

BLIND EQUALIZATION USING BAYESIAN TECHNIQUE AND WAVELET NETWORKS

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ABSTRACT

A new blind equalization algorithm based on suboptimum Bayesian symbol detector and discrete wavelet network is presented. First, the shortcomings of well-known Bayesian blind equalization is discussed, then the method is modified in order to be used for nonlinear channels. It uses discrete wavelet networks instead of linear models to cover the nonlinearities and guarantee global optimization. It is shown that the proposed method is applicable for elimination of non-symmetrical nonlinear distortions caused by even harmonics.

1. INTRODUCTION

Blind equalization algorithms attempt to determine the transmitted symbol sequence in the presence of inter-symbol interference (ISI) without prior knowledge of the channel specifications. Most efforts in the development of blind equalizers have focused on “property restoration” algorithms in which a nonlinear function of the equalizer output is forced to a constant value [1],[2]. This method involves a variety of algorithms known as Bussgang algorithms. In the constant modulus algorithm (CMA, a well known Bussgang algorithm) for example, the error between the magnitude (modulus) of the equalizer output and a constant term is recursively minimized [1]. The resulting gradient-descent method has a computational complexity similar to that of the LMS algorithm. The motivation for these methods is that by restoring the modulus of the received signal, the inversion of channel impulse response is implicitly estimated.

While Bussgang algorithms have the advantage of computational simplicity, utilizing linear models, they are not applicable for nonlinear channels. Furthermore, Bussgang methods cannot converge to estimate the inversion of non minimum phase channels in the presence of noise due to deep nulls in their characteristic functions. Thus, another group of methods called nonlinear equalizers have been developed. Decision Feedback Estimation (DFE), neural networks, and Bayesian algorithms are some well-known nonlinear methods.

DFE utilizes hard decisions for choosing symbols, based on which a linear or nonlinear model is trained in order that the

inversion of the channel is estimated. The implementation of DFE is fairly simple; however its performance is rapidly degraded when the rate of ISIs is high [3]. Neural networks are scarcely used for blind equalization since they typically have complex transfer functions to make statistical operations too difficult to result in some equations on which the blind network training bases. There are two well-known methods which utilize neural networks. The first one trains the network such that the moments of the network output coincide with the moments of the original signal; however its convergence highly depends on initial values of the network parameters to be close to the optimal values [4]. In the second method, the transfer function of the network is approximated by linear models, and then linear algorithms are used for the estimation of the network parameters [5].

On the contrary to the mentioned methods, which are based on the statistical information of original signal, Bayesian symbol-by-symbol estimation, based on parallel Kalman filters, addresses the problem by a probabilistic approach. It attempts to estimate the channel and noise likelihood by which the original signal is estimated. Although Bayesian method is rather complex, it is able to cancel ISIs faster and more effectively than the previous methods [6],[7]. The method uses some linear FIR models for channel estimation, but a nonlinear decision method to select symbols; however due to using linear models, it is not able to remove ISIs caused by nonsymmetrical nonlinearities which usually occurs in nonlinear amplifiers.

In this paper we propose a new method based on discrete wavelet networks and Bayesian symbol by symbol estimation to address the problem of nonsymmetrical nonlinearities. In this method, some discrete wavelet networks are configured based on the input signal constellation and trained using Bayesian algorithm such that some channel estimates are achieved.

The paper is organized as follows, section II introduces discrete wavelet network, section III provides an overview of Bayesian blind equalizer, section IV develops the new Bayesian method based on wavelet network model. Section V presents an assessment of the new method via simulations, and section VI concludes the paper.

2. DISCRETE WAVELET TRANSFORM

According to the wavelet Theory, it is known that each function like $f(\mathbf{x})$, $\mathbf{x} \in R^n$ which belongs to $L^2(R^n)$ space, can be approximated by a set of basis functions known as wavelets.

In one dimensional case the functions can be generated by translations and dilations of a unique wavelet called mother wavelet $\psi(x)$ which must satisfy Morlet-Grossman criteria:

$$C_{\psi_s} = \int_0^\infty \frac{|\psi_s(\omega)|^2}{\omega} d\omega < \infty \quad (1)$$

where $\psi_s(\omega)$ is the Fourier transform of $\psi(x)$.

