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COGNITIVE DECODING AND THE GOLDEN CODE

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ABSTRACT

Space time signal processing starts with a system of linear equations where signals are multiplied by channel gains, and the standard criteria for the design of space time codes focus on differences between codewords at the transmitter. The value of algebraic constructions is to transfer structure (correlation) at the transmitter to structure at the receiver, and the focus of this paper is the induced channel at the receiver. We use the Golden code to explore the idea of introducing structure at the transmitter to enable low complexity decoding at the receiver. This is an important special case, since the Golden code is incorporated in the IEEE 802.16 standard, but the value of our approach is not limited to this example. We describe a cognitive decoder for the Golden code with complexity $O(N^2)$ that comes within 3dB of full MAP/ML decoding. The decoder is cognitive in that it uses channel state information to choose between two algorithms in a way that is independent of the signal-to-noise ratio. The primary algorithm is interference cancellation which fails to perform well on a proportion of channels. We identify the channel conditions for which inteference cancellation fails and show that for these channels the decoding problem effectively reduces to a single receive antenna decoding problem for which we have developed an efficient zero forcing algorithm. Previous hybrid approaches based on sphere decoding have cubic worst case complexity and employ decision rules based on condition number of the posterior covariance matrix. Interference cancellation is different in that orientation of the covariance matters. The cognitive decoder for the Golden code provides a uniform solution to different wireless environments (Rayleigh/Rician) that combine rich scattering and line of sight components. The gap between cognitive and full MAP/ML decoding reduces to essentially ML performance as the line of sight component becomes more dominant.

1. INTRODUCTION

Maximum-Likelihood (ML) decoding of space-time codes reduces to the problem of finding the least squares solution to a system of linear equations. The entries of the unknown vector are typically drawn from a QAM constellation of size 2^m and the coefficients and the entries of the objective vector are real numbers. The problem of finding the closest lattice point to a given point is known to be NP hard, but the communications problem is more tractable since the objective vector is an unknown lattice point that has been perturbed by an additive noise vector with known statistics. In fact Hassibi and Vikalo [1] have obtained a closed form expression (given certain assumptions about the channel matrix) for the average complexity of the search algorithm of Fincke and Pohst [2] by averaging over the noise and the lattice. This algorithm performs a search over lattice points that lie in a certain sphere of radius d about an initial estimate.

However, the complexity of sphere decoding is not determined by the complexity of lattice point search. When the channel matrix is close to singular, the preprocessing stage of the sphere decoding algorithm yields a plane of possibilities rather than a single initial estimate. When this occurs, lattice point search degenerates to an exhaustive search, and the expected complexity, as shown by Jaldén et al. [3] is exponential in the constellation size and strongly dependent on the SNR. Thus sphere decoding is attractive only for some SNR regimes and for modest constellation size. By contrast, our worst case decoding complexity for the Golden code is quadratic (worst case complexity of sphere decoding is at least cubic) and the algorithm itself is more resilient to near singularity of the channel matrix.

The Golden Code [4], [5], [6] is a remarkable space-time code that employs two antennas to transmit four complex QAM symbols over two time slots while achieving full diversity. This tradeoff between rate and reliability is best possible in terms of the diversity-multiplexing bound derived by Zheng and Tse [7] and in fact the minimum determinant is bounded below by a constant that is independent of the size of the constellation.

This paper describes a cognitive decoder for the Golden code with complexity $O(N^2)$, where N is the number of symbols in the underlying QAM constellation. The receiver uses channel state information to choose between two algorithms, each with complexity $O(N^2)$, and this choice is independent of the signal-to-noise ratio. The primary algorithm is interference cancellation (IC) which fails to perform well on a proportion of channels and this failure significantly degrades overall perfomance. We identify the channel conditions for which IC fails and show that for these channels the decoding problem effectively reduces to a single receive antenna decoding problem. In this case we apply our fast decoding algorithm based on Diophantine approximation [8]. Simulation results show the performance of the cognitive decoder is within 3dB of full MAP/ML perfomance and reduces to essentially ML performance as the line of sight component becomes more dominant.

