellermann, *Frequency-domain integration of acoustic echo cancellation and a generalized sidelobe canceller with improved robustness*, European Trans. on Telecommunications, vol. 13, no. 2, pp. 123-132, Mar. 2002.

[14] P. Mowlaee and M. H. Kahaei, *Removing the GSC Noise Reduction deficiencies in Reverberant Environments by Proposing Joint AEC-GSC algorithm*, ACS/IEEE International Conference on Computer Systems and Applications, Amman, Jordan, May, 2007.

[15] P. Mowlaee, *CRLS-GSC Noise Reduction Algorithm Robust to DOA Mismatch Problem in Microphone Arrays*, 3rd International Conference on Information and Communication Technologies (ICTTA), April, 2008.

APPENDIX

The 1st and 2nd-order leakage constraints introduced in (11-13) are derived in this appendix.

A. *1st-Order Approximation*

The blocking matrix in the conventional GSC algorithm as well as in the proposed algorithms can be considered as a simple spatial filter with a null at 0 radians. The optimization problem of GSC in Fig.1 can be formulated using (5),(6) as,

$$P^z = (\mathbf{W}_f - \mathbf{B}^T\mathbf{W}_a)^T R_{ss}(\mathbf{W}_f - \mathbf{B}^T\mathbf{W}_a) \qquad \text{(A-1)}$$

note that the dependency of parameters to frequency (f) is omitted for convenience. Note also that in the following θ_{exp} is replaced with θ. The array correlation matrix of the desired signal, $\mathbf{R}_{ss}$ can be replaced from its equivalent Taylor series expansion. By only considering the first order leakage in (8), (A-1) can be rewritten as,

$$P^z = (\mathbf{W}_f - \mathbf{B}^T\mathbf{W}_a)^T \left(\mathbf{R}_{ss}(0) + \frac{\partial \mathbf{R}_{ss}}{\partial \theta}|_{\theta=0}\theta\right)(\mathbf{W}_f - \mathbf{B}^T\mathbf{W}_a) \qquad \text{(A-2)}$$

after some mathematical operations, the expected beamformer output power is obtained as,

$$P^z = A_0 + A_1 + A_2 + A_3 \qquad \text{(A-3)}$$

where $A_i, i = 0, \ldots, 3$ are as below,

$$A_0 = \mathbf{W}_f^H\mathbf{R}_{ss}(0)\mathbf{W}_f + \mathbf{W}_a^H\mathbf{B}\mathbf{R}_{ss}(0)\mathbf{W}_a \qquad \text{(A-4)}$$

$$A_1 = -\mathbf{W}_a^H\mathbf{B}\mathbf{R}_{ss}(0)\mathbf{W}_f - \mathbf{W}_f\mathbf{R}_{ss}(0)\mathbf{B}^H\mathbf{W}_a \qquad \text{(A-5)}$$

$$A_2 = \mathbf{W}_a^H\mathbf{B}\frac{\partial \mathbf{R}_{ss}}{\partial \theta}|_{\theta=0}\theta\mathbf{W}_f - \mathbf{W}_f^H\frac{\partial \mathbf{R}_{ss}}{\partial \theta}|_{\theta=0}\theta\mathbf{W}_f \qquad \text{(A-6)}$$

$$A_3 = -\mathbf{W}_f^H\frac{\partial \mathbf{R}_{ss}}{\partial \theta}|_{\theta=0}\theta\mathbf{B}^H\mathbf{W}_a + \mathbf{W}_a^H\mathbf{B}\frac{\partial \mathbf{R}_{ss}}{\partial \theta}|_{\theta=0}\theta\mathbf{B}^H\mathbf{W}_a \qquad \text{(A-7)}$$

Using (9), we can easily demonstrate that all non-leakage parts in A_0 and A_1 except the 1st term in A_0 vanish to 0. In addition, the only terms related to mismatch are those containing correlation derivative with respect to θ and blocking matrix $\mathbf{B}$, hence in order to minimize P_z it is sufficient to have,

$$\mathbf{W}_f^H\frac{\partial \mathbf{R}_{ss}}{\partial \theta}|_{\theta=0}\mathbf{B}^T\mathbf{W}_a = \mathbf{0} \qquad \text{(A-8)}$$

$$\mathbf{W}_a^H\mathbf{B}\frac{\partial \mathbf{R}_{ss}}{\partial \theta}|_{\theta=0}\mathbf{B}^T\mathbf{W}_a = \mathbf{0} \qquad \text{(A-9)}$$

In the following we prove that $\mathbf{B}\frac{\partial \mathbf{R}_{ss}}{\partial \theta_{exp}}|_{\theta_{exp}=0}\mathbf{B}^T = \mathbf{0}$ and as a result, (A-9) is always true. To proceed, note that the term $\mathbf{B}\mathbf{R}_{ss}\mathbf{B}^H$ is positive semi-definite for all θ, hence for each arbitrary vector $\mathbf{y}$ we have,

$$\mathbf{y}^H\mathbf{B}\mathbf{R}_{ss}(0)\mathbf{B}^H\mathbf{y} \geq 0 \qquad \text{(A-10)}$$

incorporating (9) we have,

$$\mathbf{y}^H\mathbf{B}\mathbf{R}_{ss}(0)\mathbf{B}^H\mathbf{y} = 0 \qquad \text{(A-11)}$$

then by taking first derivative of (A-11) with respect to θ_{exp}, and due to independence of $\mathbf{y}$ to θ_{exp} we obtain,

$$\mathbf{y}^H\frac{\partial(\mathbf{B}\mathbf{R}_{ss}(0)\mathbf{B}^H)}{\partial \theta_{exp}}|_{\theta_{exp}=0}\mathbf{y} = \mathbf{0} \qquad \text{(A-12)}$$

Noting that $\mathbf{y}$ is an arbitrarily vector, we should have

$$\mathbf{B}\frac{\partial \mathbf{R}_{ss}(0)}{\partial \theta_{exp}}|_{\theta_{exp}=0}\mathbf{B}^H = \mathbf{0} \qquad \text{(A-13)}$$

which shows that (A-9) is always true. As a consequence, the only limiting constraint for the 1st-order speech leakage cancellation is to satisfy the constraint given in (A-8) which is the same constraint in (12). As a result, this single constraint is employed in (12) for $\mathbf{C}_{1,1}$ to perform a constrained optimization of the 1st order approximation resulting in a flattened main lobe beam as demonstrated in simulation results.

B. *2nd-Order Approximation*

In this section, we derive the constraints of the 2nd order derivatives used in proposing CRLS-GSC algorithm in Section 3.A. Considering the 1st and 2nd terms of (8) inserting them into (7) and employing (9), the 2nd-order leakage constraint can be formulated as,

$$\mathbf{W}_f^H\frac{\partial^2 \mathbf{R}_{\hat{x}\hat{x}}}{\partial^2 \theta_{exp}}|_{\theta_{exp}=0}\mathbf{B}^T\mathbf{W}_a = \mathbf{0} \qquad \text{(B-1)}$$

$$\mathbf{W}_a^T\mathbf{B}\frac{\partial^2 \mathbf{R}_{\hat{x}\hat{x}}}{\partial^2 \theta_{exp}}|_{\theta_{exp}=0}\mathbf{B}^T\mathbf{W}_a = \mathbf{0} \qquad \text{(B-2)}$$

Similar to appendix. A, using (A-13) we can show that (B-2) is always true. As a result, the only constraint to be imposed for flattening the main lobe beam for 2nd order approximation is (B-2) which was employed as $\mathbf{C}_{2,1}$ in (13).