In multidimensional case, multidimensional wavelets are required, which can be created in two ways:

Direct product of scalar wavelets:

$$\psi(\mathbf{x}) = \psi(x_1) \dots \psi(x_n), \quad \mathbf{x} = [x_1 \ x_2 \ \dots \ x_n]^T \quad (2)$$

and Single scale or radial wavelets:

$$\psi(\mathbf{x}) = \psi(\mathbf{x}^T \mathbf{x}), \quad \mathbf{x} = [x_1 \ x_2 \ \dots \ x_n]^T \quad (3)$$

Considering multidimensional wavelets, any function like $f(\mathbf{x})$ can be approximated by Continuous wavelet transform as follows:

$$f(\mathbf{x}) = \frac{1}{C_\psi} \int_{R^n} \int_{R_+^n} w(\mathbf{d}, \mathbf{t}) (\det D)^{\frac{1}{2}} \psi[D(\mathbf{x} - \mathbf{t})] d\mathbf{d} d\mathbf{t} \quad (4)$$

where $w(\mathbf{d}, \mathbf{t})$ is:

$$w(\mathbf{d}, \mathbf{t}) = \int_{R^n} f(\mathbf{x}) (\det D)^{\frac{1}{2}} \psi[D(\mathbf{x} - \mathbf{t})] d\mathbf{x} \quad (5)$$

and $D = \text{diag}(\mathbf{d})$, $\mathbf{d} = [d_1 \ d_2 \ \dots \ d_n]^T$

It can be demonstrated when (4) is made discrete on a regular lattice like:

$$\Psi(\alpha, \beta) = \left\{ \alpha^{\frac{k}{2}} \psi(\alpha^k x - \beta l) : k, l \in Z \right\} \quad (6)$$

with proper choice of α and β , any function like $f(\mathbf{x}) \in L^2(R^n)$ can be represented by equation (6). This transform is called discrete wavelet transform. $\alpha = 2$ and $\beta = 1$ are the most commonly used and proper choices for the parameters.

Discrete wavelet network was proposed based on the above theory, but since equation (6) is an unlimited set of functions, some heuristic algorithms are used to select the most correlated wavelets with the target function and determine their weights [9]. One of the well-known single scale wavelet is the Mexican hat:

$$\psi(\mathbf{x}) = (n - \mathbf{x}^T \mathbf{x}) e^{-\mathbf{x}^T \mathbf{x} / 2} \quad (7)$$

With the above assumptions, the approximation of any function like $f(\mathbf{x})$ is:

$$f(\mathbf{x}) \approx \sum_{k \in Z, \mathbf{t}_i \in Z^n} \omega_{k, \mathbf{t}_i} \psi(2^k \mathbf{x} - \mathbf{t}_i) \quad (8)$$

In this paper, we use an adaptive algorithm based on Kalman filter to train the network.

3. LINEAR BAYESIAN BLIND EQUALIZATION

Bayesian equalization is a suboptimal algorithm of Maximum Likelihood Sequence Estimation (MLSE). Considering all possible states of the channel input sequence until the current time and an additive Gaussian noise at the receiver side of the channel, MLSE attempts to find the most likely input sequence. However, since the computational complexity of this method grows exponentially with respect to time, several suboptimal algorithms have been proposed, which utilize only a subsequence of input signal. Bayesian algorithms constitute a renowned group of these solutions. The approach applies Bayesian and Maximum A Posteriori (MAP) methods to subsequences with limited lengths to select the most probable symbol and channel estimates iteratively (Figure 1).