2. THE GOLDEN CODE

The Golden Code is a 2×2 block space-time code that encodes four complex symbols over two time slots yet achieves full diversity (see [4], [5] and [6]). Codewords in the Golden Code take the form

$$\frac{1}{\sqrt{5}}\begin{pmatrix} 1+i\mu & 0 \\ 0 & 1+i\tau \end{pmatrix} X, \tag{1}$$

where

$$X = \begin{pmatrix} x_1 + x_2\tau & x_3 + x_4\tau \\ i(x_3 + x_4\mu) & x_1 + x_2\mu \end{pmatrix}, \tag{2}$$

with $x_1, x_2, x_3, x_4 \in \mathscr{C} \subset \mathbb{Z}[i]$ the transmitted symbols, and $\mathscr{C}$ is a signal constellation taken to be 2^m-QAM with in-phase and quadrature components equal to $\pm 1, \pm 3, \ldots$ and m bits per symbol. The parameters τ and μ are the real roots of the polynomial $x^2 - x - 1$, that is, the Golden Ratio $\tau = \frac{1+\sqrt{5}}{2}$ and its algebraic conjugate $\mu = -1/\tau = \frac{1-\sqrt{5}}{2}$, which is the negative of the inverse of the Golden Ratio. The diagonal matrix $diag[1 + i\mu, 1 + i\tau]$ serves to equalize transmitted signal power across the two transmit antennas. The entries of Golden space-time codewords are drawn from $\mathbb{Z}[i][\sqrt{5}] \subset \mathbb{Q}(i, \sqrt{5})$. Following [9] we rewrite (2) as

$$X = \begin{pmatrix} x_1 & x_3 \\ ix_3 & x_1 \end{pmatrix} + \begin{pmatrix} \tau & 0 \\ 0 & \mu \end{pmatrix} \begin{pmatrix} x_2 & x_4 \\ ix_4 & x_2 \end{pmatrix}. \tag{3}$$

The set of integer matrices of the form

$$\begin{pmatrix} x & y \\ iy & x \end{pmatrix}, \quad x, y \in \mathbb{Z}[i]. \tag{4}$$

is a matrix representation of the cyclotomic ring $\mathbb{Z}[\zeta_8]$, where ζ_8 is a primitive 8^{th} root of unity. For details on the algebraic structure of this code, see [8].

Let (r_{11}, r_{12}) and (r_{21}, r_{22}) be the two received signal vectors where the components are the signals received over two consecutive time slots. Let $\kappa_{11}, \kappa_{21}, \kappa_{12}$ and κ_{22} be the complex channel gains from the two transmit antennas to the two receive antennas. For convenience, in what follows we will use the rescaled channel gains

$$h_1 = \frac{1}{\sqrt{5}}(1 + i\mu)\kappa_{11}, \quad h_2 = \frac{1}{\sqrt{5}}(1 + i\tau)\kappa_{21} \tag{5}$$

$$g_1 = \frac{1}{\sqrt{5}}(1 + i\mu)\kappa_{12}, \quad g_2 = \frac{1}{\sqrt{5}}(1 + i\tau)\kappa_{22}. \tag{6}$$

The received signal vector is given by

$$(r_{11}, r_{12}) = (x_1, x_3) \begin{pmatrix} h_1 & h_2 \\ ih_2 & h_1 \end{pmatrix} + (x_2, x_4\mu) \begin{pmatrix} h_1\tau & h_2\mu \\ ih_2\mu & h_1\tau \end{pmatrix} + (n_{11}, n_{12}) \tag{7}$$

$$(r_{21}, r_{22}) = (x_1, x_2) \begin{pmatrix} g_1 & g_2 \\ ig_2 & g_1 \end{pmatrix} + (x_2, x_4\mu) \begin{pmatrix} g_1\tau & g_2\mu \\ ig_2\mu & g_1\tau \end{pmatrix} + (n_{21}, n_{22}), \tag{8}$$

where n_{11}, n_{12}, n_{21} and n_{22} are complex Gaussian random variables with zero mean and covariance $2\sigma^2 I_2$.

