

# DATA-AIDED TIME-DOMAIN SYNCHRONIZATION FOR FILTER BANK MULTICARRIER SYSTEMS

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## ABSTRACT

In this paper we consider the problem of data-aided joint symbol timing and carrier-frequency offset (CFO) estimation for filter bank based multicarrier systems. In particular, we propose a new joint symbol timing and CFO synchronization algorithm exploiting the transmission of a training sequence made up of  $N_{rip}$  identical parts, each of duration  $P$ . The performance of the derived estimator assessed by computer simulation is compared with that of two data-aided synchronization algorithms previously proposed in the literature.

## 1. INTRODUCTION

In the last decade orthogonal frequency division multiplexing (OFDM) systems have attracted great interest in wireless and wireline transmissions due to their high robustness to multipath channels. However in OFDM systems the adopted pulse shaping filter is a rectangular function, that exhibits a poor frequency-decay. An alternative multicarrier modulation system, which can provide a better spectral containment and high robustness to multipath channels, is filter bank multicarrier (FBMC) system. FBMC systems referred to as Filtered Multitone (FMT) systems have been proposed for very high-speed digital subscriber line (VDSL) standards [1] and are under investigation also for broadband wireless applications [2], [3]. FBMC systems based on offset QAM modulation (OQAM), known as OFDM/OQAM systems, have been considered by the 3GPP standardization forum for improved downlink UTRAN interfaces [4].

However, as for all the multicarrier modulation schemes, one of the major disadvantages of FBMC systems is their sensitivity to carrier frequency and timing errors. Specifically, as investigated in [5] and in [6] phase noise and misalignments in time and frequency can considerably degrade the performance of FMT and OFDM/OQAM systems, giving rise to interference between successive symbols and adjacent subcarriers. Therefore, reliable and accurate timing and carrier-frequency offset (CFO) synchronization schemes must be designed for these systems.

Recently, data-aided and non-data-aided (or blind) synchronization algorithms for FBMC systems have been considered. Specifically, in [7] it is presented a blind joint CFO and symbol timing estimator exploiting the unconjugate cyclostationarity of the received OFDM/OQAM signal while in [8] it is shown that accurate CFO estimation algorithms can be obtained by using both the conjugate and the unconjugate cyclostationarity properties of the received signal. However, the derived estimators assure a satisfactory performance only when a large number of OFDM/OQAM symbols is considered and in the case of non-dispersive channel. In [12] the

problem of data-aided synchronization and channel estimation in the frequency domain for OFDM/OQAM systems has been considered while in [9] are derived data-aided joint symbol timing and frequency offset synchronization schemes in the time domain for FMT systems based on the deployment of appropriate training sequences. Specifically, in [9] the authors extend to FMT systems the synchronization techniques for OFDM systems proposed in [10] and in [11] exploiting the transmission of a training sequence with repeated parts.

Inspired to these works we develop in this paper a synchronization scheme for data-aided symbol timing and frequency offset recovery with robust acquisition properties in dispersive channels. Specifically, this algorithm exploits the known structure of a training sequence made up of  $N_{rip}$  identical parts, each of duration  $P$ . The proposed method, as illustrated by computer simulation, can assure in a multipath channel accurate symbol timing and CFO estimates outperforming the synchronization algorithms proposed in [9].

The organization of this paper is as follows. In Section 2 we describe the considered FBMC system model. In Section 3 we derive the proposed joint symbol timing and CFO estimator for multipath channel, based on the least squares (LS) approach. Numerical results are presented in Section 4 and conclusions are drawn in the final Section.

*Notation:*  $j \triangleq \sqrt{-1}$ , superscript  $(\cdot)^*$  denotes the complex conjugation,  $\Re[\cdot]$  real part,  $|\cdot|$  absolute value and  $\arg[\cdot]$  the argument of a complex number in  $[0, 2\pi)$ . Moreover  $lcm\{a, b\}$  is the least common multiple between  $a$  and  $b$ ,  $\delta(t)$  the Dirac delta and  $\otimes$  is the convolution operator.

