

BLIND IDENTIFICATION AND EQUALIZATION OF DOWNLINK TCM CODED MC-CDMA SYSTEMS USING CUMULANTS

S. Safi, M. Frikel, M. M'Saad, and A. Zeroual

Polydisciplinary Faculty, Sultan Moulay Sliman University
PoBox: 523, 23000, Beni Mellal, Morocco
phone: + (212) 78 84 44 24, fax: + (212) 23 48 52 01, email: safi.said@gmail.com

ABSTRACT

This paper describes a blind algorithm for TCM coded Multi Carrier Code Division Multiple Access (TCM coded MC-CDMA) systems equalization. The estimation of a time varying and selective frequency fading channel is considered. In order to identify, blindly, the impulse response of these channels, we have used Higher Order Statistics (HOS) to build our algorithm. We have selected two practical selective frequency fading channels called Broadband Radio Access Network (BRAN A and BRAN E) normalized for TCM coded MC-CDMA systems identification and equalization. For that, we have focussed on the BRAN A and BRAN E channels to develop a blind algorithm able to simulate the measured data with high accuracy.

1. INTRODUCTION

In this paper, we have, principally, focussed in channel impulse response estimation such as: magnitude and phase. The considered channels are with non minimum phase and selective frequency (e.i normalized channels for TCM coded MC-CDMA: BRAN A, BRAN E). In most wireless environments, there are many obstacles in the channels, such as buildings, mountains and walls between the transmitter and the receiver. Reflections from these obstacles cause many different propagation paths. This is called multi-paths propagation or a multi-path channel. The frequency impulse response, of this channel, is not flat (ideal case) but comprising some hollows and bumps, due to the echoes and reflection between the transmitter and the receiver. Another problem encountered in communication is the synchronization between the transmitter and the receiver. To solve the problem of phase estimation we will use, the Higher order cumulants (HOC) to test what's the robustness of those techniques if the channel is affected by a colored noise. HOC are a fairly topic with many applications in system theory. The HOC are only applicable to non-gaussian and non-linear process because the cumulants of a gaussian process are identically zero [1, 7]. Many real world applications are truly non-gaussian [2, 7]. Also, the Fourier transformation of HOC, which is termed higher order spectra (or polyspectra), provides an efficient tool for solving the problem of equalization technology used in communication. The major feature of HOC, from the point of view of equalization, is that the phase information of channels is present [3, 6]. Therefore they can be used to estimate the parameters of the channel model without any knowledge of the phase property (minimum phase (MP) or non-minimum phase (NMP)) of channel or the transmitted data (assuming a non-Gaussian distribution) [1, 2, 7].

In this paper, we propose an algorithm based, only, on third

order cumulants. In order to test its efficiency, we have considered practical, i.e. measured, frequency-selective fading channel, called Broadband Radio Access Network (BRAN A and BRAN E), representing respectively the transmission in indoor and outdoor scenarios. These model radio channels are normalized by the European Telecommunications Standards Institute (ETSI) in [4] and [5]. Post-equalization at the receiver for downlink MC-CDMA systems in the form of single-user detection (SUD), i.e. transmission from the base station to the mobile systems, has been investigated [8]. Recently, pre-equalization at the transmitter for downlink time division duplex (TDD), TCM coded MC-CDMA has attained increased interest and has been investigated in details [8]. In this contribution, the novel concept of blind equalization is developed and investigated for downlink TCM coded MC-CDMA systems. This paper shows that we can identify and equalize the TCM coded MC-CDMA systems blindly. However, the classical equalization of TCM coded MC-CDMA system assumes that the channel is known. The bit error rate (BER) performances of the downlink TCM coded MC-CDMA systems, using blind BRAN A and BRAN E estimation, are shown and compared with the results obtained with the classical methods (in which, the channel parameters are assumed known).