Given that the channel gains h and g are known at the receivers and each symbol is transmitted with equal probability, optimal decoding is provided by the maximum a posteriori MAP/ML estimate.

Setting $\mathbf{r}_1 = (r_{11}, r_{12})$, $\mathbf{r}_2 = (r_{21}, r_{22})$, $\mathbf{s} = (x_1, x_3)$, $\mathbf{c} = (x_2, x_4)$, $\mathbf{n}_1 = (n_{11}, n_{12})$ and $\mathbf{n}_2 = (n_{21}, n_{22})$, we rewrite equations (7) and (8) as

$$\mathbf{r}_1 = \mathbf{s}h + \mathbf{c}\tilde{h} + \mathbf{n}_1 \tag{9}$$

$$\mathbf{r}_2 = \mathbf{s}g + \mathbf{c}\tilde{g} + \mathbf{n}_2 \tag{10}$$

$$(\mathbf{r}_1, \mathbf{r}_2) = (\mathbf{s}, \mathbf{c}) \begin{pmatrix} h & g \\ \tilde{h} & \tilde{g} \end{pmatrix} + (\mathbf{n}_1, \mathbf{n}_2) \tag{11}$$

$$\mathbf{r} = \mathbf{x}A + \mathbf{n} \tag{12}$$

where $\mathbf{r} = (\mathbf{r}_1, \mathbf{r}_2)$, $\mathbf{x} = (\mathbf{s}, \mathbf{c})$, the meaning of $\tilde{}$ is clear from (7) and (8), and

$$A = \begin{pmatrix} h & g \\ \tilde{h} & \tilde{g} \end{pmatrix}. \tag{13}$$

The likelihood function of codewords $\mathbf{s}$ and $\mathbf{c}$ given the received signal $\mathbf{r}$ is given by

$$p(\mathbf{r}|\mathbf{x}) \propto \exp\{-\frac{1}{2\sigma^2}(\mathbf{x} - \mathbf{r}A^\dagger(AA^\dagger)^{-1})AA^\dagger(\mathbf{x} - \mathbf{r}A^\dagger(AA^\dagger)^{-1})^\dagger\} \tag{14}$$

Taking the prior distribution of the symbols $\mathbf{s}$ and $\mathbf{c}$ to be uniform on the constellation $\mathscr{C}_m$, we obtain the MAP/ML estimate:

$$(\hat{\mathbf{s}}, \hat{\mathbf{c}}) = \arg\max_{\mathbf{s},\mathbf{c} \in \mathscr{C}_m^2} p(\mathbf{r}|\mathbf{s}, \mathbf{c}). \tag{15}$$

3. INTERFERENCE CANCELLATION

The idea behind interference cancellation is that the computation of the MAP/ML decoder can be reduced if one of the codewords, $\mathbf{c}$ say, can be "cancelled out" in some way. We would then only need to search over $\mathbf{s} \in \mathscr{C}^2$. This implies that we should attempt to marginalize the joint posterior probability $p(\mathbf{s}, \mathbf{c}|\mathbf{r})$ with respect to $\mathbf{c}$. This leads to

$$p(\mathbf{s}|\mathbf{r}) \propto \sum_{\mathbf{c} \in \mathscr{C}^2} p(\mathbf{s}, \mathbf{c}|\mathbf{r}), \tag{16}$$

and the MAP/ML decision rule is

$$\hat{\mathbf{s}} = \arg\max_{\mathbf{s} \in \mathscr{C}^2} p(\mathbf{s}|\mathbf{r}) \tag{17}$$

