

# NEW NARROWBAND ACTIVE NOISE CONTROL SYSTEMS REQUIRING CONSIDERABLY LESS MULTIPLICATIONS

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## ABSTRACT

In a conventional narrowband active noise control system, a two-weight FIR-type magnitude/phase adjuster (MPA) is used as an adaptive controller in each frequency channel, and is updated by the so-called filtered-x LMS (FXLMS) algorithm. Each reference cosine wave has to be filtered by an estimated secondary-path with order  $\hat{M}$  before it is fed to the FXLMS. We call this part *x-filtering* block. The number of x-filtering blocks is equal to the number of targeted frequencies ( $q$ ). When  $q$  and  $\hat{M}$  become larger, the computational cost of the system due to x-filtering operations may form a bottleneck in implementation. In this paper, we propose a new narrowband ANC system structure which requires only one x-filtering block regardless of  $q$ . All the cosine waves are combined as an input to a x-filtering block whose output is decomposed by an efficient bandpass filter bank into filtered cosine waves for the FXLMS that follows. As a result, the computational cost of the system may be considerably reduced. The new structure is also implanted in a recently developed ANC system that is capable of mitigating the frequency mismatch (FM). Simulations demonstrate that the new systems present performance very similar to that of their counterparts, but enjoy considerable advantages in implementation.

## 1. INTRODUCTION

Noisy sinusoidal signals generated by rotating machines, such as diesel engines, motors, fans, factory cutting machines, etc. may be effectively reduced by narrowband active noise control (ANC) systems, especially the lower frequency portion. For example, high-power cutting machines used in factories generate such noise signals which are harmful to their operators. Narrowband active noise control (ANC) systems have been utilized in reducing these annoying noise signals.

As is well-known, research and development in the ANC area has been carried out since the early 1970s, and many promising system structures and adaptive algorithms have been developed, see [1]-[8] and the references therein. The finite-impulse-response (FIR) filters adapted by a filtered-x least mean square (FXLMS)

algorithm are usually used in the ANC systems [3]. Other techniques using recursive least squares (RLS) and Kalman filtering based algorithms have also been developed for the FIR-type ANC systems [3],[6], which generally provide better noise reduction performance at the expense of more computational cost.

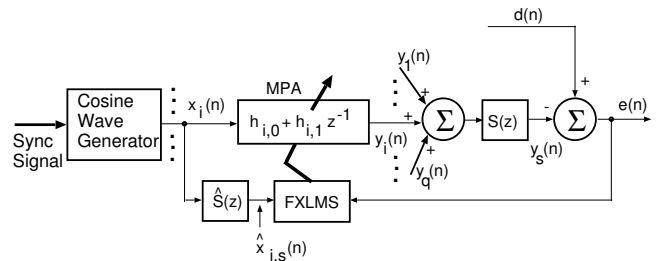


Fig. 1 A typical conventional narrowband ANC system (the  $i$ -th channel).

Several conventional narrowband ANC systems have been found effective in suppressing sinusoidal noise signals in different scenarios for many real-life applications [3]. Fig. 1 depicts a block diagram of such a typical parallel form ANC system that uses a two-weight FIR-type magnitude/phase adjuster (MPA) as controller in each frequency channel. The FXLMS algorithm has been used to adapt all the parallel channels simultaneously. In each frequency channel, one (1) x-filtering block ( $\hat{S}(z)$ ) is required. If  $q$  becomes large, the number of x-filtering blocks will increase. Because the order  $\hat{M}$  of the estimated FIR-type secondary-path ( $\hat{S}(z)$ ) can be as large as 128 or even higher in practice, the complexity due to the x-filtering blocks may become a real burden and bottleneck in system implementation. This has motivated us to explore new system structures that have less computational requirements but enjoy performance similar to that of the original system in Fig. 1.

