16th European Signal Processing Conference (EUSIPCO 2008), Lausanne, Switzerland, August 25-29, 2008, copyright by EURASIP

ROBUST TRANSMIT BEAMFORMING BASED ON PROBABILISTIC CONSTRAINT

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ABSTRACT

Transmit beamforming is a powerful technique for enhancing performance of wireless communication systems. Most existing transmit beamforming techniques require perfect channel state information at the transmitter (CSIT), which is typically not available in practice. In such situations, the design should take into account errors in the channel estimates, so that the beamformers are less sensitive to these errors. Two robust approaches are widely used. The stochastic approach optimizes the average performance of the system and assumes that the statistics, such as mean and covariance, of the errors are known. The maximin approach assumes that the errors belong to a worst-case uncertainty region and optimizes the worst-case system performance. This type of design usually leads to conservative results as the worst-case conditions may occur at a very low probability. In this paper, we propose a more flexible approach that optimizes the average beamforming performance and takes the extreme (but rare) conditions into account proportionally. Simulation results show that the proposed beamformer offers higher robustness against errors in CSIT than serval state-of-the-art beamformers.

1. INTRODUCTION

Multi-antenna diversity is well motivated in wireless communication systems because it offers significant advantages over single antenna [7]. Perfect or partial knowledge of the channel state information (CSIT) can provide further performance enhancement[10][11].

However, in practical wireless systems, the accuracy of the CSIT is impossible to know due to errors induced by imperfect (quantized, erroneous, or outdated) channel feedback. In such situations, the transmit beamforming design should take into account errors in channel estimates. Existing robust transmit beamforming designs can be categorized into stochastic and maximin approaches. The stochastic approach [6] [10] [11] assumes that statistics of errors in CSIT, such as mean and covariance, are known and optimizes the average performance of the system. On the other hand, the maximin approach considers channel estimation errors as deterministic and optimizes the worst-case system performance [1] [2]. This approach provides robustness against any error in the worst-case region. However, it is overly conservative as the worst operational condition is rare. To overcome this problem, a more flexible probabilistic constraint is introduced in [9] into the design of adaptive beamformer at the receiver side.

In this work, we propose a robust transmit beamforming technique that maximizes the average SNR performance and use probabilistic constraints to keep the worst-case performance at a very low probability. The aforementioned stochastic approach only optimizes the average performance without considering the worst-case scenario. On the other hand, although the maximin approach provides the best performance in the worst case, it is overall too conservative. To keep balance between the average and the worst-case performance, we take a more flexible approach in which the extreme (but rare) conditions are taken into account proportionally. Our approach maximizes the average SNR performance and ensures robustness against the CSIT error by keeping the probability of the worst-case performance at a very low level. Under the assumption that the CSIT error is Gaussian distributed, this stochastic optimization problem can be further simplified to equivalent deterministic forms which can be efficiently solved by modern convex optimization algorithms [3]. Simulation results show the proposed approach provides the best performance among several state-of-the-art beamforming techniques.

The paper is organized as follows. The system model is described in Section 2. We formulate the proposed method as a stochastic optimization problem in Section 3 and simplify it to an equivalent convex optimization problem in Subsection 3.1 and 3.2. Simulation results are presented and discussed in Section 4. Concluding remarks are given in Section 5.

Notation: $(\cdot)^H$ denotes Hermitian transpose; $\mathsf{E}[\cdot]$ stands for expectation; $\mathrm{tr}\{\cdot\}$ is the trace of a matrix; $\mathbf{I}_K$ denotes the identity matrix of size K; $\mathbf{0}_{K\times P}$ denotes an all-zero matrix of size $K\times P$; $diag\{\mathbf{x}\}$ stands for a diagonal matrix with $\mathbf{x}$ on its diagonal; $\{\cdot\}_j$ denotes the jth entry of a vector, $\mathbf{h}_j$ denotes the jth column of matrix $\mathbf{H}$.