## 2. FBMC SYSTEM MODEL

Let us consider an FBMC system with a signaling interval  $T$ , the received signal in multipath channel, in the presence of a timing offset  $\tau$ , a carrier phase offset  $\phi$  and a CFO  $\Delta f$  can be written as

$$r(t) = e^{j(2\pi\Delta f t + \phi)} [h(t) \otimes s(t - \tau)] + n(t) \quad (1)$$

where

$$h(t) \triangleq \sum_{q=0}^{N_m-1} a_q \delta(t - \theta_q)$$

is the multipath channel impulse response,  $s(t)$  is the transmitted OFDM/OQAM signal and  $n(t)$  denotes the zero-mean complex white Gaussian noise with power spectral density  $S_n(f) = \sigma_n^2$  and statistically independent of  $s(t)$ . The received signal  $r(t)$  is filtered with an ideal low-pass filter with a bandwidth of  $1/T_s$  and sampled with a frequency  $f_s = 1/T_s$  where  $T_s$  denotes the sampling period

$$r(kT_s) = e^{j(2\pi\Delta f T_s k + \phi)} \sum_{q=0}^{N_m-1} a_q s(kT_s - \tau - \theta_q) + n(kT_s). \quad (2)$$

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In this paper we will focus our analysis on two FBMC-based schemes: OFDM offset QAM (OFDM/OQAM) and Filtered MultiTone (FMT). In particular, into the case of OFDM/OQAM systems the useful signal  $s(kT_s)$  sampled with a sampling frequency  $f_s = N/T$  is equal to

$$s(kT_s) = \sqrt{\frac{N}{2N_u}} \left[ \sum_{p=-\infty}^{\infty} \sum_{l \in \mathcal{A}} a_l^R(p) e^{jk(\frac{2\pi}{N}l + \frac{\pi}{2})} g(kT_s - pT) + j \sum_{p=-\infty}^{\infty} \sum_{l \in \mathcal{A}} a_l^I(p) e^{jk(\frac{2\pi}{N}l + \frac{\pi}{2})} g\left(kT_s - \frac{T}{2} - pT\right) \right] \quad (3)$$

where the sequences  $a_l^R(p)$  and  $a_l^I(p)$  denote the real and imaginary parts of the complex data symbols transmitted on the  $l$ th subcarrier during the  $p$ th OFDM/OQAM symbol while  $g(t)$  is the real transmitted pulse-shaping filter, assumed to be a square-root raised cosine (SRRC) Nyquist filter with a roll-off factor  $\alpha$  ( $0 \leq \alpha \leq 1$ ) and unit energy. Moreover, in (3)  $N$  is the number of subcarriers, of which  $N_u$  are data subcarriers and  $N_v = N - N_u$  remain unmodulated (virtual subcarriers), while  $\mathcal{A}$  is the set of indices of data subcarriers.

Into the case of an FMT system the baseband discrete-time transmitted signal obtained by sampling the continuous-time signal with a sampling frequency  $f_s = K/T$  is given by

$$s(kT_s) = \sqrt{\frac{K}{N_u}} \sum_{p=-\infty}^{\infty} \sum_{l \in \mathcal{A}} a_l(p) g(kT_s - pT) e^{j\frac{2\pi}{N}kl} \quad (4)$$

where  $K = (1 + \alpha)N$  is assumed to be an integer and  $a_l(p)$  is the complex data symbol transmitted on the  $l$ th subcarrier of the  $p$ th FMT symbol.

