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REASSIGNED THREE-DIMENSIONAL PHASE SPECTROGRAM AND GROUND REACTION FORCES.

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ABSTRACT

The reassigned 3-dimensional phase spectrogram (R3DPS) is presented in this paper as a good candidate to exhibit instantaneous phase. It consists in coupling the reassigned power spectrogram (RS) and the phase of the short time Fourier transform (STFT). The resulting representation allows a direct observation of small phase variations in a time-frequency plane facing a phase reference. As a result, the instantaneous phase of each spectral component of a multicomponent signal can be estimated. The R3DPS allows quasi-stationary signals to be analysed. The herein objective is to detect small phase variations without loosing the energetic information given by the power spectrogram. To illustrate this method, the application to the analysis of a ground reaction forces signal is realized in order to characterize the human gait behaviour.

1. INTRODUCTION

Time-frequency representations are well suited to the analysis of non-stationary signals [1]. However, these time-frequency methods are not efficient for the detection of phase modulations that can occur in quasi-stationnary-signals. At this step, it seems interesting to consider the phase content of the STFT [2]. In 2000, F. Leonard [3] presented a two-dimensional discrete method named the Phase Spectrogram (PS) which is a first step to the time-frequency-phase representations. In 2007, a 3-dimensional phase spectrogram representation has been investigated and applied to musical signals [4].

This paper deals with the reassigned 3-Dimensional Phase Spectrogram which consists in coupling the reassigned power spectrogram and the phase of the short time Fourier transform (STFT). Let us recall that the STFT for a signal x(t) is expressed as:

$$STFT_x(t,f) = \int_{-\infty}^{+\infty} x(\tau)\, h^*(\tau - t)\, e^{-i2\pi f \tau} d\tau \qquad (1)$$

where $h(t)$ is a window function ($*$ denotes the conjugate symbol), t is the time, f is the frequency and $x(t)$ is the signal to be transformed.

Then, the power spectrogram (S) is defined by:

$$S_x(t,f) = |STFT_x(t,f)|^2. \qquad (2)$$

The Reassigned 3-Dimensional Phase Spectrogram main objective is to quantify small phase variations in a time-frequency plane facing a zero phase reference.

The first part of this paper presents the Phase Spectrogram (PS) and its 3D version (3DPS). It is shown that the Phase Spectrogram (PS) is built from the Frequency Spectrogram (FS). In a second part, the Reassigned 3D Phase Spectrogram is introduced. The third part is dedicated to the study of a synthetic signal in order to well understand the concept of Phase Spectrogram. The powerfulness of the R3DPS is highlighted by its application to the analysis of ground reaction forces signals, showing that the instantaneous phase estimation is accessible.

2. PHASE SPECTROGRAM

2.1 Frequency and Phase Spectrograms

The FS and PS have been recently introduced by F. Leonard [3]. In a complementary way, a continuous formulation for both PS and FS representations is proposed. The FS is presented first; since the PS construction is based on it.

The first step of the method is the calculation of the STFT [5]. The result is a complex-valued function allowing the representation of both the phase and magnitude parts of the signal over time and frequency. In order to construct the FS, only the phase of the STFT is taken into account and expressed as:

$$\phi^w_{STFT_x}(t,f) = Atan\left(\frac{Imag(STFT_x(t,f))}{Real(STFT_x(t,f))}\right) \qquad (3)$$

where ^{w} denotes the wrapped version of the phase of the STFT.

Then, the FS principle consists in the time-derivative of the unwrapped phase of the STFT (the unwrapping operation is denoted Uw):

$$\phi'_{STFT_x}(t,f) = d\phi_{STFT_x}(t,f)/dt \qquad (4)$$

where:

$$\phi_{STFT_x}(t,f) = Uw(\phi^w_{STFT_x}(t,f)) \qquad (5)$$

To control the amplitude of the phase derivative in the time-frequency plane for a given frequency modulation signal, (4) is multiplied by a gain parameter denoted P. Then, to adjust values in the $[-\pi,\pi]$ range, a wrapping operation is performed over the unit circle. In order to normalize the frequency, the result is multiplied by $1/2\pi P$. So, the FS can be expressed as:

$$FS_{x,P}(t,f) = \frac{1}{2\pi.P} Wr(Pd\phi_{STFT_x}(t,f)/dt) \qquad (6)$$

where Wr corresponds to the phase-wrapping operation [3]. Finally, a thresholding of the FS is performed conditionally to the power spectrogram energy in order to only keep and show the information for the relevant magnitudes.