A new narrowband ANC system structure is proposed that requires only a single x-filtering block regardless of the number of frequencies ( $q$ ) being targeted. The computational cost of the new system is significantly re-

duced particularly for large  $q$  and/or  $\hat{M}$ . In the new system, all the reference cosine waves are added together as an input to a x-filtering block whose output is decomposed into separated filtered cosine waves for the FXLMS by a bandpass filter bank. The cells of the filter bank are bandpass filters derived from IIR notch filters with constrained poles and zeros [9].

The new structure is also transplanted to a recently developed ANC system capable of mitigating the frequency mismatch (FM) that might exist in real-life applications due to aging and fatigue [8, 10] of the reference non-acoustic sensor such as tachometer.

Extensive simulations are provided to demonstrate that the new systems present performance very similar or identical to that of the conventional system, but enjoy great cost merit in system implementation.

## 2. NEW NARROWBAND ANC SYSTEMS

### 2.1 A typical conventional system

The primary noise signal in the typical conventional narrowband ANC system shown in Fig. 1 is given by

$$d(n) = \sum_{i=1}^q \{a_{p,i} \cos(\omega_{p,i}n) + b_{p,i} \sin(\omega_{p,i}n)\} + v(n) \quad (1)$$

where  $q$  is the number of frequency components of  $d(n)$ ,  $\omega_{p,i}$  is the (angular) frequency of the  $i$ -th component,  $v(n)$  is a zero-mean additive white Gaussian noise with variance  $\sigma_v^2$ . The  $i$ -th reference cosine wave is created by a piece of hardware called cosine wave generator, and may be expressed by

$$x_i(n) = a_{r,i} \cos(\omega_i n), \quad a_{r,i} \neq 0 \quad (2)$$

where frequency  $\omega_i$  is derived from the synchronization signal in a regression fashion [3]. Speed sensor such as tachometer is usually used as the non-acoustic reference sensor. If the sensor works perfectly,  $\omega_{p,i}$  will be exactly the same as  $\omega_i$ , and no FM exists. The output of the  $i$ -th channel is calculated as

$$y_i(n) = h_{i,0}(n)x_i(n) + h_{i,1}(n)x_i(n-1) \quad (3)$$

The block  $S(z)$  is the secondary-path or error-path and is modeled as an FIR filter with coefficients  $\{s_j\}_{j=0}^{M-1}$ , while its estimate  $\hat{S}(z)$  ( $\{\hat{s}_j\}_{j=0}^{M-1}$ ) is assumed to be known *a priori* or acquired in some way in advance. The FXLMS algorithm is utilized to update the two FIR weights in each frequency channel as follows [3].

$$h_{i,0}(n+1) = h_{i,0}(n) + \mu_i e(n) \hat{x}_{i,s}(n), \quad (4)$$

$$h_{i,1}(n+1) = h_{i,1}(n) + \mu_i e(n) \hat{x}_{i,s}(n-1) \quad (5)$$

where  $\mu_i$  is a positive step size parameter, and

$$\hat{x}_{i,s}(n) = \sum_{j=0}^{\hat{M}-1} \hat{s}_j x_i(n-j), \quad i = 1, 2, \dots, q \quad (6)$$

$$\begin{aligned} e(n) &= d(n) - y_s(n) \\ &= d(n) - \sum_{j=0}^{M-1} s_j \left\{ \sum_{i=1}^q y_i(n-j) \right\} \end{aligned} \quad (7)$$

Here, one x-filtering block ( $\hat{S}(z)$ ) must be placed in each channel. As the number of frequencies targeted gets larger, the number of x-filtering blocks will increase and the computational cost involved may become a serious cost issue in system implementation.

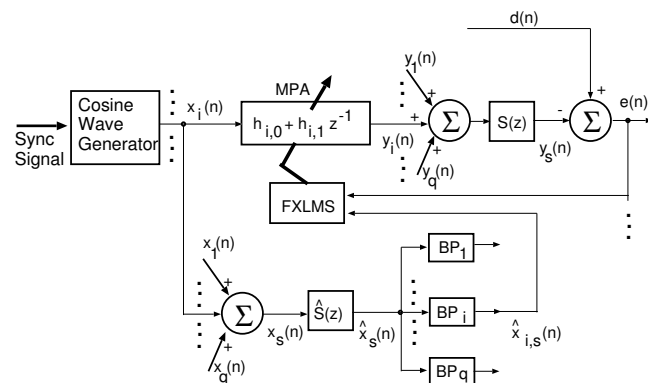


Fig. 2 A new narrowband ANC system structure (the  $i$ -th channel).