### 3. JOINT SYMBOL TIMING AND CFO LS ESTIMATOR

In this section we derive a data-aided joint CFO and symbol timing estimator based on the LS approach which exploits the transmission of a training sequence made up of  $N_{rip}$  identical blocks each of duration  $PT_s$  (see Fig. 1). Precisely, the training sequence can be obtained by transmitting a sequence of  $N_{TR}$  FBMC symbols periodic of period  $N$ , that is  $a_l(p) = a_l^{TR} \forall l \in \mathcal{A}$  and  $\forall p \in \{0, \dots, N_{TR} - 1\}$ . In this way we obtain the training sequence periodic of period  $P$

$$s_{TR}(kT_s) = s_{TR}(kT_s + PT_s) \quad (5)$$

with  $P = lcm\{N, K\}$  for FMT systems and  $P = N$  for OFDM/OQAM systems. Thus, a joint symbol timing and CFO estimator can be obtained by considering the minimization problem

$$(\Delta \hat{f}, \hat{\tau}) = \arg \min_{\Delta \hat{f}, \hat{\tau}} \left\{ \sum_{k=N_g-1}^{N_{rip}P-P-1} |r(kT_s + \hat{\tau}) - r(kT_s + PT_s + \hat{\tau}) e^{-j2\pi \Delta \hat{f} T_s P}|^2 \right\} \quad (6)$$

where  $\Delta \hat{f}$  and  $\hat{\tau}$  are trial values for CFO and timing offset, respectively, while  $N_g T_s$  is the length of the pulse shaping filter  $g(t)$ . The minimization in (6) leads to the following joint CFO and symbol timing estimator referred to as LS estimator

$$\hat{\tau}_{LS} = \arg \max_{\hat{\tau}} \{2|R(\hat{\tau})| - Q(\hat{\tau})\} \quad (7)$$

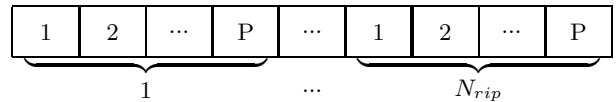


Figure 1: Training FBMC signal.

and

$$\Delta \hat{f}_{LS}(\hat{\tau}_{LS}) = \frac{1}{2\pi PT_s} \arg \{R(\hat{\tau}_{LS})\} \quad (8)$$

with

$$R(\hat{\tau}) \triangleq \sum_{k=N_g-1}^{N_{rip}P-P-1} r^*(kT_s + \hat{\tau}) r(kT_s + PT_s + \hat{\tau}) \quad (9)$$

and

$$Q(\hat{\tau}) \triangleq \sum_{k=N_g-1}^{N_{rip}P-P-1} |r(kT_s + \hat{\tau})|^2 + \sum_{k=N_g-1}^{N_{rip}P-P-1} |r(kT_s + PT_s + \hat{\tau})|^2. \quad (10)$$

Let us observe that if we divide the timing metric in (7) by  $Q(\hat{\tau})$  we obtain the joint symbol timing and CFO estimator

$$\hat{\tau}_{TR1} = \arg \max_{\hat{\tau}} \left\{ \frac{2|R(\hat{\tau})|}{Q(\hat{\tau})} \right\} \quad (11)$$

and

$$\Delta \hat{f}_{TR1}(\hat{\tau}_{TR1}) = \frac{1}{2\pi PT_s} \arg \{R(\hat{\tau}_{TR1})\}. \quad (12)$$

The joint estimator in (11) and in (12) represents a modified version of the synchronization algorithm for FMT system proposed by Tonello and Rossi in [9] and it is referred to as Tonello Rossi 1 (TR1). In [9] it is also considered a joint symbol timing and CFO estimator for AWGN channel exploiting the knowledge of a pseudo-noise sequence periodic of period  $P$ . This synchronization algorithm referred to as TR2 can be obtained by considering the minimization problem

$$(\Delta \hat{f}_{TR2}, \hat{\tau}_{TR2}) = \arg \min_{\Delta \hat{f}, \hat{\tau}} \left\{ \sum_{k=N_g-1}^{N_{rip}P-P-1} |s_{TR}^*(kT_s) s_{TR}(kT_s + PT_s) - r^*(kT_s + \hat{\tau}) r(kT_s + PT_s + \hat{\tau}) e^{-j2\pi \Delta \hat{f} T_s P}|^2 \right\}. \quad (13)$$

