16th European Signal Processing Conference (EUSIPCO 2008), Lausanne, Switzerland, August 25-29, 2008, copyright by EURASIP

ON-LINE DETECTION OF THE NUMBER OF NARROWBAND SIGNALS WITH A UNIFORM LINEAR ARRAY

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ABSTRACT

For any practical direction estimation and tracking system in array processing, estimating the number of incident signals accurately and tracking its possible changes in an on-line way is a critical requirement. In this paper, a new QR-based adaptive detection algorithm is proposed to estimate the coherent/incoherent narrowband signals impinging on a uniform linear array (ULA), where the updating of eigenvalues and the threshold setting are avoided. The effectiveness of the proposed method is verified through numerical examples, and simulation results show that the proposed method has good detection performance to track the number of suddenly appearing/disappearing incident signals or that of closely-spaced signals with time-varying directions.

1. INTRODUCTION

In sensor array processing, the problem of detecting the number of incident signals is closely linked with that of estimating their directions of arrival (DOAs) from the noisy measurements, and it is a key step in high-resolution subspace-based direction estimators. Furthermore, most of these direction estimators usually assume that the number of incident signals (i.e., the dimension of signal subspace or the effective rank of array covariance matrix) is fixed and known *a priori*. However, the number of incident signals is often unknown and/or time-varying in particularly nonstationary environment, and hence the performance of direction estimation can be adversely affected if the number of signals is not determined correctly. Therefore estimating the number of incoming signals accurately and tracking its possible changes in an on-line way is a critical requirement for any practical DOA estimation and tracking system.

The well-known batch methods for detection of the number of incident signals are eigenstructure-based nonparametric schemes because of their relatively computational simplicity without the need to estimate direction parameters, and the most popular one is the eigenvalue-based Akaike information criterion (AIC) and minimum description length (MDL) criterion [2], where the "multiplicity" of the smallest eigenvalues of a covariance matrix is utilized, and any subjective setting of a threshold required in conventional hypothesis testing is not required. Unfortunately, the computational intensiveness and time consumption of eigendecomposition will obstruct their implementation in an on-line manner (e.g., see [1], [7] and references therein).

Recently on-line detection/tracking problem has been considered by taking advantages (such as estimated eigenvalus or matrix structure) of some subspace trackers, which were mainly developed to facilitate adaptive implementations of spectral estimation or direction estimation. Firstly, by comparing the estimated eigenvalues obtained by the QR decomposition based Bauer's (bi-)iteration [3], [4] or spherical subspace tracker (e.g., [5]-[7]) with an estimated noise floor level, adaptive rank estimators were proposed to determine the number of signals [3], [4], [7], [8], but these eigenvalue-threshold comparing methods suffer from the disadvantage of setting an appropriate threshold factor. By using the estimated eigenvalues provided by the (bi-)Lanczos based fast subspace decomposition (FSD), the FSD-based modified likelihood ratio test and FSD-MDL schemes were developed [9], [10]. By combing the MDL criterion [2] with the projection approximation subspace tracking with deflation (PASTd) [11] and the four-level signal averaged (SA4) spherical subspace tracker [12], the PASTd-MDL and SA4-MDL were also proposed [13], [12]. Next, since the matrix structure provided by the QR decomposition [14] or URV/ULV decomposition [15], [16] of the data matrix and its variants [29] is utilizable in subspace tracking, some rank revealing based methods were suggested to track the number of signals [15], [17]-[19], [29], yet they usually require prespecified tolerances based on *a priori* knowledge of noise variance and/or the estimation of condition number. Additionally, a condition number based on-line detection scheme [20] and an invariant subspace updating (ISU) based parametric method with threshold comparison [28] were also presented, repectively. Unfortunately, most of these on-line detection methods perform poorly, when the incident signals are coherent (i.e., fully correlated) such as in multipath propagation environment, which is often encountered in mobile communication systems caused by various reflections.

On the other hand, some batch and nonparametric detection methods without eigendecomposition were studied [21]-[24]. The QR-based method [21] is applicable to coherent signals, nevertheless it needs *a priori* knowledge of true noise variance and subjective assessment, and its performance generally degrades in difficult scenarios. The modified MDL methods based on the QR with column pivoting and rank revealing QR decomposition (RRQR) are suitable for non-coherent signals [22], [23], while the supervised training approach [24] is restricted for no more than two incoherent/coherent signals without restrictive hypotheses and implemented with neural network.

