A NOVEL APPROACH BASED ON NON-UNITARY JOINT BLOCK-DIAGONALIZATION FOR THE BLIND MIMO EQUALIZATION OF CYCLO-STATIONARY SIGNALS

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ABSTRACT

In this paper, we address the problem of the blind separation of convolutive mixtures of sources also known as blind equalization of linear time invariant multi-input multi-output systems in digital communications. In our case, the considered input signals are cyclo-stationary processes whose cyclic frequencies are not necessarily known. To tackle that problem, we propose to use a new algorithm that performs the (non-unitary) joint block diagonalization (JBD) of a given set of complex matrices. It is based on a gradient approach. But first, the matrices set to be joint block-diagonalized has to be built: this is done using a particular linear operator on the correlation matrices of the observations and taking advantage of some specific algebraic properties of the aforementioned "transformed" matrices (for instance, the frequencies that correspond to the matrices belonging to that set happen to be the cyclic ones). Then, the JBD of this matrices set enables to estimate the mixing system and, consequently, to recover the source signals. Computer simulations are provided in order to illustrate the behavior and the usefulness of the proposed approach in the context of digital communications and a comparison with another existing method is also performed.

1. INTRODUCTION

We consider the problem of the blind equalization of Linear Time Invariant (LTI) Finite Impulse Response (FIR) Multi-Input Multi-Output (MIMO) systems. Such a problem arises in a wide variety of applications among which the multi-user wireless communications where the received signals have to be equalized in space as well as in time to eliminate both inter-symbols and co-channel interferences. These interferences are due to possible delays introduced by multi-paths propagation and/or to possible multi-users. Examples can be found in the Space Division Multiple Access (SDMA) or the Code Division Multiple Access (CDMA) communication systems [12].

The problem of the blind equalization of LTI FIR MIMO systems can be stated as follows: observing several linear (temporal and spatial) mixtures of input signals (called sources) and assuming that both the sources and the mixing MIMO system are unknown and unobservable, the goal is to recover the sources from the output signals only. As a consequence, this problem is often qualified as "blind" or "unsupervised". To perform that task, different approaches have been developed in the literature. They may differ in the assumptions made about the sources (deterministic, stochastic, mutually statistically independent or correlated, i.i.d. or not, stationary, cyclo-stationary or non-stationary...) and/or in the kind of considered mixture system (linear or non-linear, over-determined (more outputs than inputs) or undetermined mixture model). However, most of the existing methods assume that the sources are random independent stationary processes and comparatively, very few works are dedicated to the case of non stationary signals. Here, we focus on a particular class of non stationary signals: modulated ones stemming from unknown digital communication systems (involving that the baud-rates of the various transmitted signals are unknown). In such a context, the resulting signals are proved to be cyclo-stationary sequences. If W. Gardner was the first to introduce such a concept in [5] for array processing, it has proven, since, to be really useful for the modelling of communication signals. It has also led to many breakthroughs in that field among which [1][3][4][10]. Most of these works, by taking into account the very specific statistical properties of the communications signals [3][4] and the knowledge (or not) of their different cyclic frequencies, generalize techniques that were established in the context of stationary signals.

Our developments originate from the works presented in [1] (yet, those works were dedicated to the instantaneous case). Albeit taking advantage of the same property of a "transformed" correlation matrix, the treatments that follow are rather different in our case: while their algorithm is based on the optimization of a given contrast function, our approach, in the instantaneous case [11], is more direct since it combines a rank-one matrices selection procedure together with a Singular Value Decomposition (SVD) followed by a classification algorithm. It enables to tackle the under-determined case too.

Our aim, here, is to generalize some of the ideas developed in [11] to provide a solution in the case of convolutive (over-determined) mixtures of sources. Two situations are considered: the second order cyclic frequencies of the inputs are either known or unknown. Our approach consists of reformulating the convolutive mixing model into an instantaneous one (as suggested in [9]) and of fully exploiting the particular algebraic structure of the correlation matrices of the outputs after the application of a particular linear transformation. We show that the considered problem can be rewritten into a problem of JBD of a given set of matrices which leads us to introduce a new non-unitary JBD algorithm. It is based on a gradient approach but despite [6] there is no more an approximation in the calculation of the complex gradient matrix of the considered cost function. This algorithm is then applied on a particular set of matrices whose automatic selection procedure is described too. Finally, computer simulations are provided in order to illustrate the good behavior and the usefulness of the proposed approach in the context of digital telecommunications.