After simple algebraic manipulations we obtain the joint symbol timing and CFO estimator

$$\hat{\tau}_{TR2} = \arg \max_{\hat{\tau}} \left\{ \frac{2|S(\hat{\tau})|}{T(\hat{\tau})} \right\} \quad (14)$$

$$\Delta \hat{f}_{TR2}(\hat{\tau}_{TR2}) = \frac{1}{2\pi PT_s} \arg \{S(\hat{\tau}_{TR2})\} \quad (15)$$

with

$$S(\tilde{\tau}) = \sum_{k=N_g-1}^{N_{rip}P-P-1} r^*(kT_s + \tilde{\tau})s_{TR}(kT_s) \times r(kT_s + PT_s + \tilde{\tau})s_{TR}^*(kT_s + PT_s) \quad (16)$$

and

$$T(\tilde{\tau}) = \sum_{k=N_g-1}^{N_{rip}P-P-1} |s_{TR}(kT_s)|^2 |s_{TR}(kT_s + PT_s)|^2 + \sum_{k=N_g-1}^{N_{rip}P-P-1} |r(kT_s + \tilde{\tau})|^2 |r(kT_s + \tilde{\tau} + PT_s)|^2. \quad (17)$$

It is worthwhile to note that the considered LS, TR1 and TR2 CFO estimators in (8), (12) and (15), respectively, provide a closed form solution for the CFO estimate and do not require the knowledge of the signal-to-noise ratio (SNR) and of the channel. Moreover, in the case of OFDM/OQAM systems they can assure unambiguous CFO estimates if  $|\Delta f T_s| < 1/(2N)$ , while in the case of FMT systems their acquisition range is reduced to  $|\Delta f T_s| < 1/(2P)$ . On the other hand, the considered LS, TR1 and TR2 symbol timing estimators do not present a closed form solution but they require a maximization with respect to the continuous parameter  $\tilde{\tau}$ . This maximization is performed in two steps: in the first it is performed a coarse search with a step-size  $T_s/100$  followed, in the second step, by a parabolic interpolation.

It is of interest to underline that in the case of FMT systems the amount of redundancy needed to transmit the training sequence is greater than to that used in the case of OFDM/OQAM systems. In fact, in the case of OFDM/OQAM systems the training sequence is composed by  $N_{rip}$  identical OFDM/OQAM symbols while in the case of FMT systems it is necessary to transmit a training sequence of total length  $N_{rip}P$ , where  $P = l.c.m\{N, K\}$  can be much greater than  $N$ .

In figures 2 and 3 we report the cost function of the LS, TR1 and TR2 symbol timing metrics in (7), (11) and in (14), respectively, for a noiseless and distortionless transmission. Precisely, Fig. 2 shows the behavior of the considered symbol timing estimators in the case of an OFDM/OQAM system with  $N = 64$  subcarriers while in Fig. 3 an FMT system with  $N = 64$  and  $K = 72$  is considered. The results show that the TR2 and the LS symbol timing estimators exhibit a sharp peak at the actual timing value  $\tau = 10T_s$  while the TR1 cost function is more flat around its maximum.

#### 4. NUMERICAL RESULTS

In this section the performance of the proposed joint LS symbol timing and CFO estimator is compared with that of the considered modified versions of the data-aided synchronization algorithms TR1 and TR2 proposed by Tonello and Rossi in [9].

A number of  $10^4$  Monte Carlo trials has been performed under the following conditions

1. The prototype filter is obtained by truncating the sampled version of an SRRC Nyquist filter with a rolloff factor  $\alpha$ . Specifically, it is a FIR filter of length  $N_g = 4K$  for FMT systems and  $N_g = 4N$  for OFDM/OQAM systems.
2. The value of the normalized CFO, of the timing offset and of the carrier phase are fixed at  $\Delta\nu = \Delta f T_s N = 0.03$ ,  $\tau = 3T_s$  and  $\phi = \pi/8$ , respectively.