It is evident that this does not help as the sum over $\mathbf{c}$ cannot be evaluated analytically, and so the evaluation of (16) requires just as many likelihood function evaluation as (15). However, if the sum in (16) were to be replaced by a Gaussian integral the marginalisation could be computed analytically. Thus, instead of using the fact that $\mathbf{c}$ lies in QAM constellation, we simply assume that

$$E(\mathbf{c}) = 0 \quad \text{and} \quad E(\mathbf{c}\mathbf{c}^\dagger) = E_{\mathscr{C}} \tag{18}$$

Consequently, the prior for $\mathbf{c}$ is taken to be the maximum entropy distribution satisfying the constraints (18), that is, the Gaussian distribution with zero mean and variance equal to the constellation power $E_{\mathscr{C}}$. The new prior for $\mathbf{c}$ is perfectly consistent with our prior knowledge of $\mathbf{c}$, it just doesn't represent all that we know. Overall this means we allow some increase in the probability of error in detecting codewords $\mathbf{s}$ (and $\mathbf{c}$), for reduced computational load.

Consider the full posterior distribution for the symbols $\mathbf{x} = (\mathbf{s}, \mathbf{c})$:

$$p(\mathbf{x}|\mathbf{r}) \propto \exp\{-\frac{1}{2\sigma^2}(\mathbf{x} - \mathbf{r}A^\dagger(AA^\dagger)^{-1})AA^\dagger(\mathbf{x} - \mathbf{r}A^\dagger(AA^\dagger)^{-1})^\dagger\} \tag{19}$$

The covariance for $\mathbf{x}$ is

$$(AA^\dagger)^{-1} = \frac{1}{5V^2}\begin{pmatrix} \tilde{h}\tilde{h}^\dagger + \tilde{g}\tilde{g}^\dagger & -(h\tilde{h}^\dagger + g\tilde{g}^\dagger) \\ -(\tilde{h}h^\dagger + \tilde{g}g^\dagger) & hh^\dagger + gg^\dagger \end{pmatrix} \quad (20)$$

where $V = |h_1g_2 - h_2g_1|$ is the channel volume. The performance of the MAP/ML estimate for $\mathbf{x}$ is given by

$$\det\left(2\sigma^2(AA^\dagger)^{-1}\right) = \left(\frac{2\sigma^2}{\sqrt{5}V}\right)^4 \quad (21)$$

Marginalizing (19) with respect to $\mathbf{c}$, the posterior distribution for s becomes

$$p(\mathbf{s}|\mathbf{r}) \propto \exp\{-(\mathbf{s}-\mu_\mathbf{s})\Sigma_\mathbf{s}^{-1}(\mathbf{s}-\mu_\mathbf{s})^\dagger\} \quad (22)$$

where

$$\mu_\mathbf{s} = \mathbf{r}R_\mathbf{s}^{-1}\tilde{K}^\dagger\Sigma_\mathbf{s}, \quad (23)$$

$$R_\mathbf{s} = \left(E_\mathscr{C}\tilde{K}^\dagger\tilde{K} + 2\sigma^2 I_4\right), \quad (24)$$

and

$$\Sigma_\mathbf{s} = KR_\mathbf{s}^{-1}K^\dagger \quad (25)$$

with

$$K = (h,g), \quad \tilde{K} = (\tilde{h},\tilde{g}) \quad (26)$$

A similar result holds for the distribution of $\mathbf{c}$ when we marginalize $\mathbf{s}$, where K and $\tilde{K}$ switch roles. We note that in the limit $E_\mathscr{C} \to \infty$, corresponding to no prior knowledge about $\mathbf{c}$ (or $\mathbf{s}$), the marginal covariances for $\mathbf{s}$ and $\mathbf{c}$ become

$$\Sigma_\mathbf{s} = \frac{2\sigma^2}{5V^2}(\tilde{h}\tilde{h}^\dagger + \tilde{g}\tilde{g}^\dagger), \quad (27)$$

and

$$\Sigma_\mathbf{c} = \frac{2\sigma^2}{5V^2}(hh^\dagger + gg^\dagger). \quad (28)$$

As we expect $2\sigma^2 \ll E_\mathscr{C}$ in any practicable SNR regime, we will often use these expressions for analysis and thresholding in place of the full expression (25).