2. BLIND MIMO EQUALIZATION

2.1 Model and assumptions

The $m\times1$ vector of the observations is denoted $\mathbf{x}(t)$, $t\in\mathbb{Z}$. In the convolutive case, the sources are assumed mixed through a linear Finite Impulse Response (FIR) multichannel system, denoted $\mathbf{H}(t)$, implying that the system is described by the following input-output relation:

$$x_i(t)=\sum_{j=1}^{n}\sum_{\ell=0}^{L}h_{ij}(\ell)s_j(t-\ell)+n_j(t),\ \forall i=1,\dots,m \quad (1)$$

where $x_i(t)$, for all $i=1,\dots,m$ are the m observations, $s_j(t)$, for all $j=1,\dots,n$ are the n sources ($n\leq m$), $h_{ij}(t)$ is the real transfer function between the j-th source and the i-th sensor with an overall extent of $L+1$ taps and $n_i(t)$, for all $i=1,\dots,m$ are additive noises. Our developments are based on the following assumptions:

Assumption A. The sources are zero-mean and cyclo-stationary. Hence, their autocorrelation functions $R_{s_i}(t,\tau)=\mathsf{E}\{s_i(t)s_i^*(t-\tau)\}$, for all $i=1,\dots,n$ are periodic in t with a period $T_i\in\mathbb{Z}^{+*}$, for all $i=1,\dots,n$. $\mathsf{E}\{.\}$ stands for the mathematical expectation operator and T_i stands for the cyclic-period of the i-th source signal $s_i(t)$. Hence, $R_{s_i}(t,\tau)$ can be decomposed into Fourier series as

$$R_{s_i}(t,\tau)=\sum_k R_{s_i}^{\mathsf{fs}}[k,\tau]\exp(\imath2\pi\frac{k}{T_i}t), \quad (2)$$

where $\imath^2=-1$. $R_{s_i}^{\mathsf{fs}}[k,\tau)$ is the cyclic correlation function (coefficient of the Fourier series expansion). It is defined as:

$$R_{s_i}^{\mathsf{fs}}[k,\tau)=\frac{1}{T_i}\int_{-\frac{T_i}{2}}^{\frac{T_i}{2}}R_{s_i}(t,\tau)\exp(-2\imath\pi\frac{k}{T_i}t)dt. \quad (3)$$

Assumption B. The cyclic periods of the sources are (two by two) distinct, *i.e.* $T_i\neq T_j$, for all $i,j\in\{1,\dots,n\}$ and $i\neq j$. We define the set $\mathcal{V}_i$ of the whole cyclic frequencies of the i-th source signal as

$$\mathcal{V}_i=\left\{\nu_i=\frac{k}{T_i},\quad k\in\mathbb{Z}\right\}.$$

Assumption C. The noises n_j for all $j=1,\dots,m$ are stationary white zero-mean random signals, mutually uncorrelated, independent from the sources.

2.2 Some recalls

Let us now recall how the convolutive mixing model can be reformulated into an instantaneous one [9]. Denoting $M=mL'$ and $N=nQ$, let us consider three vectors: the $N\times1$ vector $\mathbf{S}(t)$ and the $M\times1$ vectors $\mathbf{X}(t)$ and $\mathbf{N}(t)$. They are respectively defined as:

$$\begin{aligned}\mathbf{S}(t)&=[s_1(t),\dots,s_1(t-Q+1),\dots,s_n(t-Q+1)]^T,\\ \mathbf{X}(t)&=[x_1(t),\dots,x_1(t-L'+1),\dots,x_m(t-L'+1)]^T,\\ \mathbf{N}(t)&=[n_1(t),\dots,n_1(t-L'+1),\dots,n_m(t-L'+1)]^T,\end{aligned}$$

where $Q=L+L'$ and $(\cdot)^T$ stands for the transpose operator. The $M\times N$ matrix $\mathbf{A}$, writes:

$$\mathbf{A}=\begin{pmatrix}\mathbf{A}_{11}&\dots&\mathbf{A}_{1n}\\ \vdots&\ddots&\vdots\\ \mathbf{A}_{m1}&\dots&\mathbf{A}_{mn}\end{pmatrix}$$

where each matrix $\mathbf{A}_{ij}$ for all $i=1,\dots,m$ and for all $j=1,\dots,n$ is a $L'\times Q$ matrix, defined as:

$$\mathbf{A}_{ij}=\begin{pmatrix}h_{ij}(0)&\dots&\dots&h_{ij}(L)&0&\dots&0\\ 0&\ddots&\ddots&\ddots&\ddots&\ddots&\vdots\\ \vdots&\ddots&\ddots&\ddots&\ddots&\ddots&0\\ 0&\dots&0&h_{ij}(0)&\dots&\dots&h_{ij}(L)\end{pmatrix}$$