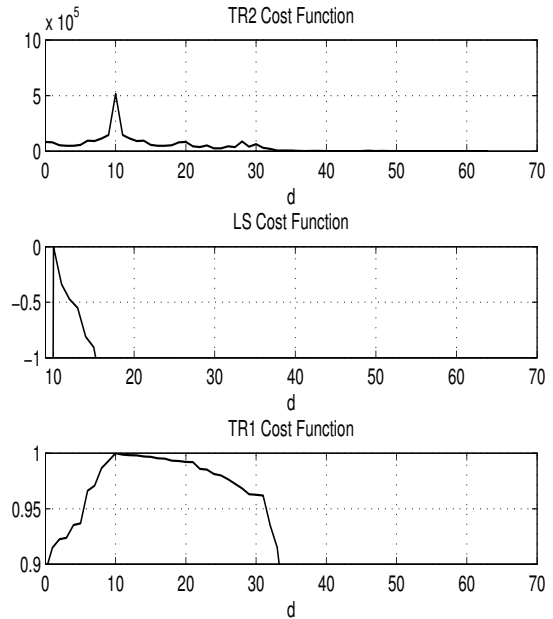


Figure 2: Cost functions of the considered symbol timing estimators for OFDM/OQAM systems in a single run and in the absence of noise.

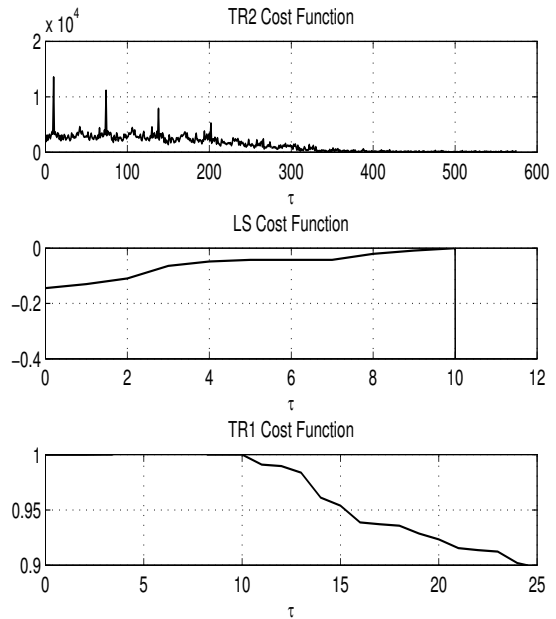


Figure 3: Cost functions of the considered symbol timing estimators in a single run, in the absence of noise and for an FMT system with  $N = 64$  and  $K = 72$ .

3. The size of the set of subcarriers and the roll-off parameter for the considered FMT system are  $N = 64$  and  $\alpha = 0.125$ , respectively.
4. The size of the set of subcarriers and the roll-off parameter for the considered OFDM/OQAM system are  $N = 64$

and  $\alpha = 0.6$ , respectively.

5. The multipath channel has been modeled using the COST 207 Hilly Terrain (HT) Rayleigh fading channel in [13].
6. The complex data symbols  $a_i(p)$ , when the FMT system is considered, belong to a QPSK constellation.
7. The data symbols  $a_i^R(p)$  and  $a_i^I(p)$ , when the OFDM/OQAM system is considered, belong to a BPSK constellation.