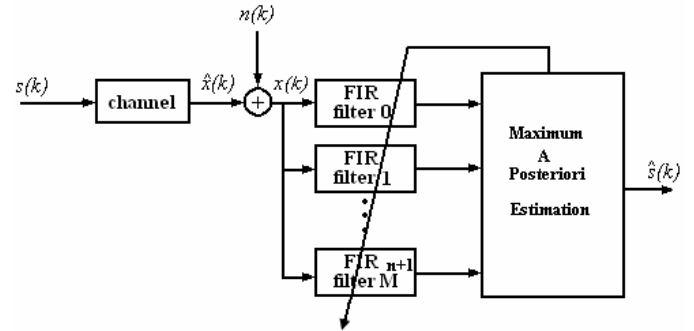


Figure 1 - Bayesian blind equalization based on linear models

Since in blind equalization case neither the channel nor the input signal is available, an iterative algorithm should be implemented, by which both the channel and signal are estimated simultaneously. Consequently, a three step algorithm has been presented in [6], as follows:

1. In each iteration, corresponding to each possible input sequence, a channel estimate is considered, and some estimated channel output values are obtained. With respect to the closeness of any of these values to the measured value, a weight (between 0 and 1) is assigned to the input sequence and the channel set. The set which provides the closest value to the real value is selected as the most likely set in the iteration and weight 1 is assigned to it; consequently, other weights are determined with respect to it.
2. The channel estimations must be updated in order to minimize the differences between estimated and the real channel output values.
3. Each of the channel estimations relatively closes the real channel with respect to its value mentioned in step 1, based

on which a new generation of channel estimations can be created to enhance the estimations.

With attention to the aforementioned steps, the formulations come as follows.

Suppose a linear FIR channel with the coefficient vector $\mathbf{h} = [h_0 \ h_1 \ \dots \ h_m]^T$, then for the input signal $s(k)$, the channel output is:

$$x(k) = \mathbf{h}^T \mathbf{s}(k) + n(k) \quad (9)$$

where $\mathbf{s}(k) = [s(k) \ s(k-1) \ \dots \ s(k-m)]^T$, and $n(k)$ is the measurement noise assumed to be a zero mean white Gaussian process with variance σ_n^2 .

Input signal is estimated from an alphabet set which consists of M symbols. Consider the following notation for the various states of the input sequence:

$$\mathbf{s}_i^{k,m} = \{s_i(k), s_i(k-1), \dots, s_i(k-m)\} \quad i = 1, \dots, M^{m+1} \quad (10)$$

where $\mathbf{s}_i^{k,m}$ shows each possible sequence with length $m+1$ made up of the input symbols estimated from the alphabet set.

The input sequence can be estimated blindly by a Maximum A Posteriori (MAP) sequence estimator in which assuming the conditional likelihood $p(\mathbf{s}_i^{k,m} | \mathbf{x}^k)$ is known, the input sequence which maximizes the likelihood is selected as the most likely sequence. However since the likelihood is not firstly known, a method should be found to estimate it. It can be shown that by using the Bayesian formula, the following recursion is achieved which provides an iterative estimation of the likelihood.

$$p(\mathbf{s}_i^{k,m} | \mathbf{x}^k) = \frac{1}{c} p(x(k) | \mathbf{s}_i^{k,m}, \mathbf{x}^{k-1}) \times \sum_{\{j: \mathbf{s}_j^{k-1,m} \in \mathbf{s}_i^{k,m}\}} p(\mathbf{s}_j^{k-1,m} | \mathbf{x}^{k-1}) \quad (11)$$

where c is a normalization constant and subsequence $\mathbf{s}_j^{k-1,m} \in \mathbf{s}_i^{k,m}$ implies that the first m symbols in subsequence $\mathbf{s}_j^{k-1,m}$ are identical to the last m symbols in $\mathbf{s}_i^{k,m}$, and the likelihood $p(x(k) | \mathbf{s}_i^{k,m}, \mathbf{x}^{k-1})$ is Gaussian:

$$p(x(k) | \mathbf{s}_i^{k,m}, \mathbf{x}^{k-1}) = N(x(k); \hat{x}_i(k | k-1), \sigma_i^2(k | k-1)) \quad (12)$$

The mean $\hat{x}_i(k | k-1)$ and $\sigma_i^2(k | k-1)$ called innovation variance are obtained as follows:

$$\hat{x}_i(k | k-1) = \hat{\mathbf{h}}_i^T(k) \mathbf{s}_i(k) \quad (13)$$

$$\sigma_i^2(k | k-1) = \mathbf{s}_i^T(k) \mathbf{P}_i(k | k-1) \mathbf{s}_i(k) + \sigma_n^2 \quad (14)$$

According to step II, an update equation is required to minimize the difference between the estimated values $\hat{x}_i(k)$ and the measurement value $x(k)$. To implement this iteration, a state space model is assumed for the channel

weights $\{h_i(k)\}$ as follows:

$$\mathbf{h}(k+1) = \mathbf{h}(k) + \mathbf{w}(k) \quad (15)$$

where $\mathbf{w}(k)$ is a Gaussian zero mean vector with covariance matrix \mathbf{Q} .