Therefore the purpose of this paper is to investigate the on-line detection of the number of narrowband signals regardless of their statistical correlations. Based on the previously proposed batch and nonparametric method for estimating the number of signals without eigendecomposition (MENSE) for the coherent signals in stationary environment [25], a new QR-based adaptive detection algorithm is proposed for incident signals impinging on a uniform linear ar-

This work was supported in part by the National Natural Science Foundation of China under Grants 60772096 and 60640440661.

ray (ULA), where the updating of eigenvalues and the threshold setting are not needed. The effectiveness of the proposed method is verified through numerical examples, and simulation results show that the proposed method has good detection performance to track the number of suddenly appearing/disappearing signals or that of the signals with time-varying directions which even cross in their trajectories.

2. DATA MODEL AND BASIC ASSUMPTIONS

Consider a ULA of M identical and omnidirectional sensors with adjacent spacing d and assume that p ($p < M/2$) narrowband signals $\{s_i(t)\}$ are in the far-field and impinge from the directions $\{\theta_i(t)\}$. The received signal $y_m(t)$ at the mth sensor is given by

$$y_m(t) = \sum_{i=1}^{p} s_i(t)\, e^{j\omega_0(m-1)\tau(\theta_i(t))} + w_m(t)$$
$$= \boldsymbol{b}_m^T(\theta)\boldsymbol{s}(t) + w_m(t) \quad (1)$$

where $\boldsymbol{b}_m(\theta) = [e^{j\omega_0(m-1)\tau(\theta_1(t))}, e^{j\omega_0(m-1)\tau(\theta_2(t))}, \cdots, e^{j\omega_0(m-1)\tau(\theta_p(t))}]^T$, $\boldsymbol{s}(t) = [s_1(t), s_2(t), \cdots, s_p(t)]^T$, while $w_m(t)$ is the additive noise, $\omega_0 = 2\pi f_0$, $\tau(\theta_i(t)) = (d/c)\sin\theta_i(t)$, and c and f_0 are the propagation speed and center frequency. Then we have a compact data model

$$\boldsymbol{y}(t) = \boldsymbol{A}(\theta(t))\boldsymbol{s}(t) + \boldsymbol{w}(t) \quad (2)$$

where $\boldsymbol{y}(t) = [y_1(t), y_2(t), \cdots, y_M(t)]^T$, $\boldsymbol{w}(t) = [w_1(t), w_2(t), \cdots, w_M(t)]^T$, and $\boldsymbol{A}(\theta(t))$ is the array response matrix given by $\boldsymbol{A}(\theta(t)) = [\boldsymbol{b}_1^T(\theta), \boldsymbol{b}_2^T(\theta), \cdots, \boldsymbol{b}_M^T(\theta)]^T = [\boldsymbol{a}(\theta_1(t)), \cdots, \boldsymbol{a}(\theta_p(t))]$ with $\boldsymbol{a}(\theta_i(t)) = [1, e^{j\omega_0\tau(\theta_i(t))}, \cdots, e^{j\omega_0(M-1)\tau(\theta_i(t))}]^T$.

Here the incident signals $\{s_i(k)\}$ are temporally complex white Gaussian random processes with zero-mean and variance given by $E\{s_i(k)s_i^*(t)\} = c_{s_i}\delta_{k,t}$ and $E\{s_i(k) \cdot s_i(t)\} = 0$, and they are mutually uncorrelated or correlated (even coherent), i.e., the rank of source signal covariance matrix $\boldsymbol{C}_s$ is given by $1 \le \mathrm{rank}(\boldsymbol{C}_s) \le p$, where $\boldsymbol{C}_s = E\{\boldsymbol{s}(t)\boldsymbol{s}^H(t)\}$. The additive noises $\{w_m(t)\}$ are temporally and spatially complex white Gaussian random processes with zero-mean and the covariance matrix given by $E\{w_m(k)w_i^*(t)\} = \sigma^2\delta_{m,i}\delta_{k,t}$ and $E\{w_m(k)w_i(t)\} = 0$, where $E\{\cdot\}$, $(\cdot)^*$, and $\delta_{n,t}$ denote the expectation, complex conjugate, and Kronecker delta, and they are uncorrelated with the incident signals.

Furthermore in order to adaptively estimate and track the number of incident signals even with time-varying directions, we assume that $\theta_i(t)$ is slowly time-varying (relative to the sampling rate $1/T_s$ [20]) so that $\theta_i(t) \approx \theta_i(nT)$ for $t \in [nT, (n+1)T)$ and $n = 0, 1, \cdots$, while N snapshots of array data are available over an interval T of parameter updating, i.e., $T = NT_s$, and N is called as the window size herein. Hence the on-line number detection is formulated as the estimating the number of signals p at the instant n for $n = 0, 1, \cdots$ from N snapshots of $\{\boldsymbol{y}(k)\}$ measured at $k = nN, nN+1, \cdots, (n+1)N-1$.

