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IDENTIFICATION OF FIFTH-ORDER BLOCK-STRUCTURED NONLINEAR CHANNELS USING I.I.D. INPUT SIGNALS

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ABSTRACT

This paper is concerned with the problem of nonlinear Wiener channel identification using input-output crossmoments. The static nonlinearity is assumed to be represented by a fifth-degree polynomial. For an i.i.d. input signal, we first derive closed-form expressions for estimating the second-order kernel of the associated fifth-order Volterra model. The parameters of the linear part of the fifth-order Wiener channel are then estimated using an eigenvalue decomposition of the associated second-order Volterra kernel, while the nonlinear subsystem is estimated in the least square sense from the reconstructed output of the linear subsystem. The proposed identification method is illustrated by means of simulation results.

1. INTRODUCTION

Nonlinear system identification is of particular importance for a lot of signal processing applications (see an extensive bibliography in [1]). A great number of models can be used for representing a nonlinear dynamical system. Among them, block-oriented models, such as Wiener and Hammerstein models, consisting of a concatenation of linear dynamic subsystems and static nonlinear elements, provide parsimonious representations.

The use of Wiener models has been considered in the literature for various applications including communications [2, 3]. For example, they have been successfully used for representing a radio over fiber channel which combines two media: radio and optical. The optical part is used to interconnect a central radio processing facility with a remote radio antenna, the latter providing coverage to wireless broadband users. In such channels, nonlinear distortion mainly occurs from the electrical to optical conversion process. This distortion is usually modelled as a static nonlinearity in polynomial form. The wireless-fiber channel can be modelled by a linear filter (the wireless channel) followed by a static nonlinearity (the nonlinear link), i.e. a Wiener model.

Several methods have been proposed for identifying such Wiener models. In [4], a nonparametric approach is used to identify a Wiener system driven by a white Gaussian input. More recently, some works have extended linear subspace identification methods to this class of nonlinear models (see [5] and references therein). In [6], Wiener systems are viewed as constrained Volterra series. Singular value decomposition (SVD) and Higher-order SVD are used for parameter estimation.

Indeed, when the nonlinear subsystem is a polynomial one, the Wiener system admits an equivalent representation under the form of a Volterra model. Unlike representations using linear and nonlinear blocks, Volterra models present the advantage of being linear in their parameters. However, the parameter number for such a model is huge. As a consequence, the parameter estimation is a very difficult task. Recently, it has been shown that, in fact, the diagonal coefficients of Volterra kernels associated with Wiener and Wiener-Hammerstein systems suffice to generate the non-diagonal coefficients [7]. This result opens new research perspectives for identifying block-oriented nonlinear systems in exploiting the algebraic structure of the associated Volterra kernels.

The input-output cross-correlation method suggested by Lee and Schetzen [8] can be used for identifying Volterra systems when the input signal is a zero-mean Gaussian one. Some improvements of the above cited method have been proposed in the literature (see [9] and references therein for example). This solution is non-trivial and due to its sequential nature, estimation errors in one kernel propagate to the estimation of the next ones. However, two particular cases are free of such drawbacks: homogeneous systems [10] and linear-quadratic systems [11]. For more general i.i.d. inputs, Tseng and Powers [12] have derived closed form expressions of the kernels of third-order Volterra systems.

In this paper, we consider a new approach based on the i.i.d. property of the input signal and on the algebraic structure of the second order Volterra kernel associated with a Wiener system. The paper is organized as follows. Section 2 gives a general problem formulation. Closed form expressions of the second-order kernel of a fifth-order Volterra system, in terms of input-output crossmoments, are derived in Section 3. A complete procedure for estimating the parameters of the linear and nonlinear parts of a fifth-order Wiener system is proposed in Section 4. Some simulation results are provided to illustrate the performances of the proposed identification method in Section 5 before concluding the paper in Section 6.

2. PROBLEM FORMULATION

Let us consider the identification of a block-structured nonlinear channel from input-output measurements $(u(.), s(.))$. The channel is modelled as a single-input single-output linear time-invariant system with finite impulse response followed by a static nonlinearity:

$$z(n) = \sum_{i=0}^{M-1} g(i)u(n-i), \quad (1)$$

$$y(n) = \mathcal{N}(z(n)), \quad (2)$$

$y(.)$ being the model output and $z(.)$ a nonmeasurable intermediate signal. We assume that the nonlinear function $\mathcal{N} : \mathbb{R} \to \mathbb{R}$ can be approximated by means of a finite degree polynomial:

$$\mathcal{N}(z(n)) = \sum_{p=1}^{P} \alpha_p z^p(n). \quad (3)$$

The output of the Volterra model associated with this Wiener nonlinear system can be written as

$$y(n) = \sum_{p=1}^{P} y_p(n) = \sum_{p=1}^{P} \sum_{i_1,\cdots,i_p=0}^{M-1} h_p(i_1,\cdots,i_p) \prod_{j=1}^{p} u(n-i_j) \quad (4)$$

where $h_p(.)$ denotes the pth-order Volterra kernel given by:

$$h_p(i_1,\cdots,i_p) = \alpha_p \prod_{k=1}^{p} g(i_k),\ i_k = 0,\cdots,M-1,\ k = 1,\cdots,p. \quad (5)$$