Eq. (1) can be rewritten in a matrix form:

$$\mathbf{X}(t)=\mathbf{A}\mathbf{S}(t)+\mathbf{N}(t) \quad (4)$$

To make sure that the considered model is an over-determined one, L' must be chosen such that $M=mL'\geq nQ=N$. We also assume, here, that the matrix $\mathbf{A}$ is full rank. The correlation matrix $\mathbf{R}_\mathbf{X}(t,\tau)$ of the observations $\mathbf{X}(t)$ is defined as:

$$\mathbf{R}_\mathbf{X}(t,\tau)=\mathsf{E}\{\mathbf{X}(t)\mathbf{X}^H(t-\tau)\}, \quad (5)$$

where $(\cdot)^H$ is the conjugate transpose operator. Using Eq. (4), it is easily seen that the correlation matrix defined in Eq. (5) admits the following decomposition:

$$\mathbf{R}_\mathbf{X}(t,\tau)=\mathbf{A}\mathbf{R}_\mathbf{S}(t,\tau)\mathbf{A}^H+\mathbf{R}_\mathbf{N}(\tau), \quad (6)$$

where $\mathbf{R}_\mathbf{S}(t,\tau)$ (resp. $\mathbf{R}_\mathbf{N}(\tau)$) is the correlation matrix of the sources (resp. the noises). Those two matrices are defined like in Eq. (5) and using Assumption C.

3. THE PROPOSED APPROACH

3.1 A useful property

Like in Eq. (3), we now define the following linear operator $(\cdot)^{\mathsf{fs}}$ operating on a matrix argument component wise

$$\mathbf{R}_\mathbf{X}^{\mathsf{fs}}(\nu,\tau)=\lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\mathbf{R}_\mathbf{X}(t,\tau)\exp(-2\imath\pi\nu t)dt. \quad (7)$$

Since this operator is linear, using Eq. (6) in Eq. (7), we directly have

$$\begin{aligned}\mathbf{R}_\mathbf{X}^{\mathsf{fs}}(\nu,\tau)&=\mathbf{A}\mathbf{R}_\mathbf{S}^{\mathsf{fs}}(\nu,\tau)\mathbf{A}^H+\mathbf{R}_\mathbf{N}^{\mathsf{fs}}(\nu,\tau)\\ &=\mathbf{A}\mathbf{R}_\mathbf{S}^{\mathsf{fs}}(\nu,\tau)\mathbf{A}^H+\mathbf{R}_\mathbf{N}(\tau)\alpha(\nu),\end{aligned} \quad (8)$$

where $\mathbf{R}_\mathbf{S}^{\mathsf{fs}}(\nu,\tau)$ and $\mathbf{R}_\mathbf{N}^{\mathsf{fs}}(\nu,\tau)$ are defined similarly to $\mathbf{R}_\mathbf{X}^{\mathsf{fs}}(\nu,\tau)$ in (7) and

$$\alpha(\nu)=\lim_{T\to\infty}\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}\exp(-2\imath\pi\nu t)dt.$$

Thus $\alpha(\nu)=1$ if $\nu=0$ and else 0. Using the fact that the sources have distinct cyclic periods as stated in Assumption B, then, there exist values of ν for which $\mathbf{R}_\mathbf{S}^{\mathsf{fs}}(\nu,\tau)$ has a particular structure. In fact, for frequencies ν_o such that:

$$\mathsf{H}_1.\ \nu_o\in\mathcal{V}_i,\ \nu_o\notin\bigcup_{j,j\neq i}\mathcal{V}_j \text{ and } R_{s_i}^{\mathsf{fs}}(\nu_o,\tau)\neq0,\quad i\in\{1,\dots,n\}$$