3. ON-LINE ALGORITHM FOR NUMBER DETECTION

3.1 Principle of QR-Based Number Detection in Batch Manner

By exploiting the array geometry and its shift invariance property with subarray averaging, we proposed a QR-based MENSE detection for the coherent signals in stationary environment [25]. In order to gain the insights into the adaptive detection algorithm, we firstly extend the MENSE to the general case of incident signals in a unified formwork. Here the directions are assumed to be distinct and time invariant, i.e. $\theta_i(k) = \theta_i$, and $\boldsymbol{A}(\theta(k))$ is denoted as $\boldsymbol{A}$ for simplicity.

By dividing the full array into L overlapping subarrays with $\bar{p}$ sensors in the forward and backward directions, where $L = M - \bar{p} + 1$ and $\bar{p} \ge p$ (see the choice of subarray size $\bar{p}$ in Section 3.2), then for $l = 1, 2, \cdots, L$, the signal vectors of the lth forward/backward subarray are given by

$$\boldsymbol{y}_{fl}(k) = [y_l(k), y_{l+1}(k), \cdots, y_{l+\bar{p}-1}(k)]^T$$
$$= \boldsymbol{A}_1\boldsymbol{D}^{l-1}\boldsymbol{s}(k) + \boldsymbol{w}_{fl}(k) \quad (3)$$
$$\boldsymbol{y}_{bl}(k) = [y_{M-l+1}(k), y_{M-l}(k), \cdots, y_{L-l+1}(k)]^H$$
$$= \boldsymbol{A}_1\boldsymbol{D}^{-(M-l)}\boldsymbol{s}^*(k) + \boldsymbol{w}_{bl}(k) \quad (4)$$

where $\boldsymbol{w}_{fl}(k) = [w_l(k), w_{l+1}(k), \cdots, w_{l+\bar{p}-1}(k)]^T$, $\boldsymbol{w}_{bl}(k) = [w_{M-l+1}(k), w_{M-l}(k), \cdots, w_{L-l+1}(k)]^H$, $\boldsymbol{A}_1$ is the submatrix of $\boldsymbol{A}$ consisting of the first $\bar{p}$ rows, $\boldsymbol{D} = \mathrm{diag}(\boldsymbol{b}_2(\theta))$, and $\mathrm{diag}(\cdot)$ denotes the diagonal operation which extracts the diagonal of a matrix as a vector or constructs a diagonal matrix with the elements of a vector. Then by defining four correlation vectors between the signal vectors in (3) and (4) and the received signals $y_1(k)$ or $y_M(k)$ as $\boldsymbol{\varphi}_{fl} = E\{\boldsymbol{y}_{fl}(k)y_M^*(k)\}$, $\bar{\boldsymbol{\varphi}}_{fl} = E\{\boldsymbol{y}_{fl}(k)y_1^*(k)\}$, $\boldsymbol{\varphi}_{bl} = E\{y_1(k)\boldsymbol{y}_{bl}(k)\}$, and $\bar{\boldsymbol{\varphi}}_{bl} = E\{y_M(k)\boldsymbol{y}_{bl}(k)\}$, we can form four Hankel correlation matrices $\boldsymbol{\Phi}_f$, $\bar{\boldsymbol{\Phi}}_f$, $\boldsymbol{\Phi}_b$, and $\bar{\boldsymbol{\Phi}}_b$ to alleviate the noise influence and to provide a basis for number detection

$$\boldsymbol{\Phi}_f = [\boldsymbol{\varphi}_{f1}, \boldsymbol{\varphi}_{f2}, \cdots, \boldsymbol{\varphi}_{fL-1}]^T \quad (5)$$
$$\bar{\boldsymbol{\Phi}}_f = [\bar{\boldsymbol{\varphi}}_{f2}, \bar{\boldsymbol{\varphi}}_{f3}, \cdots, \bar{\boldsymbol{\varphi}}_{fL}]^T \quad (6)$$
$$\boldsymbol{\Phi}_b = [\boldsymbol{\varphi}_{b1}, \boldsymbol{\varphi}_{b2}, \cdots, \boldsymbol{\varphi}_{bL-1}]^T \quad (7)$$
$$\bar{\boldsymbol{\Phi}}_b = [\bar{\boldsymbol{\varphi}}_{b2}, \bar{\boldsymbol{\varphi}}_{b3}, \cdots, \bar{\boldsymbol{\varphi}}_{bL}]^T. \quad (8)$$

Lemma: If the array is partitioned properly so that the subarray size and the number of incident signals satisfy the inequality $p \le \bar{p} < M - p$, the ranks of four Hankel correlation matrices $\boldsymbol{\Phi}_f$, $\bar{\boldsymbol{\Phi}}_f$, $\boldsymbol{\Phi}_b$, and $\bar{\boldsymbol{\Phi}}_b$ will equal the number of signals regardless of the statistical correlations between the incident signals.