In a second step, the PS is computed by integration starting from the FS. It is based on the integral relation between angular position and rotation frequency. First, the FS is indexed on a reference time denoted t_r and then unwrapped:

$$FS_{x,P,t_r}(t,f) = \frac{1}{2\pi.P}\left[Uw\left(Wr(P\phi'(t,f))\right) - Uw\left(Wr(P\phi'(t_r,f))\right)\right] \quad (7)$$

Consequently, the construction of the PS consists in the integration of (7) over the [0,t] time range:

$$\Phi_{x,P,t_r}(t,f) = 2\pi \int_0^t FS_{x,P,t_r}(\tau,f)d\tau \quad (8)$$

then, in order to clarify this expression, the "wrapping-unwrapping" operation can be simplified and by combining equations (7) and (8) it yields:

$$\Phi_{x,P,t_r}(t,f) = \frac{1}{P}\left[\int_0^t P\phi'(\tau,f)d\tau - K(f)t\right] \quad (9)$$

with:

$$K(f) = P\phi'(t_r,f) \quad (10)$$

The PS is expressed as a result of the subtraction of (9) and itself at time t_r in order to create a zero-phase reference:

$$\begin{aligned} PS_{x,P,t_r}(t,f) &= \Phi_{x,P,t_r}(t,f) - \Phi_{x,P,t_r}(t_r,f) \quad (11)\\ &= \frac{1}{P}\left[\int_0^t P\phi'(\tau,f)d\tau - K(f)t \right.\\ &\quad \left. - \int_0^t P\phi'(t_r,f)d\tau + K(f)t_r\right]\\ &= \frac{1}{P}\left[\int_0^t P\phi'(\tau,f)d\tau - 2K(f)t + K(f)t_r\right] \quad (12)\end{aligned}$$

Finally, a wrapping operation is performed in order to adjust the phase in the $[-\pi,\pi]$ range:

$$PS_{x,P,t_r}(t,f) = \frac{1}{P}Wr(\left[\int_0^t P\phi'(\tau,f)d\tau - 2K(f)t + K(f)t_r\right]) \quad (13)$$

2.2 3D Phase Spectrogram (3DPS)

It has been shown that a thresholding must be performed in order to keep only the information contained for the relevant magnitudes of the power spectrogram [3]. The 3DPS is based on the same calculus than the PS, where a thresholding is optional (Figures 1 and 4). The principle is to map the PS onto the power spectrogram. The result is a 3-dimensional time-frequency-phase representation of the signal which appears to be a time-frequency-phase surface [4]. As a comparison, quadratic time-frequency representations analyse time-frequency trajectories when this new representation makes possible to analyse geodesic trajectories of the instantaneous phase that can be computed with the Bresenam's algorithm [6] for instance.

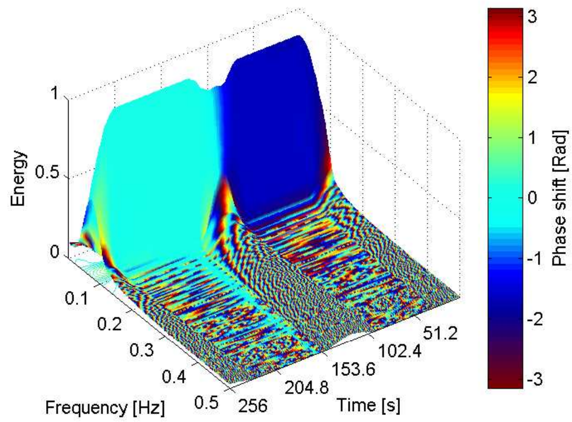

Figure 1: No-thresholded 3D phase spectrogram of a sinusoidal synthetic signal containing a 1.7 Radians phase-shift (zero phase reference: $t_r = 160s$).

3. REASSIGNED 3D PHASE SPECTROGRAM

A classical reassignment method [7] has been used to sharpen the spectrogram in order to make it more readable to measure the sinusoidality. In this paper the method of reassignment is used to improve the time and the frequency estimates. Thereby, it enhances the time-frequency resolution of the phase spectrogram representation. The principle of the reassignment is to move each given time-frequency point (t,f) to a position (t',f') seen as the energy gravity center for a selected region of interest. In our case the more interesting property concerns the fact that the reassignment vectors $(\widehat{t},\widehat{f})$ can be written by taking into account the phase of the short time Fourier transform ϕ_{STFTx}. Therefore:

$$\widehat{t}(t,f;x) = -d\phi_{STFT_x}(t,f;h)/df \quad (14)$$

and:

$$\widehat{f}(t,f;x) = f + d\phi_{STFT_x}(t,f;h)/dt. \quad (15)$$

It follows the reassigned power spectrogram (Sr) expression:

$$Sr_x(t',f';h) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} S_x(t,f;h)\delta(t'-\widehat{t}(t,f;x))\delta(f'-\widehat{f}(t,f;x))dtdf \quad (16)$$

with $\delta(.)$ denoting the Dirac distribution.