one can show that the matrix $\mathbf{R}_\mathbf{S}^{\mathsf{fs}}(\nu_o,\tau)$ possesses only $Q\times Q$ non null elements all inside the same block. This is due to the fact that $R_{s_j}^{\mathsf{fs}}(\frac{k}{T_i},\tau)=0$ for all $k\in\mathbb{Z}$ and for all $i,j\in\{1,\dots,n\}$ such that $i\neq j$. Thus, for such a frequency value, the matrix $\mathbf{R}_\mathbf{S}^{\mathsf{fs}}(\nu,\tau)$ is a block-diagonal matrix with only one non null block: the one at the position (i,i). As a consequence, one possible way to estimate the mixing matrix $\mathbf{A}$ is to joint block-diagonalize the following matrices set:

$$\mathcal{S} \stackrel{\text{def}}{=} \{\mathbf{R}^{\mathsf{fs}}_{\mathbf{X}}(\nu,\tau); \forall\tau \neq 0, \forall\nu \text{ satisfying property } \mathsf{H}_1\} \quad (9)$$

In the following, the size of this set will be denoted N_m ($N_m \in \mathbb{N}^*$). Let us notice, finally, that the recovered signals after inversion of the system are obtained up to a permutation and up to a filter which are the classical indeterminations of blind source separation in the convolutive case.
In the following subsection (3.2), we explain how to build the matrices set $\mathcal{S}$ while in the next subsection (3.3), we briefly present the principle of the (non-unitary) JBD algorithm based on a gradient descent approach that we use (the mathematical derivations of the used JBD algorithm are further detailed in [7]). The calculation of the complex gradient matrix of the cost function is no more approximated as it was the case in [6].

3.2 Building of the matrices set to be joint block-diagonalized

First, let us notice that the knowledge of the cyclic frequencies is not necessary to achieve the identification of the mixing system, even though such a knowledge can simplify this task.

1) Known cyclic frequencies
When the cyclic frequencies ν_j for all $j = 1,\dots,n$ are known (or estimated like in [2]), the matrices set $\{\mathbf{R}^{\mathsf{fs}}_{\mathbf{X}}(\nu_j,\tau); \forall\tau \neq 0, \forall j = 1,\dots,n\}$ is directly joint block-diagonalized.

2) Unknown cyclic frequencies
When the cyclic frequencies are unknown, the "transformed" correlation matrices $\mathbf{R}^{\mathsf{fs}}_{\mathbf{X}}(\nu,\tau)$ are calculated for a sufficiently large number of frequency bins to make sure that a sufficiently wide range of cyclic frequencies, *i.e.* corresponding to all the source signals, can be found. Before being able to apply a JBD algorithm, specific matrices $\mathbf{R}^{\mathsf{fs}}_{\mathbf{X}}(\nu,\tau)$ have to be selected: for instance, those that correspond to block-diagonal matrices $\mathbf{R}^{\mathsf{fs}}_{\mathbf{S}}(\nu,\tau)$ with only one non-null block matrix on their diagonal. Due to our hypothesis, one can show that matrices $\mathbf{R}^{\mathsf{fs}}_{\mathbf{X}}(\nu,\tau)$ and $\mathbf{R}^{\mathsf{fs}}_{\mathbf{S}}(\nu,\tau)$ have the same rank. The matrices set $\mathcal{S}$ is thus the set of matrices whose rank equals Q which leads to the following automatic detection procedure:

Automatic detection procedure denoted $\mathsf{Cr}^{(\mathsf{Ghe})}_{\mathsf{NU,conv}}$

- Use a Singular Value Decomposition (SVD) of $\mathbf{R}^{\mathsf{fs}}_{\mathbf{X}}(\nu,\tau)$ *i.e.* $\mathbf{R}^{\mathsf{fs}}_{\mathbf{X}}(\nu,\tau) = \mathbf{U}(\nu,\tau)\mathbf{\Delta}(\nu,\tau)\mathbf{V}(\nu,\tau)^H$ with $\mathbf{V}(\nu,\tau)$ and $\mathbf{U}(\nu,\tau)$ are $M \times M$ unitary matrices and $\delta(\nu,\tau) = \mathsf{Diag}\{\mathbf{\Delta}(\nu,\tau)\} = (\delta_1(\nu,\tau),\dots,\delta_M(\nu,\tau))^T$ is a diagonal vector with non-negative components organized in the decreasing order: $\delta_1(\nu,\tau) \geq \delta_2(\nu,\tau) \geq \dots \geq \delta_M(\nu,\tau) \geq 0$.
- Calculate the ratio $\mathcal{R} = \frac{\sum_{i=1}^{Q}\delta_i(\nu,\tau)^2}{\|\mathbf{R}^{\mathsf{fs}}_{\mathbf{X}}(\nu,\tau)\|_F^2}$ where $\|.\|_F$ stands for the Frobenius norm,
- Check if $\mathcal{R}$ is near to 1: if this assertion is true $\mathbf{R}^{\mathsf{fs}}_{\mathbf{X}}(\nu,\tau)$ is kept else it is discarded.