To obtain the best performance in IC we marginalize with respect to whichever of $\mathbf{s}$ or $\mathbf{c}$ has the smallest $\det(\Sigma)$. The MAP estimate of $\mathbf{s}$ is

$$\hat{\mathbf{s}} = \arg\min_{s\in\mathscr{C}_m} (\mathbf{s}-\mu_\mathbf{s})\Sigma_\mathbf{s}^{-1}(\mathbf{s}-\mu_\mathbf{s})^\dagger \quad (29)$$

Assuming that the symbol $\mathbf{s}$ (or $\mathbf{c}$) has been decoded correctly, the receiver then subtracts the contribution of $\mathbf{s}$ (or $\mathbf{c}$) from the received signal vector and estimates $\mathbf{c}$ (or $\mathbf{s}$):

$$\hat{\mathbf{c}} = \arg\min_{c\in\mathscr{C}} (\mathbf{r}-(\hat{\mathbf{s}},\mathbf{c})A)(\mathbf{r}-(\hat{\mathbf{s}},\mathbf{c})A)^\dagger \quad (30)$$

The performance for decoding $\mathbf{s}$ and $\mathbf{c}$ is determined by $\det(\Sigma_\mathbf{s})$ if we marginalize with respect to $\mathbf{s}$, and by $\det(\Sigma_\mathbf{c})$ if we marginalize with respect to $\mathbf{s}$.

It is clear that the performance of MAP/ML degrades when the channel approaches being rank deficient, that is, as $V \to 0$. In this situation the computational complexity of sphere decoding increases dramatically since the initial point becomes a plane and lattice point search resembles exhaustive search.

We are less interested in the absolute performance of IC, than in its performance relative to MAP/ML. The intuition

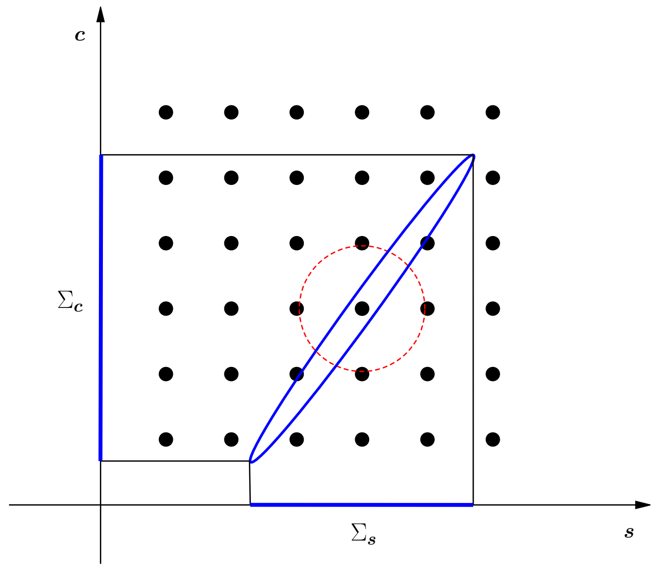

Figure 1: Geometric representation of IC perfomance.