2. SYSTEM MODEL

We consider a single-user wireless communication system with M transmit antennas and a single receive antenna. The information-bearing signal s is spread by the precoding matrix $\mathbf{C}$ and then transmitted through the flat fading channel. As we focus on symbol-by-symbol detection, the received signal $\mathbf{y}$ in the presence of additive white Gaussian noise $\mathbf{w}$ is given by

$$\mathbf{y} = \mathbf{C}\mathbf{h}s + \mathbf{w}. \quad (1)$$

In the perfect CSIT case, the estimated channel at the transmitter is error free and the output $\hat{s}$ of maximum ratio combining (MRC) at the receiver is given by

$$\hat{s} = (\mathbf{Ch})^H \mathbf{y} = \mathbf{h}^H \mathbf{C}^H \mathbf{Ch}s + \mathbf{h}^H \mathbf{C}^H \mathbf{w}. \quad (2)$$

The average signal-to-noise ratio (SNR) at MRC receiver output is

$$\mathrm{SNR} = \mathsf{E}\left[\frac{(\mathbf{h}^H\mathbf{C}^H\mathbf{Ch}s)(\mathbf{h}^H\mathbf{C}^H\mathbf{Ch}s)^H}{\mathbf{h}^H\mathbf{C}^H\mathbf{ww}^H\mathbf{Ch}}\right] = \frac{E_s}{N_0}\mathbf{h}^H\mathbf{C}^H\mathbf{Ch}, \quad (3)$$

where $E_s = \mathsf{E}\left[|s|^2\right]$ is the average energy of the signal and $N_0/2$ is the noise variance.

To extend the model to a system with N receive antennas, we assume that the channel vectors observed on different receive antennas are mutually uncorrelated. The channel vector denotes as $\mathbf{h}_j$ for jth receive antenna, and is arranged into a $M \times N$ matrix $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_N]$. Similar to the single-receive-antenna case, the received signal at the jth antenna is $\mathbf{y}_j = \mathbf{Ch}_j s + \mathbf{w}_j$. The total receiver SNR at the output of the MRC is

$$\mathrm{SNR} = \frac{E_s}{N_0}\sum_{j=1}^{N}\mathbf{h}_j^H\mathbf{C}^H\mathbf{Ch}_j = \frac{E_s}{N_0}\,\mathrm{tr}\{\mathbf{H}^H\mathbf{C}^H\mathbf{CH}\} \quad (4)$$

which includes (3) as a special case corresponding to $N = 1$.

3. ROBUST BEAMFORMING BASED ON PROBABILISTIC-CONSTRAINED OPTIMIZATION

We consider the case in which the transmitter does not have exact channel state information (CSI) but has an estimate $\hat{\mathbf{H}}$ of the channel matrix $\mathbf{H}$. The CSIT error matrix is given by

$$\mathbf{E} = [\mathbf{e}_1, \ldots, \mathbf{e}_N] = \mathbf{H} - \hat{\mathbf{H}}. \quad (5)$$

We assume that $\mathbf{e}_j$ is complex normally distributed and independent from the estimate channel $\hat{\mathbf{h}}_j$, i.e. $\mathsf{E}\left[\hat{\mathbf{h}}_j^H\mathbf{e}_j\right] = 0$. In the proposed approach, we optimize the average SNR at the output of MRC receiver and achieve the robustness by keeping the outage probability of the instantaneous SNR below a pre-specified level. For simplicity, assuming E_s/N_0 is constant in one symbol interval, we will drop the constant factor E_s/N_0 from the SNR expression.

Our objective is to derive the precording matrix $\mathbf{C}$ that maximizes the average SNR and has a low outage probability. More specifically, the design of robust beamforming matrix can be achieved by solving the following optimization problem

$$\max_{\mathbf{C}} \mathsf{E}\left[\mathrm{tr}\{(\hat{\mathbf{H}}+\mathbf{E})^H\mathbf{C}^H\mathbf{C}(\hat{\mathbf{H}}+\mathbf{E})\}\right],$$

subject to

$$P\,\{\mathrm{tr}\{(\hat{\mathbf{H}}+\mathbf{E})^H\mathbf{C}^H\mathbf{C}(\hat{\mathbf{H}}+\mathbf{E})\} \le \gamma\} \le p, \qquad \mathrm{tr}\{\mathbf{C}^H\mathbf{C}\} = 1, \quad (6)$$

where γ denotes the SNR threshold, p is a pre-specified probability value that satisfies quality of service (QoS) requirements, and $P\,\{A\}$ stands for the probability of event A. Typically we select a low probability value p and high threshold value γ. The deterministic constraint $\mathrm{tr}\{\mathbf{C}^H\mathbf{C}\} = 1$ reflects the fact that the total transmitted power is limited by the system.