In terms of numerical implementation, it leads to the following rule: choose (ν,τ) such that $\mathcal{R} < 1-\epsilon$, where ϵ is a (sufficiently) small positive constant. The matrices that belong to $\mathcal{S}$ all have a particular algebraic structure since, for $\tau \neq 0$, they all admit the same following decomposition $\mathbf{R}^{\mathsf{fs}}_{\mathbf{X}}(\nu,\tau) = \mathbf{A}\mathbf{R}^{\mathsf{fs}}_{\mathbf{S}}(\nu,\tau)\mathbf{A}^H$, with $\mathbf{R}^{\mathsf{fs}}_{\mathbf{S}}(\nu,\tau)$ being a block-diagonal matrix with only one non null block on its diagonal. One possible way to estimate the mixing matrix $\mathbf{A}$ (or its pseudo-inverse $\mathbf{B}$ also called the separation matrix) is to directly joint block diagonalize (without any unitary constraint about $\mathbf{A}$ or $\mathbf{B}$) the matrices set $\mathcal{S}$.

3.3 A non-unitary joint block diagonalisation algorithm based on a gradient approach

To solve the non-unitary JBD problem, the following cost function [6] is considered:

$$\mathcal{C}_{BD}(\mathbf{B}) = \sum_{i=1}^{N_m} \|\mathsf{OffBdiag}\{\mathbf{B}\mathbf{M}_i\mathbf{B}^H\}\|_F^2 \quad (10)$$

where $\mathbf{M}_i = (\mathbf{R}^{\mathsf{fs}}_{\mathbf{X}})_i$ is the i-th of the N_m matrices belonging to $\mathcal{S}$. Considering a square $N \times N$ matrix $\mathbf{M} = (M_{ij})$, such that:

$$\mathbf{M} = \begin{pmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} & \mathbf{M}_{1r} \\ \mathbf{M}_{21} & \ddots & \\ \mathbf{M}_{r1} & \mathbf{M}_{r2} & \mathbf{M}_{rr} \end{pmatrix}, \quad (11)$$

where $\mathbf{M}_{ij}$ for all $i,j = 1,\dots,r$ are $n_i \times n_j$ matrices. The matrix operator $\mathsf{OffBdiag}_{(\mathbf{n})}$ is then defined as:

$$\mathsf{OffBdiag}\{\mathbf{M}\} = \begin{pmatrix} \mathbf{0}_{11} & \mathbf{M}_{12} & \mathbf{M}_{1r} \\ \mathbf{M}_{21} & \ddots & \\ \mathbf{M}_{r1} & \mathbf{M}_{r2} & \mathbf{0}_{rr} \end{pmatrix}. \quad (12)$$

The case we are interested in, is characterized by $r = n$, square $Q \times Q$ block matrices $\mathbf{M}_{ij}$ for all $i,j = 1,\dots,r$ and square $Q \times Q$ null matrices $\mathbf{0}_{ii}$ for all $i = 1,\dots,r$.
We suggest to use a gradient-descent algorithm to minimize the cost function given by Eq. (10) to estimate the matrix $\mathbf{B} \in \mathbb{C}^{N\times M}$. It means that $\mathbf{B}$ is re-estimated at each iteration m and from now on denoted $\mathbf{B}^{(m)}$.
The matrix $\mathbf{B}$ is updated according to the following adaptation rule for all $m = 1,2,\dots$

$$\mathbf{B}^{(m)} = \mathbf{B}^{(m-1)} - \mu_a \nabla_a \mathcal{C}_{BD}(\mathbf{B}^{(m-1)}), \quad (13)$$

where μ_a is the step size and the complex absolute gradient matrix $\nabla_a\mathcal{C}_{BD}(\mathbf{B})$ is given by:

$$\nabla_a\mathcal{C}_{BD}(\mathbf{B}) = 2\frac{\partial\mathcal{C}_{BD}(\mathbf{B}}{\partial\mathbf{B}^*} \quad (14)$$