is represented in Figure 1, where the ellipse represents the 1σ ellipsoid of the full posterior covariance. This ellipsoid becomes progressively more eccentric as the condition number of the covariance increases. The circle represents the size of the sphere which contains the same volume total volume as the posterior distribution ellipsoid. This sphere has radius $2\sigma^2/\sqrt{5}V$. The covariance of the marginal distributions for $\mathbf{s}$ and $\mathbf{c}$ are represented by the projections of the ellipsoid on to the $\mathbf{s}$ and $\mathbf{c}$ axes. If both the condition number of the posterior covariance is large and its major axis is approximately equiangular with the $\mathbf{s}=0$ and $\mathbf{c}=0$ planes, then $\det(\Sigma_\mathbf{s})$ and $\det(\Sigma_\mathbf{c})$ will both be large, hence IC will perform poorly compared to MAP/ML. Thus, the performance of IC with respect to MAP/ML can be captured by the dimensionless quantity

$$\Delta = \min\{\det(\Sigma_\mathbf{s})/(2\sigma^2/\sqrt{5}V)^2, \det(\Sigma_\mathbf{c})/(2\sigma^2/\sqrt{5}V)^2\} \quad (31)$$

We classify channels at the receiver in terms of the expected performance of IC. Figure 2 shows the relative performance of IC on channels with $\Delta \le \gamma$, where γ is a given threshold value. We refer to these as good channels. For the 4-QAM results displayed in Figures 2 and 4, we took $\gamma = 5$. The simulations show that there are certain channels, which we refer to as bad, for which a direct application of IC performs very poorly. If we remove these channels the performance of IC improves substantially, as demonstrated in Figure 2.

If the channel is good we apply IC, and if the channel is bad we must find some alternative decoding scheme (cf. Maurer et.al [10] and Artés et.al [11] who ignore orientation and distinguish good from bad channel based only on the condition number of the covariance). To this end, we now analyze the full posterior distribution when the channel is bad.

Consider the channel matrix A from equation (19). The matrix A has the singular value decomposition

$$A = UDW^\dagger. \quad (32)$$

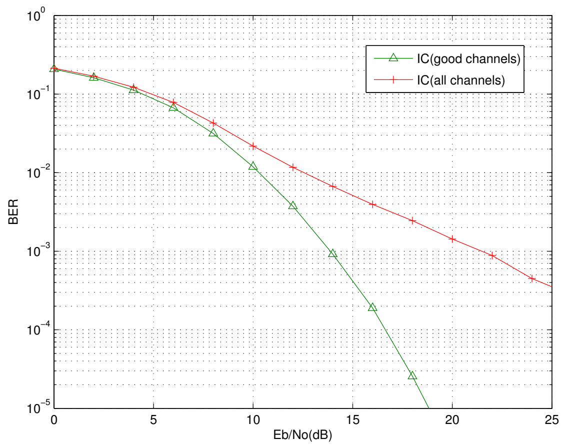

Figure 2: Performance of IC under good channels ($\Delta \leq \gamma$) and all channels conditions.

The columns of U are the eigenvectors of $AA^\dagger$ which occur in pairs of the form $\begin{pmatrix} u \\ \tilde{u} \end{pmatrix}$ where u is a 2×2 cyclotomic matrix. The columns of W are the eigenvectors of $A^\dagger A$ and $D = \text{diag}(\lambda_1, \lambda_1, \lambda_2, \lambda_2)$ where λ_1 and λ_2 are the eigenvalues of $AA^\dagger$. Both eigenvalues have multiplicity 2. Given $H = \begin{pmatrix} h \\ \tilde{h} \end{pmatrix}$, define

$$\|H\|_\tau^2 = h^\dagger h + \tilde{h}^\dagger \tilde{h}, \tag{33}$$

and the corresponding inner product by

$$(H,G)_\tau = h^\dagger g + \tilde{h}^\dagger \tilde{g}. \tag{34}$$

Then we have

$$\lambda_1, \lambda_2 = \frac{1}{2}(\|H\|_\tau^2 + \|G\|_\tau^2)\left(1 \pm \sqrt{1-4\Lambda}\right), \tag{35}$$

where

$$\Lambda = \frac{5V^2}{(\|H\|_\tau^2 + \|G\|_\tau^2)^2} \tag{36}$$

and corresponding eigenvectors

$$\mathbf{w}_j \propto \begin{pmatrix} -(H,G)_\tau \\ \|H\|_\tau^2 - \lambda_j \end{pmatrix} \quad j = 1,2. \tag{37}$$