To simplify the above problem, we consider the eigen-decomposition of $\mathbf{C}^H\mathbf{C}$

$$\mathbf{C}^H\mathbf{C} = \mathbf{U}_c\mathbf{D}_c\mathbf{U}_c^H, \quad (7)$$

where $\mathbf{U}_c = [\mathbf{u}_{c_1}, \ldots, \mathbf{u}_{c_M}]$ consists of eigenvectors of $\mathbf{C}^H\mathbf{C}$ and $\mathbf{D}_c = diag\{d_{c_1}, \ldots, d_{c_M}\}$ is a diagonal matrix with corresponding eigenvalues $d_{c_1} \ge \ldots d_{c_M} \ge 0$. The precoding matrix $\mathbf{C}$ can be viewed as a weight matrix. The error covariance $\mathbf{R}_e$ is positive definite and can be factorized as

$$\mathbf{R}_e = \mathbf{V}_e\mathbf{V}_e^H, \quad (8)$$

where $\mathbf{V}_e$ is a nonsingular matrix. Then the product $\mathbf{V}_e^H\mathbf{C}^H\mathbf{C}\mathbf{V}_e$ can be simplified as follows

$$\mathbf{V}_e^H\mathbf{C}^H\mathbf{C}\mathbf{V}_e = (\mathbf{U}_c^H\mathbf{V}_e)^H\mathbf{D}_c(\mathbf{U}_c^H\mathbf{V}_e) = \mathbf{P}^H\mathbf{D}_c\mathbf{P}, \quad (9)$$

where $\mathbf{P} = \mathbf{U}_c^H\mathbf{V}_e$.

Since the average SNR depends on the beamforming matrix $\mathbf{C}$ through $\mathbf{C}^H\mathbf{C}$, it suffices to optimize the objective function with respect to $\mathbf{U}_c$ and $\mathbf{D}_c$. Define $\check{\mathbf{H}} = \mathbf{U}_c^H\hat{\mathbf{H}}$ and $\check{\mathbf{E}} = \mathbf{U}_c^H\mathbf{E}$. The objective function in (6) can be rewritten as

$$\begin{aligned}
& \mathsf{E}\left[\mathrm{tr}\{(\hat{\mathbf{H}}+\mathbf{E})^H\mathbf{C}^H\mathbf{C}(\hat{\mathbf{H}}+\mathbf{E})\}\right] \\
= & \mathsf{E}\left[\mathrm{tr}\{(\check{\mathbf{H}}+\check{\mathbf{E}})^H\mathbf{D}_c(\check{\mathbf{H}}+\check{\mathbf{E}})\}\right] \\
= & \mathrm{tr}\left\{\mathbf{D}_c\left[\check{\mathbf{H}}\check{\mathbf{H}}^H + \mathsf{E}\left[\check{\mathbf{H}}\check{\mathbf{E}}^H\right] + \mathsf{E}\left[\check{\mathbf{H}}^H\check{\mathbf{E}}\right] + \mathsf{E}\left[\check{\mathbf{E}}\check{\mathbf{E}}^H\right]\right]\right\} \\
= & \mathrm{tr}\left\{\mathbf{D}_c(\check{\mathbf{R}}+\check{\mathbf{R}}_e)\right\}, \quad (10)
\end{aligned}$$

where $\check{\mathbf{R}} = \mathsf{E}\left[\check{\mathbf{H}}\check{\mathbf{H}}^H\right]$ and $\check{\mathbf{R}}_e = \mathsf{E}\left[\check{\mathbf{E}}\check{\mathbf{E}}^H\right]$.