As previously, the Reassigned 3D Phase Spectrogram (R3DPS) is built by mapping the PS on the reassigned power spectrogram. The herein objective when using Reassigned 3D Phase Spectrogram is to exibit an estimation of the instantaneous phase. To focus on this time-frequency-phase representation purpose, two signals have been selected. The first one concerns a synthetic signal in which occurs a known phase-shift. The second one is a Ground Reaction Forces signal where an accurate study of spectral components is crucial.

4. APPLICATION TO A SYNTHETIC SIGNAL AND GROUND REACTION FORCES

In a first step, a sinusoidal synthetic signal containing a 1.7 Radians phase-shift (Figure 2) has been generated in order to exhibit the reassigned 3D phase spectrogram (Figure 4) behaviour.

A 1.7 Radians phase-shift can be measured on the time-frequency-phase plane in reference to the phase-shift colorbar. In this case, the zero phase reference has been set to time $t_r = 160s$ (Figure 3). On the reassigned 3D phase spectrogram (Figure 4), the magnitudes of the two parts of the signal (before and after phase-shift) stay stationary.

In addition, a classical method to estimate the instantaneous phase is used; the calculus of the instantaneous phase is performed by computing the crest line of the reassigned 3D phase spectrogram. The resulting graph exhibits the same phase-shift (Figure 5) than the generated one. The interest of this method is to propose a direct reading of the instantaneous phase. The term "instantaneous phase" must be understood in this work as "phase displacement computed from an arbitrary phase reference", which is more precise.

In a second step, two real vertical ground reaction forces signals (Figure 6, 10) are analysed, a walk vertical one and a running vertical one. Ground Reaction Forces (GRF) are the forces that occur between the feet and the ground during human gait. These forces are commonly measured using an instrumented treadmill.

The utility of GRF to the analysis of human gait is not any more to show [8]. The main applications are directed towards the analysis of various pathologies like infringements of the knee or clinical analyses on patients with total knee replacement.

The use of time-frequency representations for the analysis of GRF is not very developed [9, 10], even if wavelet analysis is appreciated [11, 12, 13]. Most of the time, the analysis is done in term of high and low frequencies, but none of these studies was focused on small phase variations. These last ones could play an important part in the mechanism of osseous renewal [14, 15] for example.

The 3D phase spectrogram (Figure 7) and the reassigned 3D phase spectrogram (Figure 8) of the walk vertical GRF signal (Figure 6) are represented. Three spectral components appear on the two representations where the instantaneous phase behaviours can be observed. As previously, the instantaneous phase is estimated considering the crest lines (Figure 9) of each component. Taking into account the zero phase reference (time $t_r = 4s$), the relative phase variations can be quantified. The amount of phase-shift is related to the harmonic level; the higher the rank of the harmonic, the more important the phase-shift is. In the field of biomechanics, this kind of analysis presents a great interest to characterize the global gait behaviour linked to the walk GRF signal. In particular, the first and the third spectral components have a significant magnitude (Figure 8) and corresponds to the healthy walk frequency (first component) and the unrolling of the foot (third component).

Then, the 3D phase spectrogram (Figure 11) and the reassigned 3D phase spectrogram (Figure 12) of the running vertical GRF signal (Figure 10) are represented. Two spectral components appear on the two representations. It is interesting to notice that the running signal only exhibits two spectral components, this fact can be interpreted as a different behaviour of the feet during this kind of gait. As previously, the instantaneous phase is estimated considering the crest lines (Figure 13) of each component. Taking into account the zero phase reference (time $t_r = 4s$), the relative phase variations can also be quantified; It seems that the amount of phase-shift

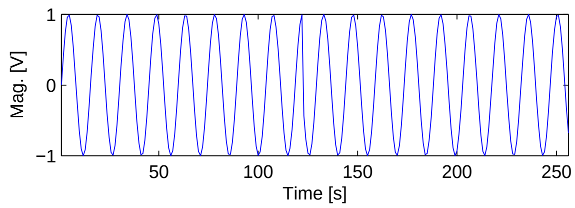

Figure 2: Sinusoidal synthetic signal containing a 1.7 Radians phase-shift.