We have to notice that all the Volterra kernels associated with a Wiener system are symmetric, i.e. for any $p \geq 2$:

$$h_p(i_1,\cdots,i_p) = h_p(\pi(i_1),\cdots,\pi(i_p)),$$

where $\pi(.)$ is any permutation of the indices $i_1,\cdots,i_p$.

We denote by $\mathbf{g}$ and $\boldsymbol{\alpha}$ the vectors containing respectively the impulse response coefficients of the linear subsystem and the polynomial coefficients:

$$\mathbf{g} = (g(0)\ \cdots\ g(M-1))^T, \quad \boldsymbol{\alpha} = (\alpha_1\ \cdots\ \alpha_P)^T$$

We consider systems with non vanishing second order kernel, i.e. $\alpha_2 \neq 0$. Taking expression (5) of the second order kernel coefficients into account, it is straightforward to verify that this kernel is a rank-one symmetric matrix that can be factored as:

$$\mathbf{H}_2 = \begin{pmatrix} h_2(0,0) & \cdots & h_2(0,M-1) \\ h_2(1,0) & \cdots & h_2(1,M-1) \\ \vdots & \ddots & \vdots \\ h_2(M-1,0) & \cdots & h_2(M-1,M-1) \end{pmatrix} = \alpha_2 \mathbf{g}\mathbf{g}^T. \quad (6)$$

Due to its symmetry, this matrix always admits an eigenvalue decomposition (EVD) and $\mathbf{g}$ can be estimated as the eigenvector associated with the nonzero eigenvalue. In fact, the obtained impulse response will be a scaled version of the actual one. Such an ambiguity is inherent to Wiener system modelling if no additional constraints are considered. A unique model can be obtained if, for example, at least one coefficient of the impulse response $g(.)$ is constrained to be equal to 1. We can also assume that the impulse response is normalized. In such a case, it remains a sign ambiguity.

We first present a method for estimating the second-order Volterra kernel using cross correlations of input-output signals. Then, we propose a procedure for parameter estimation of fifth-order Wiener systems.

3. ESTIMATION OF THE SECOND-ORDER KERNEL OF A FIFTH-ORDER VOLTERRA MODEL

Usually, given input and output signals, the estimated Volterra kernels, in the Minimum Mean Square Error (MMSE) sense, are obtained by solving a system of normal equations. The main computation burden stems from the estimation of higher order moments (up to sixth order for fifth-order Volterra systems) with various time lags. The subsequent solution of the normal equations is obtained by inverting a relatively dense auto-correlation matrix. In [12], closed form expressions of Volterra kernels of cubic systems were presented by considering a zero-mean i.i.d. input. Deriving similar expressions for higher order systems is a tough task. However, a solution was provided for complex baseband fifth-order Volterra systems using i.i.d. QAM and PSK signals [13]. Closed form expressions in terms of input-output crossmoments, for estimating the second-order kernel of a fifth-order Volterra model, are derived in this section, under stationarity and ergodicity assumptions, and in exploiting the i.i.d. property of the input.

We assume that the input signal is i.i.d. with a symmetrical probability density function (pdf) and we denote by $\mu_p = \mathbb{E}[u^p(n)]$ its pth-order moment, $\mathbb{E}[.]$ denoting the mathematical expectation.

We assume that the measured output signal $s(.)$ is centered. Therefore, we add an additional parameter h_0 in the Volterra model (4) for taking this assumption into account, i.e. the model output is given by

$$y(n) = h_0 + \sum_{p=1}^{5} y_p(n). \quad (7)$$

Our purpose is to determine the optimal second-order Volterra kernel in the MMSE sense, i.e. in minimizing the following cost function

$$\mathscr{J} = \mathbb{E}\left[\varepsilon^2(n)\right], \quad \varepsilon(n) = s(n) - y(n). \quad (8)$$