The probabilistic constraint in (6) becomes mathematically tractable if we can find a closed expression for the distribution of the random variable $\mathrm{tr}\{(\hat{\mathbf{H}}+\mathbf{E})^H\mathbf{C}^H\mathbf{C}(\hat{\mathbf{H}}+\mathbf{E})\}$. Applying a non-singular linear transformation [4], this random variable can be written as

$$\begin{aligned}
\mathrm{SNR} &= \mathrm{tr}\left\{(\hat{\mathbf{H}}+\mathbf{E})^H\mathbf{C}^H\mathbf{C}(\hat{\mathbf{H}}+\mathbf{E})\right\} \\
&= \mathrm{tr}\left\{(\hat{\mathbf{H}}+\mathbf{E})^H(\mathbf{V}_e^H)^{-1}\mathbf{V}_e^H\mathbf{C}^H\mathbf{C}\mathbf{V}_e\mathbf{V}_e^{-1}(\hat{\mathbf{H}}+\mathbf{E})\right\} \\
&= \mathrm{tr}\left\{\left[\mathbf{P}\mathbf{V}_e^{-1}(\hat{\mathbf{H}}+\mathbf{E})\right]^H\mathbf{D}_c\left[\mathbf{P}\mathbf{V}_e^{-1}(\hat{\mathbf{H}}+\mathbf{E})\right]\right\} \\
&= \mathrm{tr}\left\{(\tilde{\mathbf{H}}+\tilde{\mathbf{E}})^H\mathbf{D}_c(\tilde{\mathbf{H}}+\tilde{\mathbf{E}})\right\} \\
&= \sum_{i=1}^{M} d_{c_i}\sum_{j=1}^{N}\left(\tilde{h}_{ij}+\tilde{e}_{ij}\right)^2, \quad (11)
\end{aligned}$$

where $\tilde{\mathbf{H}} = \mathbf{P}\mathbf{V}_e^{-1}\hat{\mathbf{H}}$ and $\tilde{\mathbf{E}} = \mathbf{P}\mathbf{V}_e^{-1}\mathbf{E}$. The random matrix $\tilde{\mathbf{E}}$ has normal distribution with zero mean and covariance matrix $\mathbf{I}_{M\times M}$.

Using (10) and (11), the proposed approach can be reformulated as follows:

$$\max_{\mathbf{D}_c} \mathrm{tr}\{\mathbf{D}_c(\check{\mathbf{R}}+\check{\mathbf{R}}_e)\}, \quad (12)$$

subject to

$$P\,\left\{\mathrm{tr}\{(\tilde{\mathbf{H}}+\tilde{\mathbf{E}})^H\mathbf{D}_c(\tilde{\mathbf{H}}+\tilde{\mathbf{E}})\} \le \gamma\right\} \le p\,, \quad (13)$$

$$\mathrm{tr}\{\mathbf{D}_c\} = 1\,. \quad (14)$$

The robust beamformer design is now in the form of a probabilistic-constrained stochastic optimization problem.

Under the assumption that the error in CSIT is Gaussian, the stochastic optimization can be converted into a convex optimization problem which can be efficiently solved using modern convex optimization methods.

3.1 Relaxation of Convex Constraint

In convex programming, both the objective function and the constraints are required to be convex. We replace $\mathrm{tr}\{\mathbf{D}_c\} = 1$ with an inequality constraint which is easier to satisfy, that is

$$\mathrm{tr}\{\mathbf{D}_c\} \leq 1. \tag{15}$$

This is equivalent to relaxing the constraint (6) to $\mathrm{tr}\{\mathbf{C}^H\mathbf{C}\} \leq 1$.

Theorem The optimization problem defined in (12)-(14) is equivalent to that with the strict constraint (14) being replaced by the relaxed constraint (15)

Proof: Suppose the optimal solution $\bar{\mathbf{D}}_c$ lies in the region $\mathrm{tr}\{\mathbf{D}_c\} < 1$. This implies that the maximum of (14) is given by

$$\mathrm{tr}\left\{\bar{\mathbf{D}}_c\left(\check{\mathbf{R}}+\check{\mathbf{R}}_e\right)\right\}.$$

However, we can always construct another matrix $\underline{\mathbf{D}}_c^*$ by multiplying $\bar{\mathbf{D}}_c$ with a positive constant $c = 1/\mathrm{tr}\{\bar{\mathbf{D}}_c\} > 1$, so that the constraint $\mathrm{tr}\{\mathbf{D}_c\} = 1$ is satisfied. This leads to the following inequality:

$$\mathrm{tr}\left\{\mathbf{D}_c^*\left(\check{\mathbf{R}}+\check{\mathbf{R}}_e\right)\right\} > \mathrm{tr}\left\{\bar{\mathbf{D}}_c\left(\check{\mathbf{R}}+\check{\mathbf{R}}_e\right)\right\}. \tag{16}$$