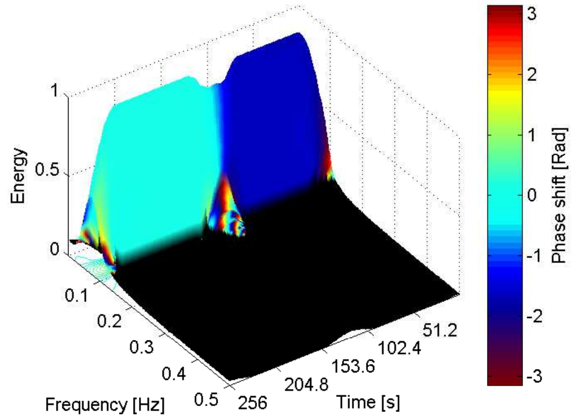

Figure 3: 3D phase spectrogram of a sinusoidal synthetic signal containing a 1.7 Radians phase-shift (zero phase reference: $t_r = 160s$).

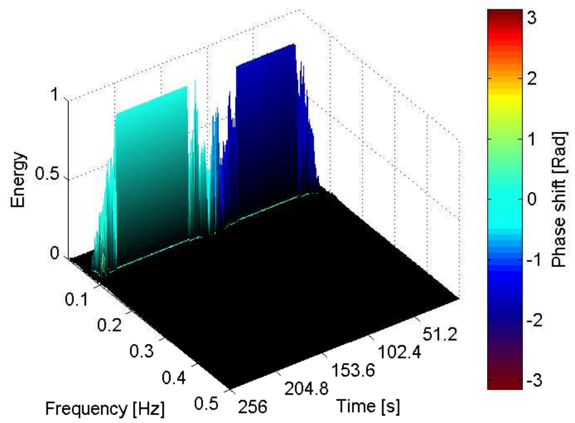

Figure 4: Reassigned 3D phase spectrogram of a sinusoidal synthetic signal containing a 1.7 Radians phase-shift (zero phase reference: $t_r = 160s$).

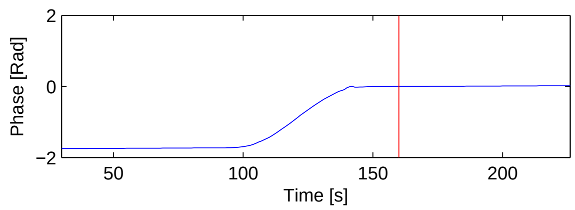

Figure 5: Phase crest of the reassigned phase spectrogram of a sinusoidal synthetic signal containing a 1.7 Radians phase-shift (zero phase reference: $t_r = 160s$).

is not related to the frequency as it is for the walk signal. In the case of the running signal, amplitude variations are more important than those found on the walk signal, whereas phase variations are less important. In terms of gait behaviour analysis, this can be interpreted as the adaptation of the human body to maintain equilibrium. During walking, one of the two feet is always in contact with the ground. During running, there is a small flight time, so the effort to maintain equilibrium must be more important.

5. CONCLUSION

The reassigned 3D phase spectrogram has been introduced in this paper. This representation leads to a direct estimation of the instantaneous phase of a time signal by calculating a crest line in the time-frequency-phase plane. Considering the first results, it appears that this representation makes a powerful tool to analyse quasi-stationary signals such as GRF and phase-shifted signals. Particularly concerning GRF signals, the reassigned 3D phase spectrogram allows a better understanding of gait behaviour. A more important clinical analysis is in preparation, in order to analyze fatigue phenomena or pathological cases.

This method belongs to a new class of representations based on the expression of the phase derivative. From an other point of view, it would be interesting to make further investigations in order to generalize this kind of method.

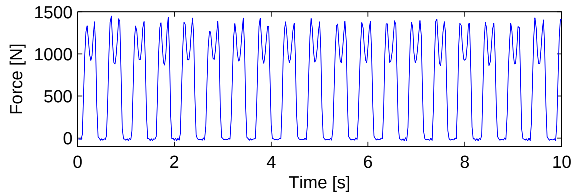

Figure 6: Walk vertical GRF signal.

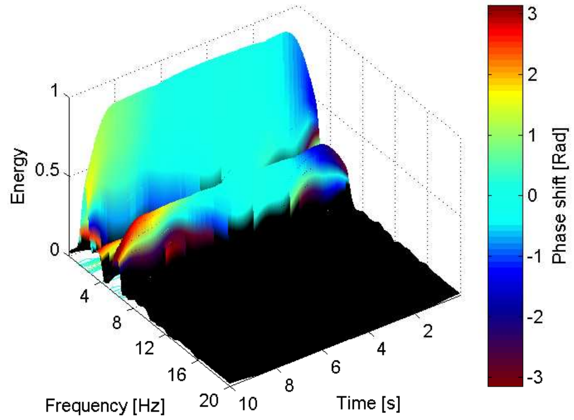

Figure 7: 3D phase spectrogram of a walk vertical GRF signal (zero phase reference: $t_r = 4s$).