$(\cdot)^*$ stands for the complex conjugate operator. $\nabla_a\mathcal{C}_{BD}(\mathbf{B})$ has to be calculated to derive the algorithm. It is found to be equal to (see the proof provided in [7]):

$$\nabla_a\mathcal{C}_{BD}(\mathbf{B}) = 2\Bigg[\sum_{i=1}^{N_m}\mathsf{OffBdiag}\{\mathbf{B}\mathbf{M}_i\mathbf{B}^H\}\mathbf{B}\mathbf{M}_i^H + \Big(\mathsf{OffBdiag}\{\mathbf{B}\mathbf{M}_i\mathbf{B}^H\}\Big)^H\mathbf{B}\mathbf{M}_i\Bigg] \quad (15)$$

Eq. (15) is then used in the gradient descent algorithm given by Eq. (13). In the following, we will denote by $\mathsf{JBD}_{\mathsf{NU},\mathsf{G}_{\mathsf{F,A}}}$ this non-unitary JBD algorithm based on a gradient approach.

3.4 Summary of the proposed method

The proposed method, namely $\mathsf{JBD}_{\mathsf{NU},\mathsf{G}_{\mathsf{F,A}},\mathsf{CM}}$, combines the non-unitary JBD algorithm $\mathsf{JBD}_{\mathsf{NU},\mathsf{G}_{\mathsf{F,A}}}$ together with the detector $\mathsf{Cr}^{(\mathsf{Ghe})}_{\mathsf{NU,conv}}$. Its principle is summed up below:

$\mathsf{JBD}_{\mathsf{NU},\mathsf{G}_{\mathsf{F},\mathsf{A}},\mathsf{CM}}$ method for the blind separation of convolutive mixtures of cyclo-stationary sources

Consider the N_m matrices of set $\mathcal{S}$: $\{(\mathbf{R}^{\mathsf{fs}}_{\mathbf{X}})_1, \ldots, (\mathbf{R}^{\mathsf{fs}}_{\mathbf{X}})_{N_m}\}$.

Given an initial estimate $\mathbf{B}^{(0)}$ (for example in the square case $M = N$ one can choose $\mathbf{B}^{(0)} = \mathbf{I}_M{}^{a}$).

For $m = 1, 2, \ldots$

Compute $\nabla_a \mathcal{C}_{BD}^{(m)}(\mathbf{B})$ given in Eq. (15).

Set $\mathbf{B}^{(m)} = \mathbf{B}^{(m-1)} - \mu_a \nabla_a \mathcal{C}_{BD}(\mathbf{B}^{(m-1)})$.

Eventually normalize $\mathbf{B}^{(m)} = \mathbf{B}^{(m)}/\|\mathbf{B}^{(m)}\|_F$.

Stop after a fixed number of iterations or when $\|\mathbf{B}^{(m)} - \mathbf{B}^{(m-1)}\|_F \leq \varepsilon$ where ε is a small positive threshold.

EndFor

[a] Let us notice that a good initial estimate remains important to ensure the convergence to the true solution. Other ways could be considered to start in the neighborhood of the solution among which the solution given by an orthogonal JBD algorithm or the SVD of the observations' cyclic correlation matrices.

4. COMPUTER SIMULATIONS

We consider $m = 4$ mixtures of $n = 2$ digital carrier-modulated source signals written as:

$$s(t) = \Re\{v(t)\exp(2\imath\pi f_c t)\}, \qquad (16)$$

where $\Re\{\cdot\}$ stands for the real part of a complex signal, f_c is the reduced carrier frequency which is equal to 0.2 for the first source (respectively 0.25 for the second one) and $v(t)$ is the complex envelop of $s(t)$. It can be expressed as:

$$v(t) = \sum_{k\in\mathbb{Z}} a(k)h(n - kT), \qquad (17)$$

with $a(n)$ is an i.i.d. zero-mean complex random binary sequence referred to as the transmitted symbols, T is an integer related to the period symbol and $h(n)$ is a deterministic waveform signal. The waveform $h(n)$ is a triangular one. It is defined for an even cyclic period as: $h(n) = \frac{2}{T}n$ if $0 \leq n \leq \frac{T}{2}$; $h(n) = -\frac{2}{T}n + 2$ if $\frac{T}{2} + 1 \leq n \leq T - 1$ and $h(n) = 0$ otherwise. The cyclic period of the two considered sources are respectively equal to $T_1 = 10$, $T_2 = 8$. The noiseless case is studied and the sources are mixed according to the following transfer function matrix (we use $L = 3$ and $L' = 3$) $\mathbf{A}[z] = (\mathbf{A_1}[\mathbf{z}]\ \mathbf{A_2}[\mathbf{z}])$, where $\mathbf{A}[z]$ stands for the z transform of $\mathbf{A}(t)$ and:

$$\mathbf{A_1}[\mathbf{z}] = \begin{pmatrix} 0.5400 + 0.9936z^{-1} - 0.6051z^{-2} + 0.8040z^{-3} \\ -0.3335 - 0.0827z^{-1} - 0.2809z^{-2} - 0.8249z^{-3} \\ -0.0849 + 0.7305z^{-1} + 0.5760z^{-2} - 0.1143^{-3} \\ -0.4646 - 0.9072z^{-1} - 0.4450z^{-2} - 0.9106z^{-3} \end{pmatrix},$$

$$\mathbf{A_2}[\mathbf{z}] = \begin{pmatrix} 0.6933 - 0.7387z^{-1} - 0.4140z^{-2} - 0.4746z^{-3} \\ -0.3995 + 0.8315z^{-1} - 0.2386z^{-2} - 0.6213z^{-3} \\ -0.5921 - 0.6382z^{-1} - 0.8122z^{-2} + 0.4283z^{-3} \\ -0.0052 + 0.6556z^{-1} + 0.1437z^{-2} - 0.9450z^{-3} \end{pmatrix}.$$

To measure the quality of the estimation, the following performance index is used [8]:

$$I_{conv}(\mathbf{G}) = \frac{1}{r(r-1)}\left[\sum_{i=1}^{r}\left(\sum_{j=1}^{r}\frac{\|(\mathbf{G})_{i,j}\|_F^2}{\max_{\ell}\|(\mathbf{G})_{i,\ell}\|_F^2} - 1\right) + \sum_{j=1}^{r}\left(\sum_{i=1}^{r}\frac{\|(\mathbf{G})_{i,j}\|_F^2}{\max_{\ell}\|(\mathbf{G})_{\ell,j}\|_F^2} - 1\right)\right],$$

where $(\mathbf{G})_{i,j}$ for all $i, j \in \{1, \ldots, r\}$ is the (i,j)-th block matrix of $\mathbf{G} = \hat{\mathbf{B}}\mathbf{A}$. The nest results are obtained when the index performance $I_{conv}(\cdot)$ is found to be close to 0 in linear scale ($-\infty$ in logarithmic scale). The proposed method is also compared with another one (for instance the one suggested in [6]) based on an "approximate gradient" and denoted $\mathsf{JBD}_{\mathsf{NU},\mathsf{G}_{\mathsf{O}},\mathsf{CM}}$. On the Fig. 1 (resp. Fig. 2), the performance index is displayed versus the realizations. The results have been sorted in the decreasing order of the obtained performances in both cases *i.e.* when the cyclic frequencies are known or unknown.

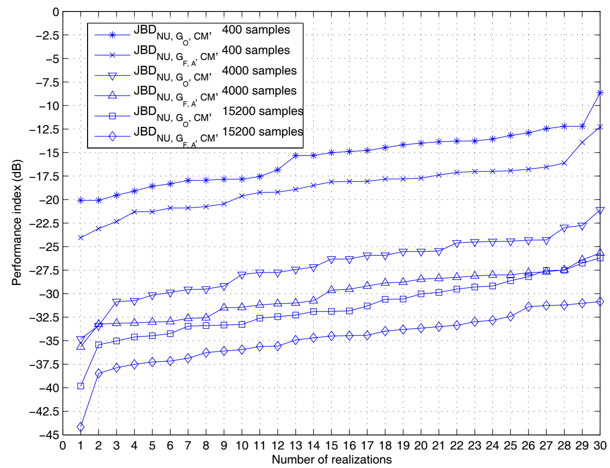

Figure 1: I_{conv} versus realizations (chosen among 30 Monte-Carlo trials) when the cyclic frequencies are assumed known. The realizations are sorted in the decreasing order of the obtained performances.