Substituting (32) into (12), we then obtain

$$(\mathbf{r}_1, \mathbf{r}_2)W = (\mathbf{s}, \mathbf{c})UD + (\mathbf{n}_1, \mathbf{n}_2)W \tag{38}$$

It follows from (35) that for a given SNR, the smaller eigenvalue $\lambda_2 \to 0$ as the channel volume $V \to 0$. Retaining only the larger eigenvalue λ_1 and its corresponding eigenvectors $\mathbf{w}_1 = (w_1, w_2)^T$ and $\mathbf{u}_1 = (u_1, \tilde{u}_1)^T$ leads to

$$\mathbf{r}' = \mathbf{s}u_1 + \mathbf{c}\tilde{u}_1 + \mathbf{n}', \tag{39}$$

where

$$\mathbf{r}' = \mathbf{r}_1 w_1 + \mathbf{r}_2 w_2, \quad \mathbf{n}' = \mathbf{n}_1 w_1 + \mathbf{n}_2 w_2, \tag{40}$$

and

$$\mathbf{u}_1 = \begin{pmatrix} u_1 \\ \tilde{u}_1 \end{pmatrix} = \frac{1}{\lambda_1}\begin{pmatrix} hw_1 + gw_2 \\ \tilde{h}w_1 + \tilde{g}w_2 \end{pmatrix}. \tag{41}$$

Equivalently, (39) can be written in terms of cyclotomic numbers:

$$r' = h's + \tilde{h}'c + n' \tag{42}$$

where

$$r' = \begin{pmatrix} r'_{11} & r'_{12} \\ ir'_{12} & r'_{11} \end{pmatrix} \tag{43}$$

and

$$h' = (hw_1 + gw_2)/\lambda_1, \quad \tilde{h}' = (\tilde{h}w_1 + \tilde{g}w_2)/\lambda_1. \tag{44}$$

As $V \to 0$, the return from the virtual channel corresponding to the smaller of the two eigenvalues contains almost no information. Thus, we can drop this channel and (12) is reduced to (42), which is equivalent to a single receive antenna system with virtual channel h'.

In [8] we give an $O(N^2)$ algorithm for decoding the Golden code with a single receive antenna. Channel state information is used to select between two zero-forcing equalizers inverting h' or $\tilde{h}'$. The selection is made to maximize effective SNR and the underlying Diophantine geometry guarantees that at least one of the choices is good. Simulation results for the Golden Code show performance within 1 dB of full MAP/ML decoding.

4. THE COGNITIVE DECODER

We have observed that the poor performance of IC is due to a small proportion of channels for which the associated covariance matrix $AA^\dagger$ is both close to rank deficient and oriented in such a way that the marginal covariances for $\mathbf{s}$ and $\mathbf{c}$ are both large . The performance of IC relative to MAP/ML is measured by the signal-to-noise ratio independent quantity

$$\Delta = \min\left\{\det(\Sigma_\mathbf{s})/(2\sigma^2/\sqrt{5}V)^2, \det(\Sigma_\mathbf{c})/(2\sigma^2/\sqrt{5}V)^2\right\}$$

given in (31).

We propose the following decoding algorithm, where γ is a threshold parameter to be chosen:

1. If $\Delta \leq \gamma$ apply interference cancellation to the decoding problem (12).
2. If $\Delta > \gamma$ apply the single receiver quadratic decoder [8] to the virtual single receiver decoding problem (39).

That is, when the channel is good, we apply IC and when channel is bad we fall back to single antenna decoding (39).