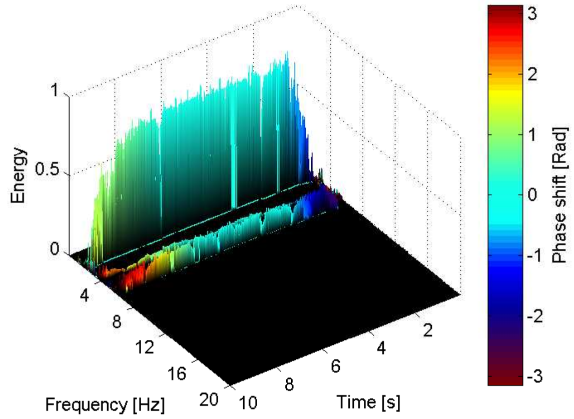

Figure 8: Reassigned 3D phase spectrogram of a walk vertical GRF signal (zero phase reference: $t_r = 4s$).

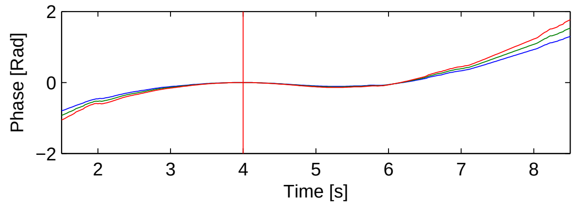

Figure 9: Phase crests of the reassigned phase spectrogram of a walk vertical GRF signal (zero phase reference: $t_r = 4s$).

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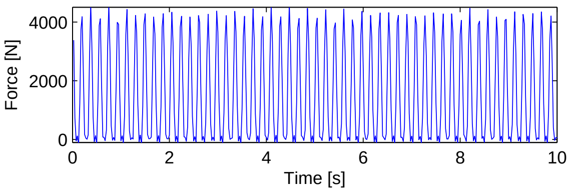

Figure 10: Running vertical GRF signal.

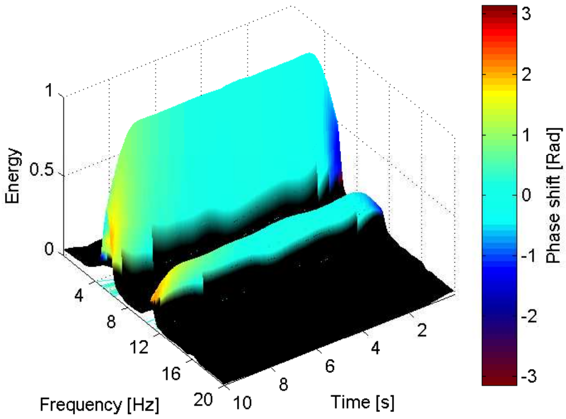

Figure 11: 3D phase spectrogram of a running vertical GRF signal (zero phase reference: $t_r = 4s$).

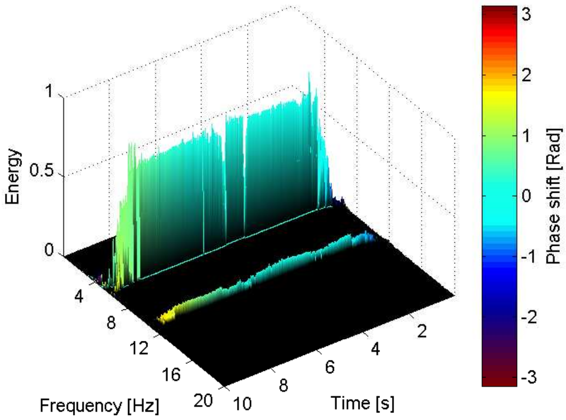

Figure 12: Reassigned 3D phase spectrogram of a running vertical GRF signal (zero phase reference: $t_r = 4s$).

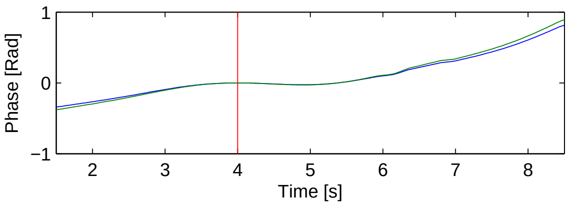

Figure 13: Phase crests of the reassigned phase spectrogram of a running vertical GRF signal (zero phase reference: $t_r = 4s$).

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