This inequality (16) contradicts our assumption that $\bar{\mathbf{D}}_c$ maximizes (12). Thus, a matrix $\mathbf{D}_c$ satisfying the constraint $\mathrm{tr}\{\bar{\mathbf{D}}_c\} < 1$ can not be the optimal solution. In other words, the optimal solution always satisfies the original constraint $\mathrm{tr}\{\mathbf{D}_c\} = 1$. Hence, the objective function (12)-(14) can be equivalently transformed into a convex optimization problem by relaxing the constraint $\mathrm{tr}\{\mathbf{D}_c\} = 1$ to $\mathrm{tr}\{\mathbf{D}_c\} \leq 1$. □

3.2 Reformulation of Probabilistic Constraint

To make the proposed approach tractable, we apply Imhof's results [5] to approximate the distribution of the quadratic form $\mathrm{tr}\{(\tilde{\mathbf{H}}+\tilde{\mathbf{E}})^H\mathbf{D}_c(\tilde{\mathbf{H}}+\tilde{\mathbf{E}})\}$ and transform the probabilistic constraint into a deterministic constraint.

We consider the quadratic form (11) as a linear combination of noncentral χ^2-distributed random variables

$$\sum_{i=1}^{M} d_{c_i} \sum_{j=1}^{N} \left(\tilde{h}_{i,j}+\tilde{e}_{i,j}\right)^2 = \sum_{i=1}^{M} d_{c_i}\chi^2_{n_i,\delta_i^2}, \tag{17}$$

where $\chi^2_{n_i,\delta_i^2}$, $i = 1,\ldots,M$ are independent noncentral χ^2-distributed random variables with degree of freedom $n_i = N$ and non-centrality parameter $\delta_i^2 = \sum_{j=1}^{N}\tilde{h}_{i,j}^2$. Imhof has derived an integral form of the cumulative distribution function for random variables in the form of (17). Based on the results of [5], the probabilistic constraint can be rewritten as

$$\begin{aligned} & P\ \{\ \sum_{i=1}^{M} d_{c_i}(n_i+\delta_i)^2 \leq \gamma\} \\ = \quad & 1-\left(\frac{1}{2}+\frac{1}{\pi}\int_0^\infty \frac{\sin\theta(u)}{u\rho(u)}du\right) \\ = \quad & \frac{1}{2}-\frac{1}{2\pi}\gamma+\frac{1}{2\pi}\sum_{i=1}^{M} d_{c_i}(n_i+\delta_i^2) \end{aligned} \tag{18}$$

where

$$\lim_{u\to 0}\frac{\sin\theta(u)}{u\rho(u)} = \frac{1}{2}\sum_{i=1}^{M} d_{c_i}(n_i+\delta_i^2)-\frac{1}{2}\gamma$$

$$\lim_{u\to\infty} u\rho(u) = +\infty$$

$$\lim_{u\to\infty}\theta(u) = \begin{cases} -\infty & : \text{ if } \gamma > 0 \\ +\infty & : \text{ if } \gamma < 0 \\ \frac{\pi}{4}\sum_{i=1}^{m} n_i d_{c_i}|d_{c_i}|^{-1} & : \text{ if } \gamma = 0 \end{cases}$$

With (18), the probabilistic constraint (11) can be transformed into a convex constraint as follows

$$\frac{1}{2}-\frac{1}{2\pi}\gamma+\frac{1}{2\pi}\sum_{i=1}^{M} d_{c_i}(n_i+\delta_i^2) \leq p. \tag{19}$$

With the relaxation (15) and the expression (19), the original stochastic optimization problem (6) is now converted into the convex optimization problem defined as follows.

$$\max_{\mathbf{D}_c}\ \mathrm{tr}\{\mathbf{D}_c(\check{\mathbf{R}}+\check{\mathbf{R}}_e)\}, \tag{20}$$

subject to

$$\frac{1}{2}-\frac{1}{2\pi}\gamma+\frac{1}{2\pi}\sum_{i=1}^{M} d_{c_i}(n_i+\delta_i^2) \leq p,$$

$$\mathrm{tr}\{\mathbf{D}_c\} \leq 1$$

The optimal solution can be efficiently found by modern convex optimization algorithms, such as CVX [3]. CVX software package is a Matlab-based modeling system for convex optimization that allows constraints and objective functions to be specified using standard Matlab expression syntax.