### 2.2 A new system structure

To reduce the number of x-filtering blocks, we propose a new idea; first filter all the cosine waves by a single x-filtering block and then decompose the block output into separated filtered cosine waves by a proper bandpass filter bank. This idea is implemented in a new system structure shown in Fig. 2, where

$$x_s(n) = \sum_{i=1}^q x_i(n), \quad \hat{x}_s(n) = \sum_{j=0}^{\hat{M}-1} \hat{s}_j x_s(n-j) \quad (8)$$

The transfer function of  $i$ -th cell of the bandpass filter bank is given by

$$H_{bp_i}(z) = \frac{(\rho-1)c_i z^{-1} + (\rho^2-1)z^{-2}}{1 + \rho c_i z^{-1} + \rho^2 z^{-2}} \quad (9)$$

where the frequency-dependent filter coefficient  $c_i$  is defined by

$$c_i = -2 \cos(\omega_i) \quad (10)$$

and  $\rho$  is a pole attraction factor (or pole radius) over  $(0, 1)$ . This bandpass filter is derived from an IIR notch filter with constrained poles and zeros [9]. The filtered signals  $\hat{x}_{i,s}(n)$  for FXLMS that follows is the output of

**Table 1** Comparison of complexity between the conventional and proposed systems shown in Figs. 1 and 2 ( blk: number of x-filtering blocks, mult: number of multiplications).

		Conventional		Proposed	
		blk	mult	blk	mult
$q$	$\hat{M}$	$q$	$q\hat{M}$	1	$\hat{M} + 4q$
3	41	3	123	1	53
3	64	3	192	1	76
3	128	3	384	1	140
10	41	10	410	1	81
10	64	10	640	1	104
10	128	10	1280	1	168

the above bandpass filter.

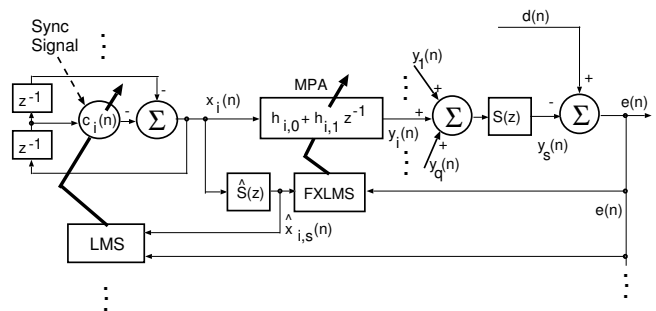
$$\hat{x}_{i,s}(n) = -\rho c_i \hat{x}_{i,s}(n-1) - \rho^2 \hat{x}_{i,s}(n-1) + (\rho-1)c_i \hat{x}_s(n-1) + (\rho^2-1)\hat{x}_s(n-2), \quad (11)$$

where only four (4) additional multiplications are involved. It is not difficult to prove that  $H_{bpi}(e^{j\nu\omega_i}) = 1$  ( $\nu = \sqrt{-1}$ ) for any  $\rho$ , which implies that the proposed system will have properties very similar to those of the conventional system.  $\rho$  may be chosen according to the spacing of the signal frequencies considered. If they are closely spaced, a  $\rho$  very close to unit may be selected to allow the bandpass filters to produce clean reference waves for the FXLMS. However, this may bring some delay to the system's dynamics, because the bandpass filter with larger  $\rho$  has a longer time constant. Note that this delay does not directly contribute to the dynamics of the system and thus will not be so severe. See simulation results in next Section for the resultant delay that is actually quite small even for a  $\rho$  very close to unit. But this delay of convergence is truly the only sacrifice we have to make, even though it may be not so painful and may be tolerated in practice.