2. PROPOSED ALGORITHM

2.1 Problem Statement

The channel output of a FIR channel, excited by an unobservable input sequences, i.i.d zero-mean symbols with unit energy, belonging to some alphabet A , across a selective channel with memory p and additive colored noise. The output time series is described by the following equation

$$r(t) = h_p x(t) + n(t)$$

where $h_p = (h(1), h(2), \dots, h(p))$ represents the channel impulse response, $x(k)$ and $n(t)$ is the additive colored Gaussian noise with energy $E\{n^2(t)\} = \sigma^2$. The completely blind channel identification problem is to estimate h_p based only on the received signal $r(t)$ and without any knowledge of the energy of the transmitted data $x(t)$ nor the energy of noise. The output of the channel is characterized by its impulse response $h(n)$, which we identify "blindly" its parameters, is given in the equation

$$y(k) = \sum_{i=0}^p x(i)h(k-i); \quad r(t) = y(t) + n(t). \quad (1)$$

Let us suppose that: The additive noise $n(t)$ is Gaussian, colored or with symmetric distribution, zero mean, with variance σ^2 , i.i.d with the m^{th} order cumulants vanishes for

$m > 2$. In addition we suppose that the noise $n(t)$ is independent to $x(t)$ and $y(t)$. The channel order p is supposed to be known and $h(0) = 1$. Then the m^{th} order cumulant of the output signal is given by the following equation [1, 2, 7]

$$C_{my}(t_1, \dots, t_{m-1}) = \gamma_{mx} \sum_{i=-\infty}^{+\infty} h(i)h(i+t_1)\dots h(i+t_{m-1}) \quad (2)$$

with γ_{mx} represent the m^{th} order cumulants of the excitation signal ($x(t)$) at origin. If $m = 3$ the equation (2) yield to

$$C_{3y}(t_1, t_2) = \gamma_{3x} \sum_{i=0}^p h(i)h(i+t_1)h(i+t_2) \quad (3)$$

the same, if $m = 2$ the equation (2) becomes

$$C_{2y}(t_1) = \sigma^2 \sum_{i=0}^p h(i)h(i+t_1) \quad (4)$$

the Fourier transformation of the equations (3) and (4) gives us the spectre and bispectra respectively

$$S_{3y}(\omega_1, \omega_2) = \gamma_{3x} H(\omega_1)H(\omega_2)H(-\omega_1 - \omega_2) \quad (5)$$

$$S_{2y}(\omega) = \sigma^2 H(\omega)H(-\omega) \quad (6)$$

if we suppose that $\omega = (\omega_1 + \omega_2)$ in the equation (6), and using the equation (5) we obtain the following equation

$$H(\omega_1 + \omega_2)S_{3y}(\omega_1 + \omega_2) = \varepsilon H(\omega_1)H(\omega_2)S_{2y}(\omega_1 + \omega_2) \quad (7)$$

with $\varepsilon = \left(\frac{\gamma_{3x}}{\sigma^2}\right)$. The inverse Fourier transformation of the equation (7) demonstrates that the 3^{rd} order cumulants, the Auto-Correlation Function (ACF) and the impulse response channel parameters are combined by the following equation

$$\sum_{i=0}^p h(i)C_{3y}(t_1 - i, t_2 - i) = \varepsilon \sum_{i=0}^p h(i)h(i+t_2 - t_1)C_{2y}(t_1 - i) \quad (8)$$

if we use the property of the ACF of the stationary process, such as $C_{2y}(t) \neq 0$ only for $(-p \leq t \leq p)$ and vanish elsewhere. In addition if we take $t_1 = -p$, the equation (8) takes the form

$$\sum_{i=0}^p h(i)C_{3y}(-p - i, t_2 - i) = \varepsilon h(0)h(t_2 + p)C_{2y}(-p) \quad (9)$$

else if we suppose that $t_2 = -p$, the equation (9) becomes $C_{3y}(-p, -p) = \varepsilon h(0)C_{2y}(-p)$, so, the equation (9) yield to:

$$\sum_{i=0}^p h(i)C_{3y}(-p - i, t_2 - i) = h(t_2 + p) \quad (10)$$

else if we suppose that the system is causal, i.e. that $h(i) = 0$ if $i < 0$. So, for $t_2 = -p, \dots, 0$, the system of equations (10) can be written in compact form as

$$Mh_p = d \quad (11)$$

with M the matrix of size $(p+1) \times (p)$ element, h_p a column vector constitute by the unknown impulse response parameters $h(n) : n = 1, \dots, p$ and d is a column vector of size $(p+1) \times (1)$ as indicated in the equation (10). The Least Squares solution (LS) of the system of equation (11), permit an identification of the parameters $h(n)$ blindly and without any 'information' of the input selective channel. So, the solution will be written under the following form