4. SIMULATION RESULTS

The proposed beamformer is tested by simulation. We consider a single-user MIMO system with $M = 4$ transmit antennas and $N = 3$ receive antennas. A hundred Monte Carlo trials were performed in each experiment. The proposed beamformer is compared with existing techniques, such as the worst-case one-directional, equal-power loading beamformer and robust beamformer [1]. Without any loss of generality, we assume the following:

- Channel paraments: Angle of spread Δ is related to the channel state information. The angle of spread determines the spatial correlations of the channel. For the small angle spread, the correlation coefficient between the pth and qth transmit antenna can be presented as [8]

$$[\mathbf{R}]_{p,q} \approx \frac{1}{2\pi}\int_0^\pi \exp\left[-j2\pi(p-q)\Delta\frac{d_t}{\lambda}\sin\theta\right]d\theta,$$

  where λ is the wavelength of a narrow-band signal, d_t the antenna spacing and Δ the angle of spread.

- Sample covariance matrix: The channel covariance matrix $\hat{\mathbf{R}}$ is estimated by sampling the instantaneous channels

$$\hat{\mathbf{R}} = \frac{1}{N}\sum_{i=1}^{N}\hat{\mathbf{H}}\hat{\mathbf{H}}^H.$$

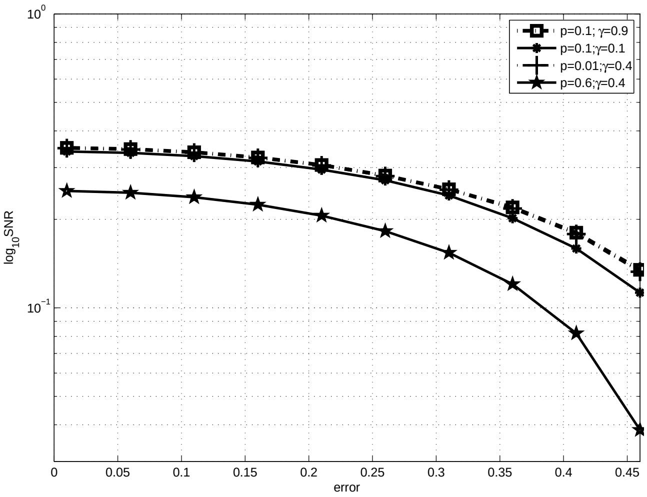

Figure 1: SNR performance under the different parament selection, where $\Delta = 45^{\circ}$

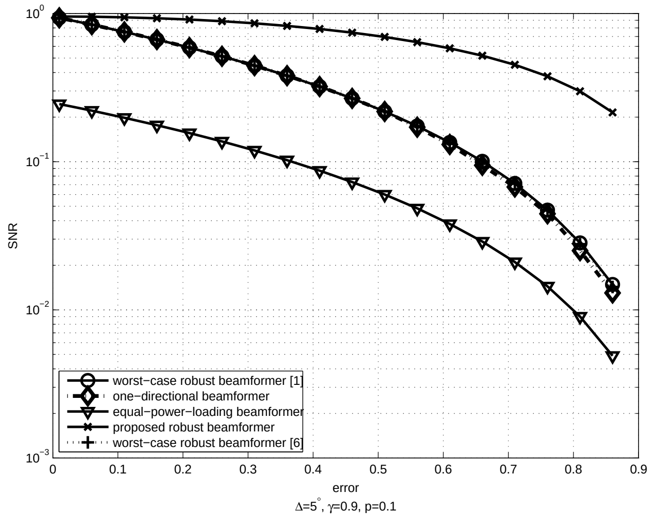

Figure 2: SNR performance of one-directional beamformer, equal-power-loading beamformer, worst-case robust beamformers [1] and [6], proposed beamformer versus error: $\gamma = 0.9$, $p = 0.1$, $\Delta = 5^{\circ}$

- Estimated error at the transmitter: We assume that the error is Gaussian distributed with zero mean and covariance matrix $\sigma^2\mathbf{I}$, that is,

$$\mathbf{E}_{M\times M} \sim \mathscr{C}\mathscr{N}(0, \sigma^2\mathbf{I}).$$

In our simulation, the error is varied from 0.01 to 0.9.