We now consider the choice of threshold value in the algorithm. Figure 3 displays the perfomance of cognitive decoding for 4-QAM as a function of the threshold parameter γ. We observe that the best performance for all the SNR was achieved at $\gamma = 5$, ($\approx 7dB$). Thus, both Δ and the threshold parameter are independent of SNR, and depend only on the "shape" and "orientation" of the ellipsoid associated with posterior convariance and not on its size.

We compared the performance of the cognitive decoder with the MAP/ML decoder in simulation, assuming the channel is known at the receiver. The SNR at a receive antenna is defined as

$$\text{SNR(dB)} = 10\log_{10}\left(\frac{P_b}{2\sigma^2}\right), \tag{45}$$

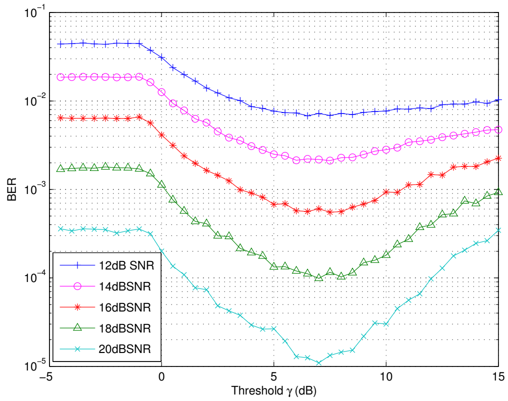

Figure 3: Performance against threshold values.

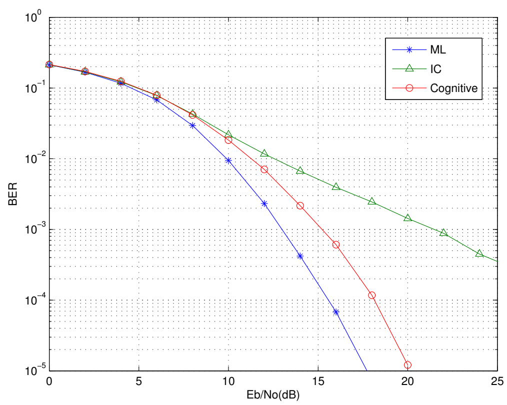

Figure 4: Performance comparison between MAP/ML decoding, IC and cognitive decoding for two receive antennas.

where P_b is the (average) signal power per bit at each receive antenna which is defined as

$$P_b = E_b(\|h\|^2 + \|\tilde{h}\|^2 + \|g\|^2 + \|\tilde{g}\|^2)/2. \quad (46)$$

where E_b is the average energy per bit. Figure 4 and 5 show the performance of the cognitive decoder comes within 3dB of full MAP/ML for Rayleigh channel and 1dB of full MAP/ML for pure line of sight respectively.

5. CONCLUSION

We presented a single cognitive decoder for the Golden code with complexity $O(N^2)$ that comes within 3dB of full MAP/ML decoding on Rayleigh channels and reduces to essentially ML performance for pure line of sight channels. The cognitive decoder uses channel state information to choose between two algorithms, each with complexity $O(N^2)$, and this choice is independent of the signal-to-noise ratio.

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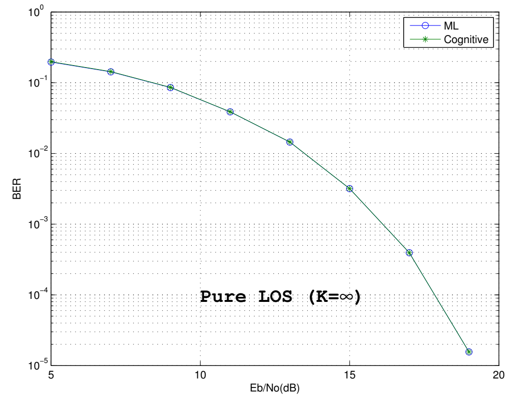

Figure 5: Performance comparison between MAP/ML and cognitive decoding for a pure LOS (Rician K-factor $K = \infty$) channel.

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