$$h_p = (M^T M)^{-1} M^T d \quad (12)$$

2.2 Equalization of TCM coded MC-CDMA system

The principles of TCM coded MC-CDMA [8] is that a single data symbol is transmitted at multiple narrow band sub-carriers. Indeed, in MC-CDMA systems, spreading codes are applied in the frequency domain and transmitted over independent sub-carriers. However, multicarrier systems are very sensitive to synchronization errors such as symbol timing error, carrier frequency offset and phase noise. Synchronization errors cause loss of orthogonality among subcarriers and considerably degrade the performance especially when large number of subcarriers presents. There have been many approaches on synchronization algorithms in literature [8]. In this part, we describe a blind equalization techniques for TCM coded MC-CDMA systems using the algorithm presented above.

2.2.1 TCM coded MC-CDMA Transmitter

The TCM coded MC-CDMA modulator spreads the data of each user in frequency domain. In addition, the complex symbol g_i of each user i is, first, multiply by each chips $c_{i,k}$ of spreading code, and then apply to the modulator of multi-carriers. Each sub-carrier transmits an element of information multiply by a code chip of that sub-carrier.

We consider, for example, the case where the length L_c of spreading code is equal to the number N_p of sub-carriers. The optimum space between two adjacent sub-carriers is equal to inverse of duration T_c of chip of spreading code in order to guaranty the orthogonality between sub-carriers. The occupied spectral band is, then equal: $B = \frac{(N_p+1)}{T_c}$. The TCM coded MC-CDMA emitted signal is

$$x(t) = \frac{a_i}{\sqrt{N_p}} \sum_{k=0}^{N_p-1} c_{i,k} e^{2jfk t} \quad (13)$$

where $f_k = f_0 + \frac{1}{T_c}$, N_u : is the users number and N_p is the number of sub-carriers.

We suppose that, the channel is time invariant and it's impulse response is characterized by: P paths of magnitudes β_p and phase θ_p . So the impulse response is given by

$$h(\tau) = \sum_{p=0}^{P-1} \beta_p e^{j\theta_p} \delta(\tau - \tau_p)$$

The relationship between the emitted signal $s(t)$ and the received signal $r(t)$ is given by: $r(t) = h(t) * x(t) + n(t)$

$$\begin{aligned} r(t) &= \int_{-\infty}^{+\infty} \sum_{p=0}^{P-1} \beta_p e^{j\theta_p} \delta(\tau - \tau_p) x(t - \tau_p) d\tau + n(t) \\ &= \sum_{p=0}^{P-1} \beta_p e^{j\theta_p} x(t - \tau_p) + n(t) \end{aligned} \quad (14)$$

where $n(t)$ is an additive white gaussian noise (AWGN).

2.2.2 MC-CDMA Receiver

The downlink received TCM coded MC-CDMA signal at the input receiver is given by the following equation :

$$r(t) = \frac{1}{\sqrt{N_p}} \sum_{p=0}^{P-1} \sum_{k=0}^{N_p-1} \sum_{i=0}^{N_u-1} \Re \left\{ \beta_p e^{j\theta} a_i c_{i,k} e^{2j\pi(f_0+k/T_c)(t-\tau_p)} \right\} + n(t) \quad (15)$$

The equalization goal, is to obtain a good estimation of the symbol a_i . At the reception, we demodulate the signal according the

N_p sub-carriers, and then we multiply the received sequence by the code of the user.

After the equalization and the spreading operation, the estimation \hat{a}_i of the emitted user symbol a_i , of the i^{th} user can be written by the following equation

$$\hat{a}_i = \underbrace{\sum_{k=0}^{N_p-1} c_{i,k}^2 g_k h_k a_i}_{I} + \underbrace{\sum_{q=0}^{N_u-1} \sum_{k=0}^{N_p-1} c_{i,k} c_{q,k} g_k h_k a_q}_{II \quad (q \neq i)} + \underbrace{\sum_{k=0}^{N_p-1} c_{i,k}^2 g_k n_k}_{III} \quad (16)$$

where the term I, II and III of the equation (16) are, respectively, the desired signal (signal of the considered user), a multiple access interferences (signals of the others users) and the additive white gaussian noise (AWGN) ponderated by the equalization coefficient and by spreading code of the chip. We suppose that the user data are independents and the h_k are ponderate by the g_k equalization coefficient are independent to the indices k .