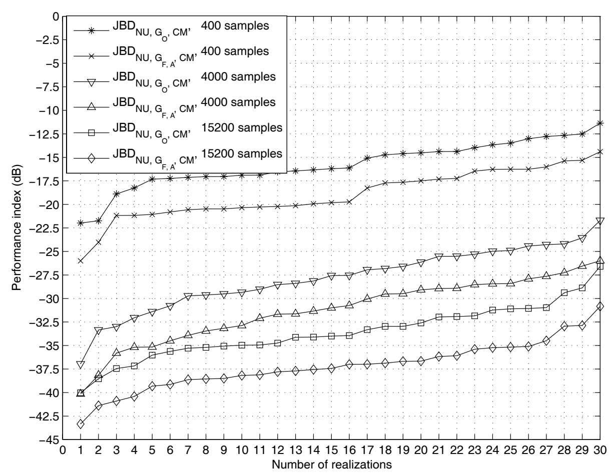

Figure 2: I_{conv} versus realizations (chosen among the 30 Monte-Carlo trials) when the cyclic frequencies are unknown. The realizations are sorted in the decreasing order of the obtained performances.

One can notice that the proposed method always outperforms the method based on an approximated gradient. Moreover the more samples we use, the better the results are. Finally, it can be observed too that satisfying performances are reached whether the cyclic frequencies are known or unknown.

On the Fig. 3, we have displayed the performance index versus the number of sources time samples in the case of known and unknown cyclic frequencies: the displayed results have been averaged over 30 Monte-Carlo trials. On the Fig. 4, we have plotted the number of matrices selected by the detector $\mathsf{Cr}^{(\mathsf{Ghe})}_{\mathsf{NU,conv}}$ versus the number of time samples when the cyclic frequencies are unknown. This chart illustrates the

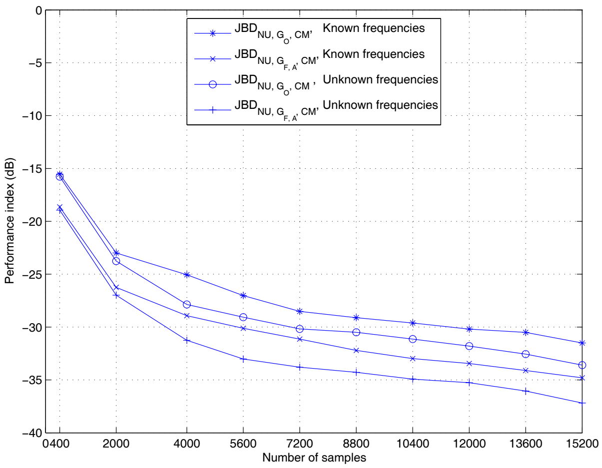

Figure 3: I_{conv} versus the number of time samples in both cases of known and unknown cyclic frequencies.

good behavior of the proposed method which always outperforms the method based on an approximated gradient. The better results are obtained with unknown cyclic frequencies since the size of the considered matrices set is then higher.

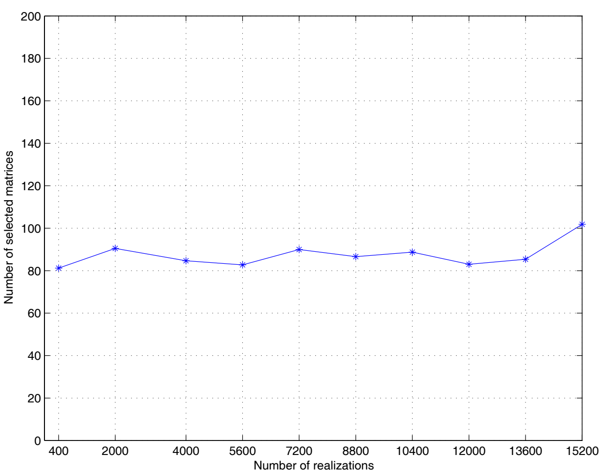

Figure 4: The number of selected matrices when the cyclic frequencies are unknown versus the number of samples.

5. CONCLUSION

We have presented a new approach for the blind separation of LTI MIMO systems in digital communications. The inputs are cyclo-stationary signals. It operates in three stages: first a linear operator is applied on the observations correlation matrices, then, a detection procedure (based on a rank property) is used to select matrices with a specific algebraic structure and finally a (non unitary) JBD algorithm is applied onto the selected matrices set. It enables to estimate the mixing system and to restore the sources.

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