The number of x-filtering blocks and the corresponding multiplications is compared in Table 1 for the new and the conventional systems. The computational merit of the new system is obvious and significant. For example, if  $q = 10$  and  $\hat{M} = 128$ , the number of multiplications of the new system reduces approximately to only 13% of the conventional structure.

### 2.3 A modified robust system in presence of FM

When there is a frequency mismatch (FM) between the reference waves and the primary noise due to the non-acoustic sensor aging and fatigue, the performance of conventional system will degrade severely [8, 10]. To make the system robust to the existence of FM, we have recently proposed an ANC system shown in Fig. 3,

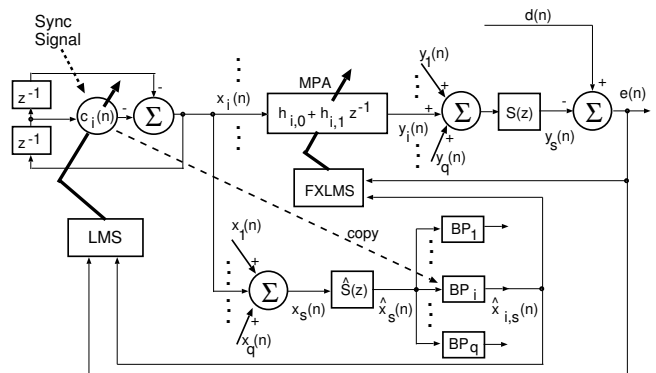


**Fig. 3** A robust narrowband ANC system in the presence of FM, recently developed in [8] ( $i$ -th channel).

where the frequency-dependent coefficient  $c_i(n)$  is updated by an LMS-like algorithm as follows [8]

$$c_i(n+1) = c_i(n) - \mu_{c_i} e(n) h_{i,0}(n) \hat{x}_{i,s}(n-1) \quad (12)$$

where  $\mu_{c_i}$  is a positive step size parameter, and this update implements the minimization of the residual noise (error) signal power ( $e^2(n)$ ). The idea used in the new system of Fig. 2 is implanted in the system of Fig. 3 to obtain a modified system shown in Fig. 4 which is not only computationally efficient but also robust to the FM. The computational advantage of the modified system over that shown in Fig. 3 is still preserved to the same extent. Only two (2) more additional multiplications for each bandpass filter are required as  $c_i$  is now adaptively updated (see (11) for the number of multiplications involved in a bandpass filter).



**Fig. 4** A modified robust narrowband ANC system in the presence of FM ( $i$ -th channel).

## 3. SIMULATIONS

It has been made clear that the two proposed systems have significant computational advantages over the con-

ventional [3] and a recently developed system [8], and are anticipated to present performance very similar to that of their counterparts. Extensive simulations have been performed. Some representative simulation results are provided below to peek at the performance of the new systems.

Figs. 5 and 6 present comparisons between the conventional system in Fig. 1 and the new system in Fig. 2 for both short and long secondary-path. The step size parameters of the FXLMS in both systems were carefully adjusted such that their residual noise signals have the same amount of power at their steady states for the sake of fair comparisons. In both figures, the order of the estimated secondary-path ( $\hat{M}$ ) was set significantly lower than that of the true secondary-path ( $M$ ). This is because we prefer short  $\hat{S}(z)$  in real applications. Obviously both systems indicate very similar dynamic behaviors and almost identical steady-state residual noise power. The convergence of the new system is delayed a little bit. This is the only sacrifice we have to make. We add here that scenarios where  $\hat{M} = M$  or  $\hat{M} > M$  were also extensively simulated and the same observations were obtained.