3. TECHNIQUES OF EQUALIZATION

3.1 Zero forcing (ZF)

The principle of the ZF, is to completely cancel the distortions brought by the channel. The gain factor of the ZF equalizer, is given by the equation

$$g_k = \frac{1}{|h_k|}$$

by that manner, each symbol is multiplied by a unit magnitude. So, the estimated received symbol, \hat{a}_i of symbol a_i of the user i is described by:

$$\hat{a}_i = \underbrace{\sum_{k=0}^{N_p-1} c_{i,k}^2 a_i}_{I} + \underbrace{\sum_{q=0}^{N_u-1} \sum_{k=0}^{N_p-1} c_{i,k} c_{q,k} a_q}_{II \quad (q \neq i)} + \underbrace{\sum_{k=0}^{N_p-1} c_{i,k} \frac{1}{h_k} n_k}_{III} \quad (17)$$

if we suppose that the spreading code are orthogonal, this fact, the equation (17) becomes

$$\hat{a}_i = \sum_{k=0}^{N_p-1} c_{i,k}^2 a_i + \sum_{k=0}^{N_p-1} c_{i,k} \frac{1}{h_k} n_k \quad (18)$$

thus, the performance obtained using this detection technique is independent of the users number, in condition that the spreading codes are orthogonal. But, if the h_k value is very weak, (great fading cases), the values g_k increases and the noise will be amplified (second term of the equation (18)).

3.2 Minimum Mean Square Error, (MMSE)

The MMSE techniques combine the minimization the multiple access interference and the maximization of signal to noise ratio. Thus as its name indicates, the MMSE techniques minimize the mean squares error for each sub-carrier k between the transmitted signal x_k and the output detection $g_k r_k$:

$$\varepsilon[|\varepsilon|^2] = \varepsilon[|x_k - g_k r_k|^2] \quad (19)$$

the minimisation of the function $\varepsilon[|\varepsilon|^2]$, give us the optimal equalizer coefficient, under the minimization of the mean square error criterion, of each sub-carrier as:

$$g_k = \frac{h_k^*}{|h_k|^2 + \frac{1}{\gamma_k}} \quad (20)$$

where $\gamma_k = \frac{E[|s_k h_k|^2]}{E[n_k]^2}$.

If the values h_k are small, the SNR for each sub-carrier is minimal. So, the use of the MMSE criterion avoid the noise amplification. On the other hand, the greatest values of the h_k and g_k being, inversely, proportional, allows to restore orthogonality between users. So, the estimated received symbol, \hat{a}_i of symbol a_i of the user i is described by:

$$\hat{a}_i = \sum_{k=0}^{N_p-1} c_{i,k}^2 \frac{|h_k|^2}{|h_k|^2 + \frac{1}{\gamma_k}} a_i + \sum_{k=0}^{N_p-1} c_{i,k} \frac{h_k^*}{|h_k|^2 + \frac{1}{\gamma_k}} n_k \quad (21)$$

4. SIMULATION

4.1 BRAN A radio channel

In this paragraph we consider the BRAN A model representing the fading radio channels, the data corresponding to this model is measured in an indoor case for Multi-Carrier Code Division Multiple Access (MC-CDMA) systems. The following equation (22) describes the impulse response of BRAN A radio channel.

$$h_A(n) = \sum_{i=0}^{N_T} h_i \delta(n - \tau_i) \quad (22)$$

where $\delta(n)$ is Dirac function, h_i the magnitude of the target i , $N_T = 18$ the number of target and τ_i is the time delay (from the origin) of target i . In the Table 1 we have summarized the values corresponding the BRAN A radio channel impulse response [4, 5].