Proof: By applying the facts that $\boldsymbol{b}_m(\theta) = \boldsymbol{D}^{m-1}\boldsymbol{b}_1(\theta)$, $\boldsymbol{s}^T(k)\boldsymbol{D} = \boldsymbol{b}_2^T(\theta)\boldsymbol{S}(k)$, and $E\{\boldsymbol{S}(k)\boldsymbol{s}^H(k)\boldsymbol{b}_m^*(\theta)\} = \mathrm{diag}(\boldsymbol{C}_s\boldsymbol{b}_m^*(\theta))$, where $\boldsymbol{S}(k) = \mathrm{diag}(\boldsymbol{s}(k))$, from (3)-(8), under the assumptions on data model, we can obtain after some manipulations

$$\boldsymbol{\Phi}_f = E\{\boldsymbol{Y}_f(k)y_M^*(k)\} = \bar{\boldsymbol{A}}E\{\boldsymbol{S}(k)\boldsymbol{s}^H(k)\boldsymbol{b}_M^*(\theta)\}\boldsymbol{A}_1^T$$
$$= \bar{\boldsymbol{A}}\mathrm{diag}(\boldsymbol{C}_s\boldsymbol{b}_M^*(\theta))\boldsymbol{A}_1^T \quad (9)$$
$$\bar{\boldsymbol{\Phi}}_f = E\{\bar{\boldsymbol{Y}}_f(k)y_1^*(k)\} = \bar{\boldsymbol{A}}\boldsymbol{D}\mathrm{diag}(\boldsymbol{C}_s\boldsymbol{b}_1^*(\theta))\boldsymbol{A}_1^T \quad (10)$$
$$\boldsymbol{\Phi}_b = E\{\boldsymbol{Y}_b(k)y_1(k)\}$$
$$= \bar{\boldsymbol{A}}\boldsymbol{D}^{-(M-1)}(\mathrm{diag}(\boldsymbol{C}_s\boldsymbol{b}_1^*(\theta)))^*\boldsymbol{A}_1^T \quad (11)$$
$$\bar{\boldsymbol{\Phi}}_b = E\{\bar{\boldsymbol{Y}}_b(k)y_M(k)\}$$
$$= \bar{\boldsymbol{A}}\boldsymbol{D}^{-(M-2)}(\mathrm{diag}(\boldsymbol{C}_s\boldsymbol{b}_M^*(\theta)))^*\boldsymbol{A}_1^T \quad (12)$$

where $\boldsymbol{Y}_f(k) = [\boldsymbol{y}_{f1}(k), \boldsymbol{y}_{f2}(k), \cdots, \boldsymbol{y}_{fL-1}(k)]^T$, $\bar{\boldsymbol{Y}}_f(k) = [\boldsymbol{y}_{f2}(k), \boldsymbol{y}_{f3}(k), \cdots, \boldsymbol{y}_{fL}(k)]^T$, $\boldsymbol{Y}_b(k) = [\boldsymbol{y}_{b1}(k), \boldsymbol{y}_{b2}(k),$

$\cdots, \boldsymbol{y}_{bL-1}(k)]^T$, $\bar{\boldsymbol{Y}}_b(k) = [\boldsymbol{y}_{b2}(k), \boldsymbol{y}_{b3}(k), \cdots, \boldsymbol{y}_{bL}(k)]^T$, and $\bar{\boldsymbol{A}}$ is the submatrix of $\boldsymbol{A}$ consisting of its first $M-\bar{p}$ rows.

Clearly the ranks of two Vandermonde matrices $\bar{\boldsymbol{A}}$ and $\boldsymbol{A}_1$ are given by $\text{rank}(\bar{\boldsymbol{A}}) = \min(M-\bar{p}, p) = p$ and $\text{rank}(\boldsymbol{A}_1) = \min(\bar{p}, p) = p$ iff $p \le \bar{p} < M-p$, while the $p \times p$ diagonal matrices $\text{diag}(\boldsymbol{C}_s \boldsymbol{b}_M^*(\theta))$, $\text{diag}(\boldsymbol{C}_s \boldsymbol{b}_1^*(\theta))$, and $\boldsymbol{D}$ have full rank no matter if the incident signals are coherent or not in view of the facts that $\boldsymbol{C}_s \ne \boldsymbol{O}_{p\times p}$ and $\boldsymbol{b}_i(\theta) \ne \boldsymbol{0}_{p\times 1}$. Hence from (9)-(12), we find that the ranks of matrices $\boldsymbol{\Phi}_f$, $\bar{\boldsymbol{\Phi}}_f$, $\boldsymbol{\Phi}_b$, and $\bar{\boldsymbol{\Phi}}_b$ equal the number of signals even though the incident signals are coherent. ■