#### 4.1 OFDM/OQAM System

In this first set of simulations we have tested the performance of the considered algorithms for an OFDM/OQAM system in AWGN (solid lines) and multipath channel (dashed lines). Precisely, figures 4 and 5 show the mean squared error (MSE) of the considered joint CFO and symbol timing estimators as a function of the SNR  $\triangleq \frac{1}{\sigma_n^2}$  for a training sequence with  $N_{rip} = 5$  identical blocks each of length  $N$ . As indicated in figure 4 in AWGN channel the TR2 symbol timing estimator exhibits the best performance for all the considered SNR values, while in the considered COST 207 HT channel the proposed LS symbol timing estimator assures the lowest MSE and, moreover, its performance does not present a floor for high SNR values. It is worthwhile to emphasize that the considered TR1 and LS symbol timing estimators exhibit in multipath channel a contained performance loss with respect to that achieved in AWGN while the TR2 symbol timing estimator presents a severe performance degradation in the case of dispersive channel. As shown in Fig. 5 in AWGN channel the considered CFO estimators exhibit nearly the same performance while in multipath channel the greater accuracy of the proposed LS symbol timing estimator has beneficial effects also on the performance of the LS CFO estimator that assures for high SNR values the lowest MSE.

#### 4.2 FMT System

In this subsection we present the performance of the considered algorithms for FMT systems in AWGN (solid lines) and multipath channel (dashed lines). Precisely, figures 6 and 7 show the MSE of the considered joint CFO and symbol timing estimators as a function of the SNR for a training sequence with  $N_{rip} = 2$  identical blocks each of duration  $P = l.c.m.\{64, 72\} = 576$ . We can note that, as in the OFDM/OQAM case, in the FMT case and in AWGN channel the TR2 symbol timing estimator exhibits the best performance, while in multipath channel the proposed LS symbol timing estimator assures the lowest MSE. In regard to the performance of the CFO estimators, we can note that as in the case of OFDM/OQAM systems, the LS CFO estimator guarantees practically the same performance in AWGN and multipath channel, outperforming all the other estimators for high SNR values.

### 5. SUMMARY AND CONCLUSIONS

In this paper the problem of data-aided symbol timing and CFO estimation in FBMC systems has been considered. A synchronization scheme based on a training sequence made up of  $N_{rip}$  identical parts each of duration  $P$  has been considered. The proposed method is based on the LS approach and operates in the time domain before running the receiver filter bank. Moreover, it does not require the knowledge of the channel impulse response and of the SNR. The performance of the derived LS estimator has been assessed via computer simulation and compared with that of modified versions of two joint symbol timing and CFO estimators previously proposed by Tonello and Rossi in [9]. The numerical results have shown that in a multipath channel it can outperform the estimators proposed in [9].

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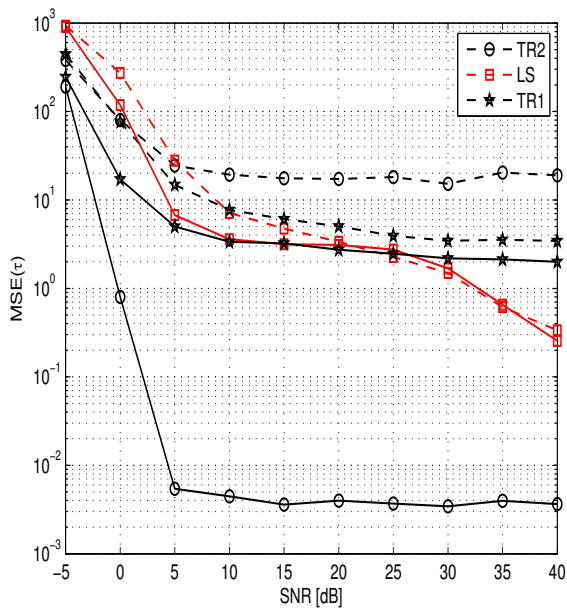


Figure 4: Performance of the considered symbol timing estimators in AWGN (solid lines) and multipath channel (dashed lines) as a function of SNR and for an OFDM/OQAM system with  $N = 64$  and  $\alpha = 0.6$ .