Table 1: Delay and magnitudes of 18 targets of BRAN A channel (τ_i : delay (ns), h_i : magnitude (dB))

τ_i (ns)	0	10	20	30	40	50
h_i (dB)	0	-0.9	-1.7	-2.6	-3.5	-4.3
τ_i (ns)	60	70	80	90	110	140
h_i (dB)	-5.2	-6.1	-6.9	-7.8	-4.7	-7.3
τ_i (ns)	170	200	240	290	340	390
h_i (dB)	-9.9	-12.5	-13.7	-18	-22.4	-26.7

4.1.1 Blind channel impulse response estimation of BRAN A

Although, the BRAN A radio channel is constituted by $N_T = 18$ parameters and seeing that the latest parameters are very small. So, in order to estimate the parameters -of BRAN A radio channel impulse response- using the maximum information obtained by calculating the cumulants function, we take the following procedure:

- We decompose the BRAN A radio channel impulse response into four sub-channels as follow:

$$h(n) = \sum_{j=1}^4 h_j(n); \quad \left(h_j(n) = \sum_{i=j}^{P_j} C_j \delta(n - \tau_j); \sum_{j=1}^4 P_j = N_T \right) \quad (23)$$

- We estimate the parameters of each sub-channel independently, using the proposed algorithm [section II].

- We add all sub channel parameters, to construct the full BRAN A radio channel impulse response.

This procedure gives us a good estimation of the impulse response channel. In the case of $SNR = 24dB$ and for the data length $N = 4096$, we represent estimated magnitude and phase of the impulse response BRAN A radio channel (Fig. 1)

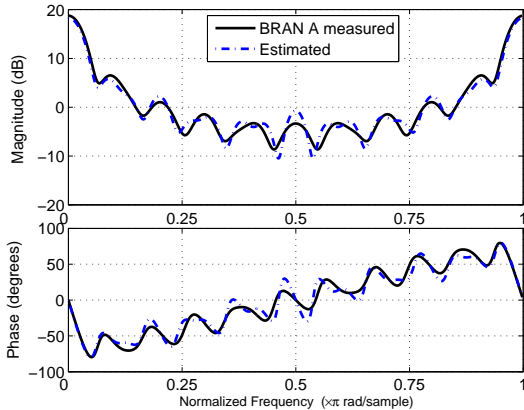


Figure 1: Estimated magnitude and phase of BRAN A radio channel, for $N = 4096$ and for $SNR = 24dB$.

From the figure 1 we observe that the estimated magnitude and phase have the same form and we have not more difference between the estimated and the true ones. In conclusion, if we increase the data length, i.e. $N \geq 4096$, the estimated magnitude and phase will be more closed to the true ones. This result is very interest such as the estimation of impulse response selective frequency channel impulse response in noisy environment. If the data sample increase, we remark that the noise is ‘approximately’ without influence on the BRAN A radio channel impulse response estimation.

4.2 BRAN E radio channel

We have considered in the previous paragraph the BRAN A model representing the fading radio channels, where the data corresponding to this model are measured in a scenario of transmission in indoor environment. But in this section we consider the BRAN E model representing the fading radio channels, where the model parameters are measured in outdoor scenario. The equation (24) describes the impulse response of BRAN E radio channel.

$$h_E(n) = \sum_{i=0}^{N_T} h_i \delta(n - \tau_i) \quad (24)$$

where $\delta(n)$ is Dirac function, h_i the magnitude of the target i , $N_T = 18$ the number of target and τ_i is the delay of target i . In the Table 1 we have represented the values corresponding to the BRAN E radio channel impulse response.

Table 2: Delay and magnitudes of 18 targets of BRAN E channel (τ_i : delay (ns), h_i : magnitude (dB))

τ_i (ns)	0	10	20	40	70	100
h_i (dB)	-4.9	-5.1	-5.2	-0.8	-1.3	-1.9
τ_i (ns)	140	190	240	320	430	560
h_i (dB)	-0.3	-1.2	-2.1	0	-1.9	-2.8
τ_i (ns)	710	880	1070	1280	1510	1760
h_i (dB)	-5.4	-7.3	-10.6	-13.4	-17.4	-20.9

4.2.1 Blind channel impulse response estimated of BRAN E

In the same manner, such as BRAN A, we estimate the parameters of the BRAN E radio channel impulse response. This procedure gives a good estimation of the impulse response channel. In the Fig. 5 we represent the magnitude and phase estimation of the impulse response BRAN E radio channel, In the case of $SNR = 24dB$ and $N = 4096$. From the (Fig. 2) we observe that the estimated magnitude and phase are the same allure and we have not more difference between the estimated and the true ones. In conclusion, if the data length increases, i.e. $N > 4096$, the estimated magnitude and phase will be more closed to the true ones. If the SNR is superior to $24dB$ we remark that the noise is ‘approximately’ without influence on the BRAN A radio channel impulse response estimation.