Therefore when $p \le \bar{p} < M-p$, it is possible to estimate the number of incident signals from the ranks of the $(M-\bar{p}) \times \bar{p}$ Hankel correlation matrices $\boldsymbol{\Phi}_f$, $\bar{\boldsymbol{\Phi}}_f$, $\boldsymbol{\Phi}_b$, and $\bar{\boldsymbol{\Phi}}_b$ in spite of the statistical correlations between the incident signals, which can cause the source signal covariance matrix $\boldsymbol{C}_s$ to be rank-deficient sometimes and consequently complicates detection problem. Furthermore, these Hankel correlation matrices are insensitive to the additive noise even with the spatially inhomogenous noise model.

Remark: It is worthy to note that the Hankel correlation matrices $\boldsymbol{\Phi}_f$, $\bar{\boldsymbol{\Phi}}_f$, $\boldsymbol{\Phi}_b$, and $\bar{\boldsymbol{\Phi}}_b$ are derived in (9)-(12) irrespective of the statistical correlations between the incident signals. By introducing the source signal covariance matrix $\boldsymbol{C}_s = c_{s_1}\boldsymbol{\beta}\boldsymbol{\beta}^H$ (for coherent case) or $\boldsymbol{C}_s = \text{diag}([c_{s_1}, c_{s_2}, \cdots, c_{s_p}]^T)$ (for uncorrelated case) into (9)-(12), the expressions of such matrices in the MENSE [25] can be obtained straightforwardly, where $\boldsymbol{\beta} = [\beta_1, \beta_2, \cdots, \beta_p]^T$, β_i is the complex attenuation coefficient of $s_i(k)$ with respect to $s_1(k)$ with $\beta_i \ne 0$ and $\beta_1 = 1$. □

Then by defining an $(M-\bar{p}) \times 4\bar{p}$ correlation matrix $\boldsymbol{\Phi}$ as $\boldsymbol{\Phi} = [\boldsymbol{\Phi}_f, \bar{\boldsymbol{\Phi}}_f, \boldsymbol{\Phi}_b, \bar{\boldsymbol{\Phi}}_b]$, after some algebraic manipulations, we obtain an auto-product $\boldsymbol{\Psi}$ of matrix $\boldsymbol{\Phi}$ as

$$\boldsymbol{\Psi} = \boldsymbol{\Phi}\boldsymbol{\Phi}^H = \bar{\boldsymbol{A}}\boldsymbol{F}\boldsymbol{F}^H\bar{\boldsymbol{A}}^H \tag{13}$$

where

$$\begin{aligned}\boldsymbol{F} = [&\text{diag}(\boldsymbol{C}_s\boldsymbol{b}_M^*(\theta))\boldsymbol{A}_1^T, \boldsymbol{D}\text{diag}(\boldsymbol{C}_s\boldsymbol{b}_1^*(\theta))\boldsymbol{A}_1^T,\\ &\boldsymbol{D}^{-(M-1)}(\text{diag}(\boldsymbol{C}_s\boldsymbol{b}_1^*(\theta)))^*\boldsymbol{A}_1^T,\\ &\boldsymbol{D}^{-(M-2)}(\text{diag}(\boldsymbol{C}_s\boldsymbol{b}_M^*(\theta)))^*\boldsymbol{A}_1^T].\end{aligned} \tag{14}$$

Clearly the number of signals p equals the rank of $\boldsymbol{\Psi}$ irrespective of the signal coherency iff the detectability condition that $p \le \bar{p} < M-p$ is satisfied and is revealed in the rank of the QR upper-trapezoidal factor $\boldsymbol{R}$ of $\boldsymbol{\Psi}$ given by

$$\begin{aligned}\boldsymbol{\Psi} &= \boldsymbol{Q}\boldsymbol{R}\\ &= \boldsymbol{Q}\begin{bmatrix}\boldsymbol{R}_{11}, & \boldsymbol{R}_{12}\\ \boldsymbol{O}_{(M-\bar{p}-p)\times(M-\bar{p})} & \end{bmatrix}\begin{matrix}\}p\\ \}M-\bar{p}-p\end{matrix}\end{aligned} \tag{15}$$

where $\boldsymbol{Q}$ is the $(M-\bar{p}) \times (M-\bar{p})$ unitary matrix, $\boldsymbol{R}_{11}$ is the $p\times p$ upper-triangular and nonsingular matrix, and $\boldsymbol{R}_{12}$ is the $p \times (M-\bar{p}-p)$ matrix with non-zero elements.