Comparisons between the systems shown in Figs. 1-4 are provided in Figs. 7 and 8. Fig. 7(a) and (b) present the residual noise signals produced by systems in Figs. 1 and 2, respectively. Fig. 7(c) and (d) show the residual noise signals generated by systems in Figs. 3 and 4. All of the four (4) systems were simulated in the presence of an FM of 1%. Fig. 8 provides the simulated results for a longer secondary-path. From these simulation results, the proposed and modified systems work almost the same as their counterparts do. Systems in Figs. 1 and 2 failed completely due to an FM of 1%. However, the modified systems of Figs. 3 and 4 have the capabilities of mitigating the influence of the FM very effectively. It should be noted that when the FM is larger than 1% (up to 10% [8]) and the secondary path is longer one has to let the proposed and modified systems in Figs. 2 and 4 to adapt much slowly compared with the convergence in Figs. 7 and 8. However, we have reached quite similar conclusions from extensive simulations.

#### 4. CONCLUSIONS

In this paper, two narrowband ANC systems with new structures have been proposed, which provide performance very similar to that of the conventional and a recently established systems, while requiring considerably less computational cost. Extensive simulations have been conducted to demonstrate that the proposed systems are very promising even in the presence of FM. DSP-based implementation of the proposed systems, their detailed performance analysis and application to

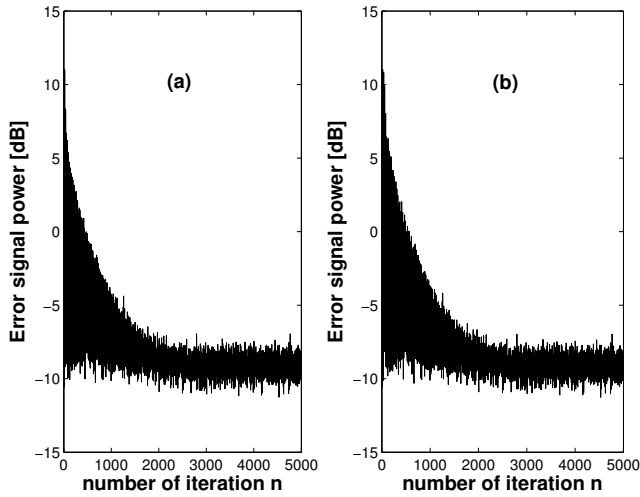
real-life systems are future research topics. Applying the new idea used in this work to other existing narrowband ANC systems is also a future topic.