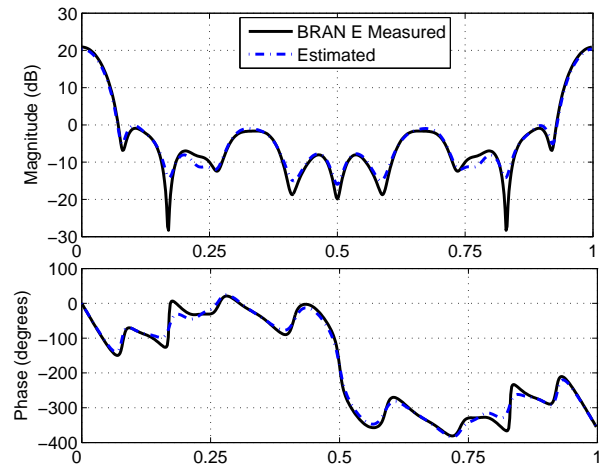


Figure 2: Estimated magnitude and phase of BRAN E radio channel, for $N = 4096$ and for $SNR = 24dB$.

5. TCM CODED MC-CDMA SYSTEM PERFORMANCE

In order to evaluate the performance of the TCM coded MC-CDMA systems, using the proposed blind algorithm. These performances are evaluated by calculation of the Bit Error Rate (BER), for the two equalizers ZF and MMSE, using the measured and estimated (using the proposed blind algorithm) BRAN A and BRAN E channel impulse response. The results are evaluated for different values of SNR .

5.1 ZF and MMSE equalizers: case of BRAN A channel

In the (Fig. 3), we represent the BER for different SNR , using the measured and estimated BRAN A channel. The equalization is performed using the ZF equalizer. The same, in (Fig. 4) we represent the BER for different SNR , using the measured and estimated BRAN A channel. The equalization is performed using the MMSE equalizer.

The BER simulation for different SNR demonstrates that the results obtained by the blind algorithm have the same form comparing to those obtained using the measured data. From the figure (3 and 4) we conclude that: if the $SNR \geq 20dB$ we have only a BER of 10^{-4} . In real case, and in abrupt channel, the proposed blind identification techniques can be useful as remarked in the figure (3 and 4).

5.2 ZF and MMSE equalizers : case of BRAN E channel

We represent in the Fig. 10, the simulation results of BER estimation using the measured and blind estimated of the BRAN E channel impulse response. The equalization is performed using ZF equalizer. The Fig. 5 demonstrates clearly that the BER obtained using blind ZF equalization gives good results like these obtained using measured values for ZF equalization. Both of the two techniques

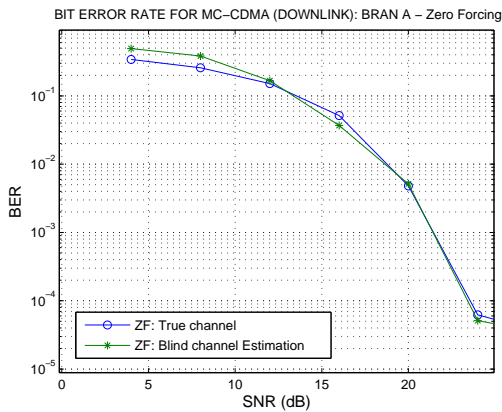


Figure 3: BER of the estimated and measured BRAN A channel, ZF equalizer.

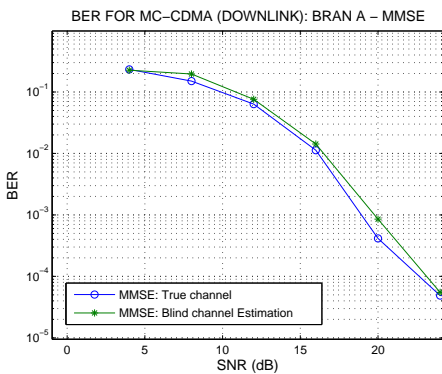


Figure 4: BER of the estimated and measured BRAN A channel, MMSE equalizer.

gives the 1 bit error if we receive 10^4 if the $SNR \geq 24dB$. In the same manner, we represent in Fig. 6 the simulation results of BER estimation using the measured and blind estimated of the BRAN E channel impulse response. The equalization is performed using MMSE equalizer.