Proof: Omitted (see [25] for reference). ■

3.2 On-Line Algorithm for Number Detection

Now based on the aforementioned QR-based number detection, we consider the on-line algorithm to estimate the number of signals p at the instant n for $n = 0, 1, \cdots$ from N snapshots of $\{\boldsymbol{y}(k)\}$ measured at $k = nN, nN+1, \cdots, (n+1)N-1$.

From (9)-(12), we can get the instantaneous Hankel correlation matrices at the instant n as

$$\hat{\boldsymbol{\Phi}}_f(n) = \frac{1}{N}\sum_{k=nN}^{(n+1)N-1} \boldsymbol{Y}_f(k) y_M^*(k) \tag{16}$$

$$\hat{\bar{\boldsymbol{\Phi}}}_f(n) = \frac{1}{N}\sum_{k=nN}^{(n+1)N-1} \bar{\boldsymbol{Y}}_f(k) y_1^*(k) \tag{17}$$

$$\hat{\boldsymbol{\Phi}}_b(n) = \frac{1}{N}\sum_{k=nN}^{(n+1)N-1} \boldsymbol{Y}_b(k) y_1(k) \tag{18}$$

$$\hat{\bar{\boldsymbol{\Phi}}}_b(n) = \frac{1}{N}\sum_{k=nN}^{(n+1)N-1} \bar{\boldsymbol{Y}}_b(k) y_M(k). \tag{19}$$

Then by performing the QR decomposition with column pivoting to the instantaneous estimate $\hat{\boldsymbol{\Psi}}(n)$ of matrix $\boldsymbol{\Psi}$ in (13) at the instant n, we get

$$\begin{aligned}\hat{\boldsymbol{\Psi}}(n)\boldsymbol{\Pi} &= \hat{\boldsymbol{Q}}(n)\hat{\boldsymbol{R}}(n)\\ &= \hat{\boldsymbol{Q}}(n)\begin{bmatrix}\hat{\boldsymbol{R}}_{11}(n), & \hat{\boldsymbol{R}}_{12}(n)\\ \boldsymbol{O}_{(M-\bar{p}-p)\times p}, & \hat{\boldsymbol{R}}_{22}(n)\end{bmatrix}\begin{matrix}\}p\\ \}M-\bar{p}-p\end{matrix}\end{aligned} \tag{20}$$

where $\boldsymbol{\Pi}$ is an $(M-\bar{p}) \times (M-\bar{p})$ permutation matrix, which is introduced to remedy the effect of additive noise and that of finite window size, and which can be determined by the column index maximum-difference bisection rule based scheme (called QRPP) [22], [26]. Then by introducing an auxiliary quantity $\zeta(n,i)$ in terms of the non-zero elements of the ith row of QR factor $\hat{\boldsymbol{R}}(n)$ as

$$\zeta(n,i) = \sum_{m=i}^{M-\bar{p}} |\hat{r}_{im}| + \varepsilon, \quad \text{for } i = 1, 2, \cdots, M-\bar{p} \tag{21}$$

we can define a ratio criterion $\xi(n,i)$ as

$$\xi(n,i) = \frac{\zeta(n,i)}{\zeta(n,i+1)}, \quad \text{for } i = 1, 2, \cdots, M-\bar{p}-1 \tag{22}$$

where ε is an arbitrary and positive small constant (e.g., $\varepsilon = 10^{-10}$) for avoiding the possibly undetermined ratio of $0/0$ in (22). Thus the number of incident signals at the instant n is determined as the value of the running index $i \in \{1, 2, \cdots, M-\bar{p}-1\}$ for which the criterion $\xi(n,i)$ is maximized, i.e.,

$$\hat{p}(n) = \arg\max_i \xi(n,i). \tag{23}$$

Therefore the implementation of the proposed on-line detection algorithm is summarized as:

Step 1: Set the subarray size to $\bar{p} = \lfloor M/2 \rfloor$, which satisfies the condition that $p_{\max} \le \bar{p} = \lfloor M/2 \rfloor < M - p_{\max}$, where $p_{\max} = \lceil M/2 \rceil - 1$, while $\lceil x \rceil$ or $\lfloor x \rfloor$ denotes the smallest integer not less than x or the largest integer not greater than x, respectively.