We know that Kalman filter is applied to find the least variance estimation of the states of a state space model iteratively, therefore using the Kalman filter equations, (15) and (16), a channel update equation can be obtained as follows:

$$\hat{\mathbf{h}}_i(k | k) = \hat{\mathbf{h}}_i(k | k-1) + \frac{1}{\sigma_i^2(k | k-1)} \mathbf{P}_i(k | k-1) \mathbf{s}_i(k) [x(k) - \hat{x}_i(k | k-1)] \quad (17)$$

and

$$\mathbf{P}_i(k | k) = \left[\mathbf{I} - \frac{1}{\sigma_i^2(k | k-1)} \mathbf{P}_i(k | k-1) \mathbf{s}_i(k) \mathbf{s}_i^T(k) \right] \mathbf{P}_i(k | k-1) \quad (18)$$

where $\mathbf{P}_i(k | k-1)$ s are covariance matrices corresponding to $\hat{\mathbf{h}}_i(k)$ s, and represent measures for the uncertainty of the channel estimations.

To implement step III, the likelihood of the channel weights conditioned on the input and measurement vectors is considered, which can be written as follows:

$$p(\mathbf{h}(k) | \mathbf{s}_i^{k,m}, \mathbf{x}^{k-1}) = \sum_{\{j: \mathbf{s}_j^{k,m} = \mathbf{s}_i^{k,m}\}} p(\mathbf{h}(k) | \mathbf{s}_j^k, \mathbf{x}^{k-1}) \frac{p(\mathbf{s}_j^k | \mathbf{x}^{k-1})}{p(\mathbf{s}_i^{k,m} | \mathbf{x}^{k-1})} \quad (19)$$

where $\mathbf{s}_j^k = [s_j(k) \ s_j(k-1) \ \dots \ s_j(0)]$ and $p(\mathbf{h}(k) | \mathbf{s}_j^k, \mathbf{x}^{k-1})$ is a Gaussian vector.

$$p(\mathbf{h}(k) | \mathbf{s}_i^k, \mathbf{x}^{k-1}) = N(\mathbf{h}(k); \hat{\mathbf{h}}_i(k | k-1), \mathbf{P}_i(k | k-1)) \quad (20)$$

where $\hat{\mathbf{h}}_i(k | k-1)$ is the prior channel estimation. It is obvious that (19) provides the equation which produces a new generation of channel estimations with respect to the previous channel estimations and their metrics.

It is known that the best estimation of a stochastic variable is its expectation value, which can be written for $\hat{\mathbf{h}}_i(k+1 | k)$ as follows:

$$\hat{\mathbf{h}}_i(k+1 | k) = E(\mathbf{h}(k+1) | \mathbf{s}_i^{k,m}, \mathbf{x}^k) \quad (21)$$

using (19),(20) in (21), the following equations are obtained:

$$\hat{\mathbf{h}}_i(k+1 | k) = \sum_{\{j: \mathbf{s}_j^{k,m} \in \mathbf{s}_i^{k+1,m}\}} \hat{\mathbf{h}}_j(k | k) \frac{p(\mathbf{s}_j^{k,m} | \mathbf{x}^k)}{\sum_{\{r: \mathbf{s}_r^{k,m} \in \mathbf{s}_i^{k+1,m}\}} p(\mathbf{s}_r^{k,m} | \mathbf{x}^k)} \quad (22)$$

$$\mathbf{P}_i(k+1|k) = \sum_{\{j: \mathbf{s}_j^{k,m} \in \mathbf{s}_i^{k+1,m}\}} (\mathbf{P}_j(k|k) + \mathbf{Q} + \mathbf{v}_{j,i}(k) \mathbf{v}_{j,i}^T(k)) \times \frac{p(\mathbf{s}_j^{k,m} | \mathbf{x}^{k-1})}{\sum_{\{r: \mathbf{s}_r^{k,m} \in \mathbf{s}_i^{k,m}\}} p(\mathbf{s}_r^{k,m} | \mathbf{x}^k)} \quad (23)$$

where

$$\mathbf{v}_{j,i}(k) = [\hat{\mathbf{h}}_i(k+1|k) - \hat{\mathbf{h}}_j(k|k)] \quad (24)$$