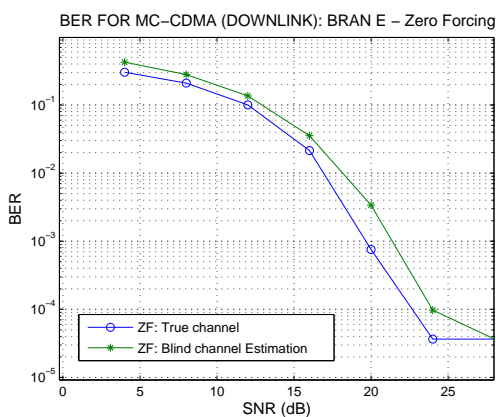


Figure 5: BER of the estimated and measured BRAN E channel, ZF equalizer.

From the figure 6, we observe that the blind MMSE equalization gives approximately the same results obtained using the measured BRAN E values for MMSE equalization. So, if the SNR values are superior to $20dB$, we observe that 1 bit error occurred when we receive 10^3 bit, but if the $SNR \geq 24dB$ we obtain only one bit error for 10^4 bit received.

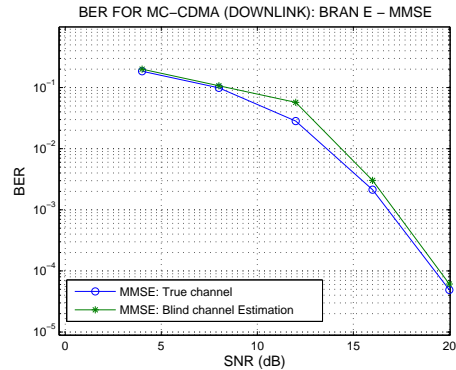


Figure 6: BER of the estimated and measured BRAN E channel, MMSE equalizer.

6. CONCLUSION

In this paper we have presented an algorithm based on third order cumulants. The proposed algorithm is used to identify the parameters of the impulse response of the frequency selective channel such as the experimental channels, BRAN A and BRAN E. The simulation results show the efficiency of our algorithm, mainly if the data input is sufficient. The phase estimation of the channel impulse response is estimated with higher precision, this is because the HOC constitute the best element to estimate the system phases. Also the magnitude of the impulse response is estimated with an acceptable precision in noisy environment in the case of small number of samples. In part of two experimental channels for TCM coded MC-CDMA systems application, we have estimated (with good precision) the impulse response radio channels of the BRAN A and BRAN E.

REFERENCES

- [1] S. Safi, A. Zeroual, "Blind identification in noisy environment of non-minimum phase Finite Impulse Response (FIR) using higher order statistics," *International Journal of Systems Analysis Modelling Simulation, Taylor Francis* vol. 43 no. 5 pp. 671–681, May 2003.
- [2] S. Safi, A. Zeroual, "Blind Parametric identification of linear stochastic Non-Gaussian FIR Systems using higher order cumulants," *International Journal of Systems Sciences Taylor Francis*. vol. 35, no. 15, pp. 855-867, Dec. 2004.
- [3] J.G. Proakis, *Digital Communications*, 4th edition : Mc Graw Hill, New York 2000.
- [4] ETSI, "Broadband Radio Access Networks (BRAN); HIPERLAN Type 2; Physical Layer", Dcembre 2001.
- [5] ETSI, "Broadband Radio Access Networks (BRAN); High Performance Radio Logical Area Network (HIPERLAN) Type 2; Requirements and architectures for wireless broadband access", Janvier 1999.
- [6] L. Ju and H. Zhenya, "Blind identification and equalization using higher-order cumulants and ICA algorithms," *Proceeding int. conf. Neural Networks and Brain (ICNN B'98)*, Beijing, October 1998.
- [7] S. Safi and A. Zeroual, "MA system identification using higher order cumulants: Application to modelling solar radiation," *International Journal of Statistical Computation and Simulation, Taylor Francis*, vol. 72, no. 7, pp. 533–548, October 2002.
- [8] Jean-Paul M. G. Linnartz, "Performance Analysis of Synchronous MC-CDMA in Mobile Rayleigh Channel With Both Delay and Doppler Spreads". *IEEE Transactions on Vehicular Technology*, Vol. 50, No. 6, November 2001.