Step 2: Calculate the instantaneous correlation vector $\hat{\boldsymbol{\varphi}}(n)$ between $\boldsymbol{y}(k)$ and $y_M^*(k)$ and those of $\hat{\bar{\boldsymbol{\varphi}}}(n)$ between

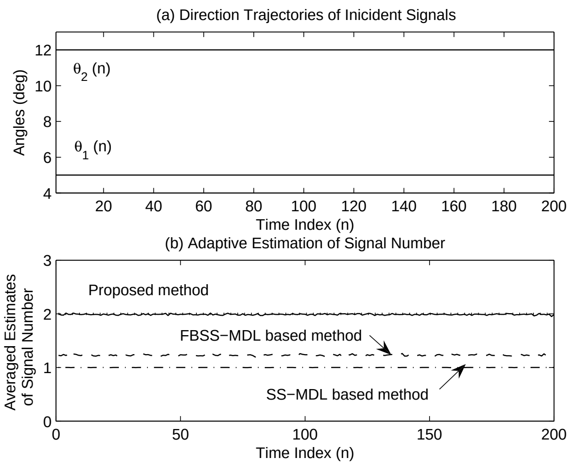

Figure 1: Adaptive estimation performance for the coherent signals with constant directions: (a) scenario and (b) estimated number of signals (dotted line: true value) in Example 1.

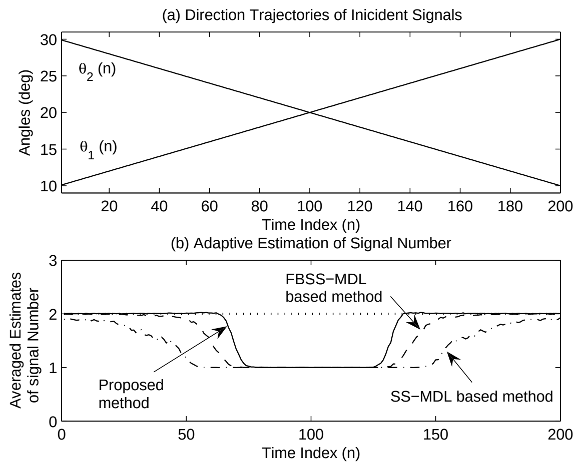

Figure 2: Tracking performance for the coherent time-varying signals: (a) scenario and (b) estimated number of signals (dotted line: true value) in Example 2.

$\boldsymbol{y}(k)$ and $y_1^*(k)$ by using N snapshots of $\{\boldsymbol{y}(k)\}$ measured at $k = nN, nN+1, \cdots, (n+1)N-1$ as

$$\hat{\boldsymbol{\varphi}}(n) = \frac{1}{N}\sum_{k=nN}^{(n+1)N-1} \boldsymbol{y}(k)y_M^*(k) \quad (24)$$

$$\hat{\bar{\boldsymbol{\varphi}}}(n) = \frac{1}{N}\sum_{k=nN}^{(n+1)N-1} \boldsymbol{y}(k)y_1^*(k) \quad (25)$$

where $\hat{\boldsymbol{\varphi}}(n) = [\hat{c}_{1M}(n), \hat{c}_{2M}(n), \cdots, \hat{c}_{MM}(n)]^T$, and $\hat{\bar{\boldsymbol{\varphi}}}(n) = [\hat{c}_{11}(n), \hat{c}_{21}(n), \cdots, \hat{c}_{M1}(n)]^T$.

Step 3: Form the instantaneous Hankel correlation matrices $\hat{\boldsymbol{\Phi}}_f(n)$, $\hat{\bar{\boldsymbol{\Phi}}}_f(n)$, $\hat{\boldsymbol{\Phi}}_b(n)$, and $\hat{\bar{\boldsymbol{\Phi}}}_b(n)$ at the instant n from $\hat{\boldsymbol{\varphi}}(n)$ and $\hat{\bar{\boldsymbol{\varphi}}}(n)$ obtained above without the use of $\hat{c}_{11}(n)$ and $\hat{c}_{MM}(n)$.

Step 4: Form the instantaneous correlation matrix $\hat{\boldsymbol{\Phi}}(n)$ as

$$\hat{\boldsymbol{\Phi}}(n) = [\hat{\boldsymbol{\Phi}}_f(n), \hat{\bar{\boldsymbol{\Phi}}}_f(n), \hat{\boldsymbol{\Phi}}_b(n), \hat{\bar{\boldsymbol{\Phi}}}_b(n)]. \quad (26)$$

Step 5: Calculate the auto-product $\hat{\boldsymbol{\Psi}}(n)$ of $\hat{\boldsymbol{\Phi}}(n)$ as (13) and perform its QR decomposition with the permutation matrix $\boldsymbol{\Pi}$ as (20).