4. BLIND EQUALIZATION USING DESCRETE WAVELET NETWORK

Considering the Bayesian blind equalization presented in the previous section, it was shown that the method can be extended for any dynamic system structure which is a weighting sum of some fixed basis. It was also shown in Section II that the discrete wavelet network meets such a requirement.

Using (8), the output of any FIR channel (linear or nonlinear) can be written as follows:

$$x(k) = \sum_i \omega_i \psi(2^d \mathbf{s}^{k,m} - \mathbf{t}_i) + n(k) \quad ; \quad i = 1, \dots, M^{m+1} \quad (25)$$

where $\mathbf{s}^{k,m} = [s^k \ s^{k-1} \ \dots \ s^{k-m}]^T$ is the input vector and $\mathbf{t}_i = [t_{i,0} \ t_{i,1} \ \dots \ t_{i,m}]^T$ are translation vectors.

Like linear model, we consider a state space model for the weights of the network:

$$\boldsymbol{\omega}(k+1) = \boldsymbol{\omega}(k) + \mathbf{w}(k) \quad ; \quad \boldsymbol{\omega} = [\omega_1, \dots, \omega_{M^{m+1}}] \quad (26)$$

and then (25) can be rewritten in the following vector form:

$$\mathbf{x}^k = \boldsymbol{\omega}^T \mathbf{c}(k) + n(k) \quad (27)$$

where

$$\mathbf{c}(k) = \begin{bmatrix} \psi(2^d \mathbf{s}^k - \mathbf{t}_1) \\ \vdots \\ \psi(2^d \mathbf{s}^k - \mathbf{t}_{M^{m+1}}) \end{bmatrix} \quad (28)$$

Similar to the Bayesian method based on linear models, the value of the channel output, for all possible states of input sequences $\mathbf{s}_i^{k,m}$, must be calculated. So if $\mathbf{s}_i^{k,m}$ s are put in (28), $\mathbf{c}_i(k)$ s are:

$$\mathbf{c}_i(k) = \mathbf{c}(k) \Big|_{\mathbf{s}_i^{k,m}} \quad (29)$$

Now, with no limit, equation (11) can also be used for the wavelet network. Thus using this equation, the input signal is estimated. Based on Kalman filter equations, (26) and (27) the following update equations are derived:

$$\hat{\boldsymbol{\omega}}_i(k|k) = \hat{\boldsymbol{\omega}}_i(k|k-1) + \frac{1}{\sigma_i^2(k|k-1)} \mathbf{P}_i(k|k-1) \mathbf{c}_i(k) (x(k) - \hat{x}(k|k-1)) \quad (30)$$

and

$$\mathbf{P}_i(k|k) = \left[\mathbf{I} - \frac{1}{\sigma_i^2(k|k-1)} \mathbf{P}_i(k|k-1) \mathbf{c}_i(k) \mathbf{c}_i^T(k) \right] \mathbf{P}_i(k|k-1) \quad (31)$$

where

$$\sigma_i^2(k|k-1) = \mathbf{c}_i^T(k) \mathbf{P}_i(k|k-1) \mathbf{c}_i(k) + \sigma_n^2 \quad (32)$$