#### Acknowledgements:

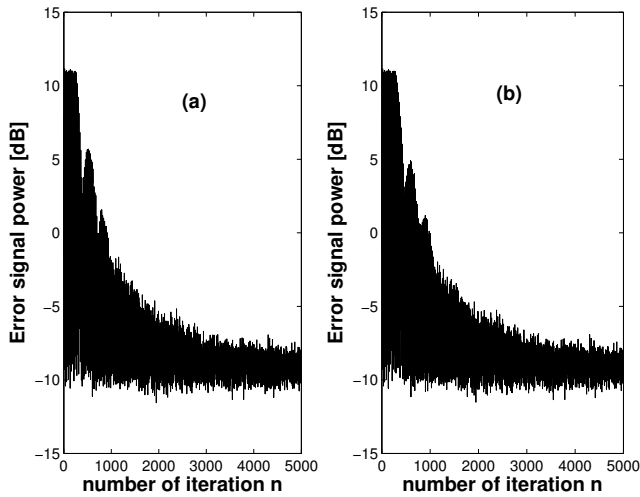
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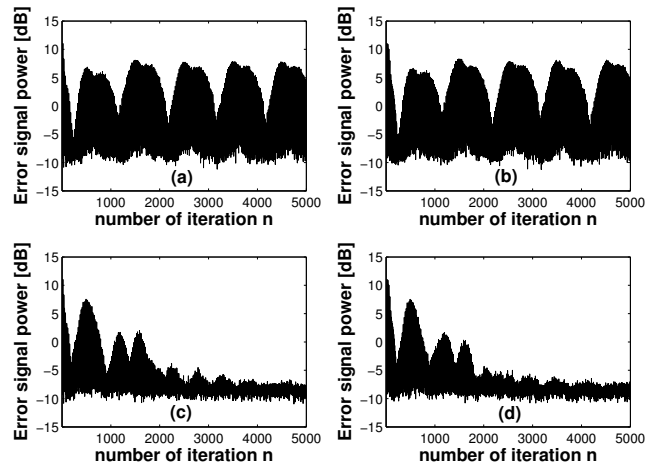
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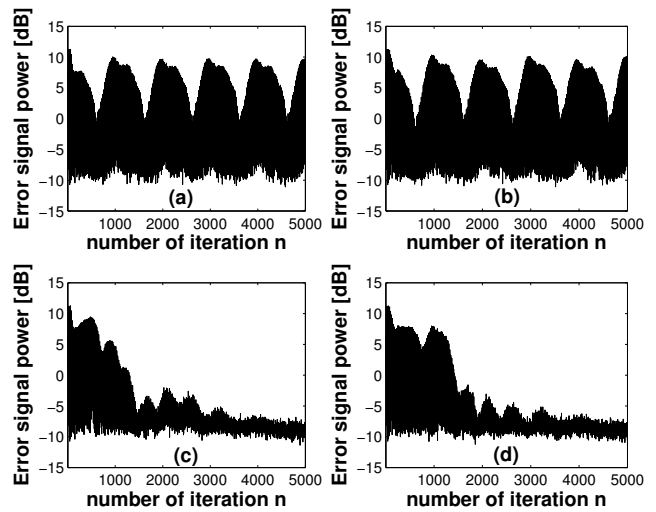
**Fig. 5** Comparisons between systems in Figs. 1 [(a)] and 2 [(b)]. Simulation conditions: signal frequencies =  $0.10\pi$ ,  $0.20\pi$ ,  $0.30\pi$ ;  $a_1 = 2.0$ ,  $b_1 = -1.0$ ,  $a_2 = 1.0$ ,  $b_2 = -0.5$ ,  $a_3 = 0.5$ ,  $b_3 = 0.1$ ;  $\mu_1 = \mu_2 = \mu_3 = 0.05$ ;  $\sigma_v = 0.33$ ,  $M = 32$ ,  $\hat{M} = 21$ ,  $\rho = 0.985$ , 100 runs.



**Fig. 6** Comparisons between systems in Figs. 1 [(a)] and 2 [(b)]. Simulation conditions: signal frequencies =  $0.10\pi$ ,  $0.20\pi$ ,  $0.30\pi$ ;  $a_1 = 2.0$ ,  $b_1 = -1.0$ ,  $a_2 = 1.0$ ,  $b_2 = -0.5$ ,  $a_3 = 0.5$ ,  $b_3 = 0.1$ ;  $\mu_1 = \mu_2 = \mu_3 = 0.015$ ;  $\sigma_v = 0.33$ ,  $M = 256$ ,  $\hat{M} = 128$ ,  $\rho = 0.985$ , 100 runs.



**Fig. 7** Performance comparisons among systems in Figs. 1-4 [(a), (b), (c), (d)] that were all simulated in presence of FM. Simulation conditions: frequency mismatch ( $\Delta\omega_i = \frac{\omega_{p,i} - \omega_i}{\omega_i} \times 100\%$ ) is 1% for all the frequencies targeted;  $\mu_{c1} = 0.00005$ ,  $\mu_{c2} = 0.0001$ ,  $\mu_{c3} = 0.00015$ ;  $M = 32$ ,  $\hat{M} = 21$ ,  $\rho = 0.985$ , other simulation conditions the same as in Fig.5.



**Fig. 8** Performance comparisons among systems in Figs. 1-4 [(a), (b), (c), (d)] that were all simulated in presence of FM. Simulation conditions: frequency mismatch ( $\Delta\omega_i = \frac{\omega_{p,i} - \omega_i}{\omega_i} \times 100\%$ ) is 1% for all the frequencies targeted;  $\mu_{c1} = 0.00005$ ,  $\mu_{c2} = 0.0001$ ,  $\mu_{c3} = 0.00015$ ;  $M = 64$ ,  $\hat{M} = 41$ ,  $\rho = 0.985$ , other simulation conditions the same as in Fig.5.