Step 6: Calculate the ratio criterion $\xi(n,i)$ as (22) and determine the number of incident signals by using (23).

4. NUMERICAL EXAMPLES

The ULA with $M = 10$ sensors is separated by a half-wavelength, and the signal-to-noise ratio (SNR) is defined as the ratio of the power of incident signal c_{s_i} to that of the additive noise σ^2 at each sensor, i.e., $\mathrm{SNR} = c_{s_i}/\sigma^2$. The spatial smoothing (SS) and forward-backward SS (FBSS) based MDL methods [30], [2] are also carried out for comparison, where the instantaneous array covariance matrix at the instant n is calculated from the N snapshots $\{\boldsymbol{y}(k)\}_{k=nN}^{(n+1)N-1}$ and the its eigenvalue decomposition (EVD) is performed. The simulation results shown below are obtained by the ensemble-averaged based on 1000 independent trials.

Example 1—Adaptive Detection for Coherent Signals with Constant Directions: Two coherent signals with equal power ($c_{s_i} = 1$) arriving from $\theta_1 = 5^{\mathrm{o}}$ and $\theta_2 = 12^{\mathrm{o}}$, and their SNR is set at 2.5dB. The number of snapshots during the interval of parameter updating (i.e., window size) is set at $N = T/T_s = 10$. As shown in Fig. 1, the proposed on-line detection algorithm with QRPP can estimate the number of signals adaptively and accurately, and it outperforms that with the ordinary SS- and FBSS-based MDL methods, where the effects of the additive noise and finite window size N are alleviated efficiently.

Example 2—Tracking for Closely-Spaced and Time-Varying Coherent Signals: Two coherent signals initially located at 10^{o} and 30^{o}, and their directions change linearly and cross at $n = 100$. The other simulation parameters are similar to those of Example 1. As observed from (13), when two incident signals become close, the ranks of $\bar{\boldsymbol{A}}$ and $\boldsymbol{A}_1$ (i.e., $\boldsymbol{A}(\theta(n))$) will begin to collapse and result in the rank-deficiency of $\boldsymbol{\Psi}$, consequently only one signal will be detected. When the signals become close and 6^{o} apart, the rank of $\hat{\boldsymbol{\Psi}}(n)$ (i.e., $\boldsymbol{A}(\theta(n))$) first begins to collapse, the proposed algorithm with QRPP fails to estimate the number of two coherent signals as one, but it can track the number of signals accurately when the angular separation becomes larger than 8^{o}. Furthermore as shown in Fig. 2, it performs well than the proposed ones with SS- and FBSS-based methods.

Example 3—Tracking for Appearing and Disappearing Coherent Signals: There are three coherent signals imping-ing on the array from $\theta_1(n)$, $\theta_2(n)$, and $\theta_3(n)$, where $s_2(n)$ appears at $n = 30$, and it disappears during the interval between $n = 80$ and $n = 120$ and from $n = 160$, while $s_3(n)$ disappears from $n = 80$ and $n = 120$. The other simulation conditions are the same as those in Example 1. As shown Fig. 3, the proposed on-line algorithm with QRPP has better tracking performance than the SS- and FBSS-based MDL methods even with the EVD, and it can estimate the decreasing and increasing number of incident signals accurately and immediately.

5. CONCLUSION

In this paper, a new QR-based on-line algorithm was proposed for estimating the number of coherent and/or incoherent narrowband signals impinging on a ULA, where the up-

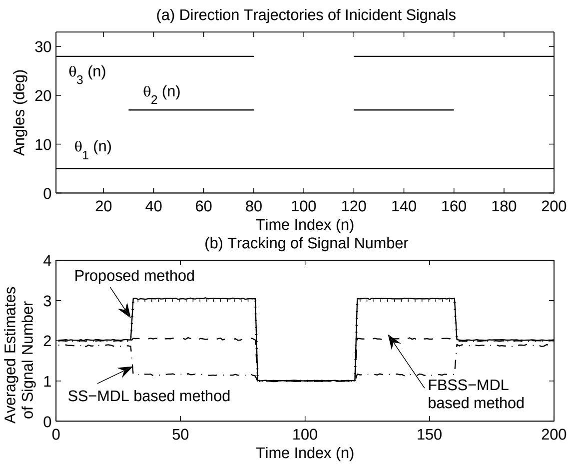

Figure 3: Tracking performance for the coherent appearing and disappearing signals: (a) scenario and (b) estimated number of signals (dotted line: true value) in Example 3.

dating of eigenvalue and threshold setting are not needed. The proposed algorithm has good detection performance to track the number of suddenly appearing/disappearing incident signals or that of incoming signals with time-varying directions which even cross in their trajectories.

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