Using (21), (22) and (23) one step prediction of network weights and their corresponding covariance matrices are obtained as follows:

$$\hat{\boldsymbol{\omega}}_i(k+1|k) = \sum_{\{j: \mathbf{s}_j^{k,m}, \mathbf{s}_i^{k+1,m}\}} \hat{\boldsymbol{\omega}}_j(k|k) \frac{p(\mathbf{s}_j^{k,m} | \mathbf{x}^k)}{\sum_{\{r: \mathbf{s}_r^{k,m}, \mathbf{s}_i^{k+1,m}\}} p(\mathbf{s}_r^{k,m} | \mathbf{x}^k)} \quad (33)$$

$$\mathbf{P}_i(k+1|k) = \sum_{\{j: \mathbf{s}_j^{k,m} \in \mathbf{s}_i^{k+1,m}\}} (\mathbf{P}_j(k|k) + \mathbf{Q} + \mathbf{v}_{j,i}(k) \mathbf{v}_{j,i}^T(k)) \times \frac{p(\mathbf{s}_j^{k,m} | \mathbf{x}^{k-1})}{\sum_{\{r: \mathbf{s}_r^{k,m} \in \mathbf{s}_i^{k,m}\}} p(\mathbf{s}_r^{k,m} | \mathbf{x}^k)} \quad (34)$$

where

$$\mathbf{v}_{j,i}(k) = [\hat{\boldsymbol{\omega}}_i(k+1|k) - \hat{\boldsymbol{\omega}}_j(k|k)] \quad (35)$$

The overall algorithm is summarized in Table II.

5. SIMILUATION

In order to evaluate the performance of the proposed approach in comparison to the Bayesian method based on linear models, two nonlinear channels were considered. The channels comprised of the same linear part which was a non-minimum phase FIR filter with three taps as follows:

$$x_i(k) = 0.3482s(k) + 0.87(k-1) + 0.3482s(k-2)$$

and different nonlinear parts; the first one created a symmetric nonlinearity which is popular in satellite channels as follows:

$$\hat{x}(k) = \frac{1.32x_i(k)}{1 + 0.2x_i^2(k)}$$

The second nonlinearity was selected as an out case study since the channel created second harmonies.

$$\hat{x}(k) = 0.82(x_i(k) + 0.2x_i^2(k))$$

Input signal is binary and drawn from $\{1, -1\}$.

The comparison between linear model and wavelet network was carried out based on SNR at the receiver side of the channel.

With attention to the memory of the channels, there were 2^3 possible states for the input sequence. Therefore, the wavelet

networks comprised of eight wavelets. The input vector of the network was:

$$\mathbf{s}(k) = [s(k) \ s(k-1) \ s(k-2)]^T$$

Considering the discrete wavelet transform, described in Section II, the wavelets were centred on all possible input sequences, and their scales were selected to be -1. Eight vectors with length eight were also created and initiated with random values between [-0.5 0.5]. For the Bayesian method based on linear models, eight vectors with length three were assumed.

The BER was evaluated after 300 iterations for both channels. The results related to the first channel are shown in Figure 2; as we can see, both methods managed to equalize the channel; however, the new methods had a better performance to some extent. On the contrary, as Figure 3 shows, only the new method managed to equalize the second channel which created even harmonics.