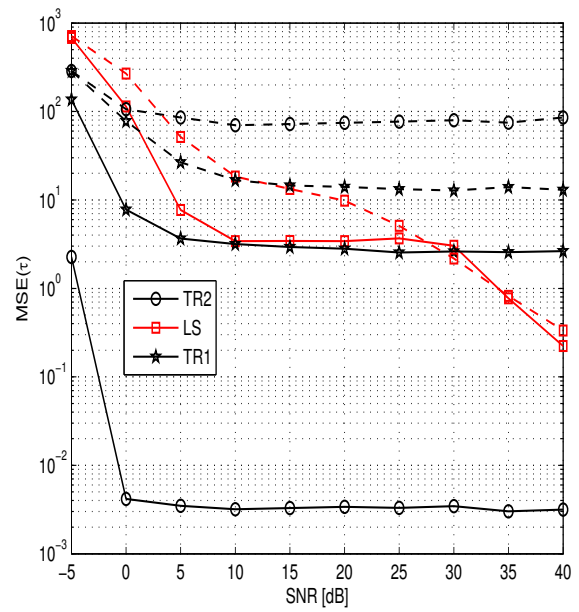


Figure 6: Performance of the considered symbol timing estimators in AWGN (solid lines) and multipath channel (dashed lines) as a function of SNR and for an FMT system with  $N = 64$  and  $K = 72$ .

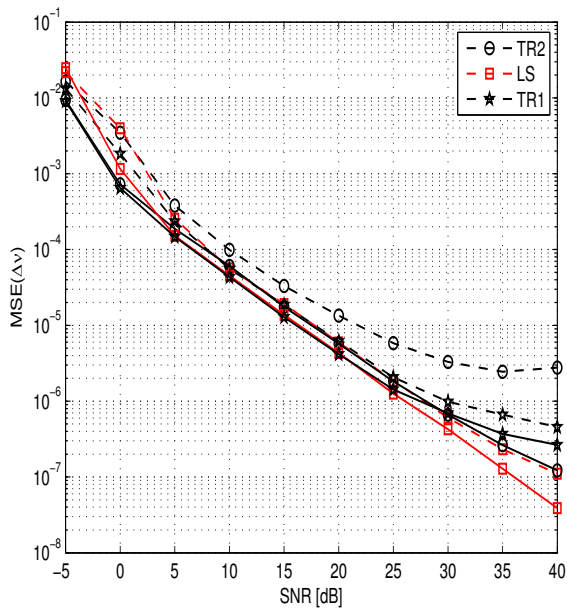


Figure 5: Performance of the considered CFO estimators in AWGN (solid lines) and multipath channel (dashed lines) as a function of SNR and for an OFDM/OQAM system with  $N = 64$  and  $\alpha = 0.6$ .

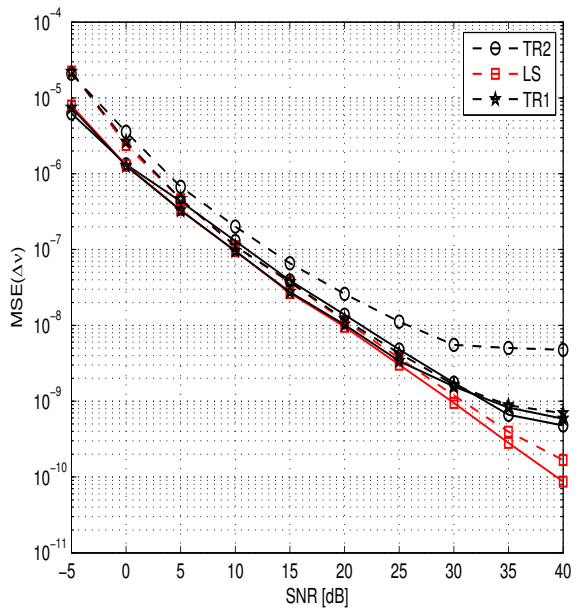


Figure 7: Performance of the considered CFO estimators in AWGN (solid lines) and multipath channel (dashed lines) as a function of SNR and for an FMT system with  $N = 64$  and  $K = 72$ .