Firstly, we compare performances under varies choices of parameters γ and p, shown in Fig. 1. We set the spread angle $\Delta = 45^{\circ}$. With the same probability $p = 0.1$, the high-threshold beamformer ($\gamma = 0.9$) outperforms the low-threshold one ($\gamma = 0.4$). One the other hand, under the same SNR threshold $\gamma = 0.4$, the beamformer with $p = 0.01$ achieves an overall higher SNR than $p = 0.6$. This implies that a low outage probability ensures robustness against errors. In Fig. 1, we can also observe that the proposed transmit beamformer is sensitive to the selection of the outage probability p.

Then we compare the average SNR performance of the proposed transmit beamformer and four other existing methods. According to the quality of service (QoS) requirements, we select a low probability value $p = 0.1$ and a high SNR threshold $\gamma = 0.9$.

In Fig.2, the angle of spread is 5° and the correlation between pth and qth channel is high. That means less knowledge of CSIT can be obtained and the MRC output of SNR is more sensitive to the error. In this case, worst-case robust beamformers [1] [6] and one-directional beamformer [7] prefer to focus all available power on the channel's strongest direction. And the equal-power-loading beamformer equally loads the transmit power without considering CSIT. However, in the proposed beamformer, the instantaneous SNR is controlled by the probabilistic constraint and the proposed robust design offers the best performance over other beamformers.

In Fig.3, the spread angle is $\Delta = 25^{\circ}$ and the channel environment is better than the channel in the previous experiment. In this case, for the maximum MRC output of SNR, the transmit power trends to be loaded equally. The performances of both worst-case robust beamformers tend to that of the equal-power-loading robust beamformer. Meanwhile, the one-directional beamformer offers the worst performance as the error increases. On the other hand, the proposed beamformer still offers the highest average SNR in the entire error range.

5. CONCLUSION

In this work, we propose a novel transmit beamformer design that maximizes average SNR performance and also guarantees robustness against the CSIT errors. The robust transmit beamformer design is formulated as a stochastic optimization problem. Under the assumption that the CSIT error is Gaussian distributed, the underlying stochastic optimization problem is transformed into a convex optimization problem which can be efficiently solved by modern software packages. Simulation results show that the proposed robust transmit beamformer is less sensitive to the errors in CSIT and outperforms several state-of-the-art robust beamforming algorithms.

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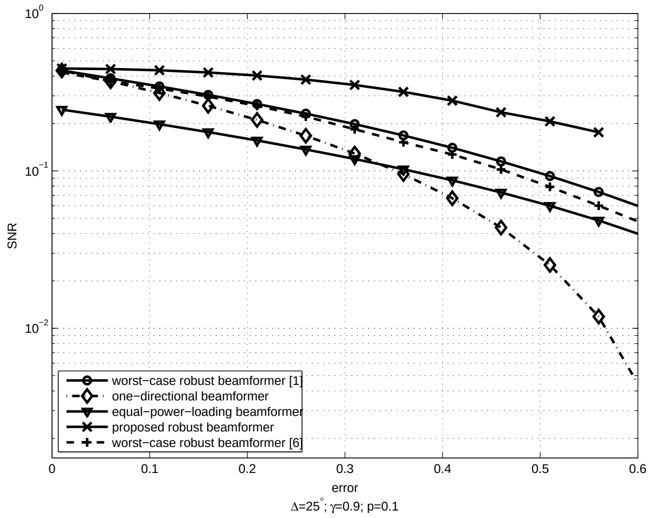

Figure 3: SNR performance of one-directional beamformer, equal-power-loading beamformer, worst-case robust beamformers [1] and [6], proposed beamformer versus error: $\gamma = 0.9$; $p = 0.1$; $\Delta = 25^\circ$

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