TABLE I:
BAYESIAN BLIND EQUALIZATION ALGORITHM

1. Define input signal vectors:
$\mathbf{s}_i(k) = [s_i(k), \dots, s_i(k-m)] \quad , \quad i = 1 \dots M^{m+1}$
2. compute \mathbf{c}_i vectors
$\mathbf{c}_i(k) = \mathbf{c}(k) _{\mathbf{s}_i^{k,m}}$
3. Compute conditional covariance
$\sigma_i^2(k k-1) = \mathbf{c}_i^T(k) \mathbf{P}_i(k k-1) \mathbf{c}_i(k) + \sigma_n^2$
4. Compute channel output estimates
$\hat{x}_i(k k-1) = \sum_j \omega_j \psi(2^d s_i^k - \mathbf{t}_j)$
5. Update conditional measurement estimates
$\hat{\omega}_i(k k) = \hat{\omega}_i(k k-1) + \frac{1}{\sigma_i^2(k k-1)} \mathbf{P}_i(k k-1) \mathbf{c}_i(k) (x(k) - \hat{x}_i(k k-1))$
6. Update conditional error covariance
$\mathbf{P}_i(k k) = \left[\mathbf{I} - \frac{1}{\sigma_i^2(k k-1)} \mathbf{P}_i(k k-1) \mathbf{c}_i \mathbf{c}_i^T \right] \mathbf{P}_i(k k-1)$
7. Update weighting probabilities
$p(\mathbf{s}_i^{k,m} \mathbf{x}^k) = \frac{1}{c} N(x(k); \hat{x}_i(k k-1), \sigma_i^2(k k-1)) \prod_{\{j: s_j^{k-1,m} \in \mathbf{s}_i^{k,m}\}} p(\mathbf{s}_j^{k-1,m} \mathbf{x}^{k-1})$
8. Choose symbol
$\hat{\mathbf{s}}_i^{k,m} = \arg \max_{\mathbf{s}_i^{k,m}} p(\mathbf{s}_i^{k,m} \mathbf{x}^k)$
$\hat{s}(k-m) = \hat{\mathbf{s}}_i(k-m)$
9. Update one step predictions
$\hat{\omega}_i(k+1 k) = \sum_{\{j: \mathbf{s}_j^{k,m}, \mathbf{s}_i^{k+1,m}\}} \hat{\omega}_i(k k) \frac{p(\mathbf{s}_j^{k,m} \mathbf{x}^k)}{\sum_{\{r: \mathbf{s}_r^{k,m}, \mathbf{s}_i^{k+1,m}\}} p(\mathbf{s}_r^{k,m} \mathbf{x}^k)}$
10. Compute covariance outer product vectors
$\mathbf{v}_{j,i}(k) = [\hat{\omega}_i(k+1 k) - \hat{\omega}_j(k k)]$
11. Update one step error covariance
$\mathbf{P}_i(k+1 k) = \sum_{\{j: \mathbf{s}_j^{k,m} \in \mathbf{s}_i^{k+1,m}\}} \left\{ \mathbf{P}_j(k k) \mathbf{F}^T + \mathbf{Q} + \mathbf{v}_{j,i}(k) \mathbf{v}_{j,i}^T(k) \times \frac{p(\mathbf{s}_j^{k,m} \mathbf{x}^{k-1})}{\sum_{\{r: \mathbf{s}_r^{k,m} \in \mathbf{s}_i^{k+1,m}\}} p(\mathbf{s}_r^{k,m} \mathbf{x}^k)} \right\}$

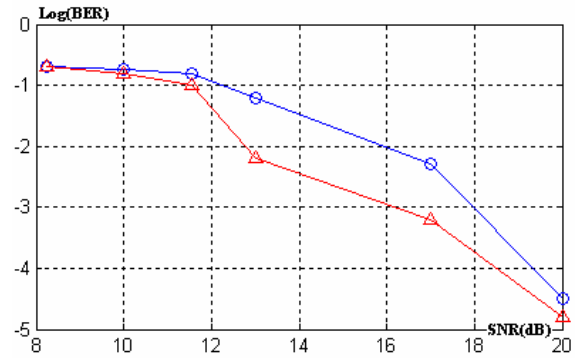


Figure 2 – First channel results: Bit Error Rate vs. SNR at the channel output for Bayesian method based on linear model(O) and wavelet model(Δ).

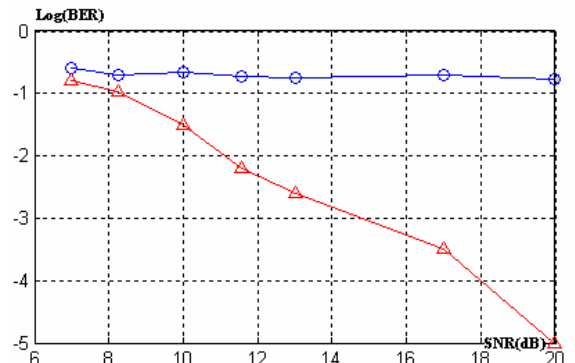


Figure 3 – Second channel results: Bit Error Rate vs. SNR at the channel output for Bayesian method based on linear model(O) and wavelet model(Δ).

6. CONCLUSION

A new method for nonlinear blind equalization based on discrete wavelet network and Bayesian estimation has been presented in this paper. The previous Bayesian methods (based on linear models) are not able to equalize channels with even nonlinearities, whereas the proposed method, as shown in the simulation results, can effectively remove these kinds of distortions.

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