The MMSE solution is obtained in solving the orthogonality relations

$$\mathbb{E}\left[\varepsilon(n)\prod_{k=1}^{q} u(n-\tau_k)\right] = 0, \quad q = 0,\cdots,5, \quad (9)$$

which can be equivalently written as

$$\begin{aligned} C_{s,u,q+1}(\tau_1,\cdots,\tau_q) &= \mathbb{E}\left[y(n)\prod_{k=1}^{q} u(n-\tau_k)\right] \quad (10) \\ &= h_0\kappa_{\tau_1,\cdots,\tau_q} + \sum_{p=1}^{5} C_{y_p,u,q+1}(\tau_1,\cdots,\tau_q) \end{aligned}$$

where $C_{x,u,q}(\tau_1,\cdots,\tau_{q-1})$ denotes the qth-order cross-moment of the signal $x(.)$ with the input signal delayed by time lags $\tau_1,\cdots,\tau_{q-1}$:

$$C_{x,u,q}(\tau_1,\cdots,\tau_{q-1}) = \mathbb{E}\left[x(n)\prod_{k=1}^{q-1} u(n-\tau_k)\right], \quad q > 1,$$

$$C_{x,1} = \mathbb{E}[x(n)],$$

and

$$\kappa_{\tau_1,\cdots,\tau_q} = \mathbb{E}\left[\prod_{k=1}^{q} u(n-\tau_k)\right]. \quad (11)$$

Due to the symmetry of the input signal pdf, the cross-moments $C_{y_p,u,q+1}(.)$ are zero if $p = 1,3,5$ and $q = 0,2,4$. Therefore we have the following basic relation, for $q = 0,2,4$:

$$\begin{aligned} C_{s,u,q+1}(\tau_1,\cdots,\tau_q) &= h_0\kappa_{\tau_1,\cdots,\tau_q} + C_{y_2,u,q+1}(\tau_1,\cdots,\tau_q) \\ &\quad + C_{y_4,u,q+1}(\tau_1,\cdots,\tau_q). \quad (12) \end{aligned}$$

In the sequel, we express the above equation in terms of the Volterra kernel coefficients and then we solve the resulting set of equations. We first define the following parameters:

$$\begin{aligned} a &= \mu_4 - \mu_2^2 \quad (13) \\ b &= \mu_6 - \mu_2\mu_4 \quad (14) \\ c &= \mu_8 - \mu_4^2 \quad (15) \\ d &= \mu_4^2 + \mu_2\mu_6 \quad (16) \\ e &= \mu_4^2 - \mu_2\mu_6. \quad (17) \end{aligned}$$

Theorem: *If the input signal is i.i.d. with a symmetrical pdf and if its moments are such as $a \neq 0$, $e \neq 0$, and $b^2 - ac \neq 0$, then the optimal coefficients of the second order kernel of a fifth-order Volterra system are given by:*

$$h_2(\tau,\tau) = \frac{bC_{s,u,5}(\tau,\tau,\tau,\tau) - cC_{s,u,3}(\tau,\tau)}{b^2 - ac} - 6\mu_2 G_{2,2}(\tau,\tau) \quad (18)$$

and

$$h_2(\tau_1,\tau_2) = \frac{\mu_2\mu_4 D_{s,u,5}(\tau_1,\tau_2) - dC_{s,u,3}(\tau_1,\tau_2)}{2e\mu_2^2} - 6\mu_2 G_{2,2}(\tau_1,\tau_2) \quad (19)$$

with

$$D_{s,u,5}(\tau_1,\tau_2) = C_{s,u,5}(\tau_1,\tau_1,\tau_1,\tau_2) + C_{s,u,5}(\tau_1,\tau_2,\tau_2,\tau_2), \quad (20)$$

$$G_{2,2}(\tau,\tau) = \sum_{i\neq\tau} \frac{C_{s,u,5}(\tau,\tau,i,i) - \mu_2 C_{s,u,3}(i,i) - \mu_2 C_{s,u,3}(\tau,\tau)}{6a^2}, \quad (21)$$

$$G_{2,2}(\tau_1,\tau_2) = \sum_{i\neq\tau_1, i\neq\tau_2} \frac{C_{s,u,5}(\tau_1,\tau_2,i,i) - \mu_2 C_{s,u,3}(\tau_1,\tau_2)}{12a\mu_2^2} \quad (22)$$

Proof: By solving equations (39) and (41), derived in Appendix B, we get:

$$h_2(\tau,\tau) = \frac{bC_{s,u,5}(\tau,\tau,\tau,\tau) - cC_{s,u,3}(\tau,\tau)}{b^2 - ac} - 6\mu_2 G_{2,2}(\tau,\tau),$$

with $G_{2,2}(\tau,\tau) = \sum_{i\neq\tau} h_4(\tau,\tau,i,i)$. We now derive the closed-form expression yielding $h_4(\tau,\tau,i,i)$. Taking (39) into account, (43) yields:

$$C_{s,u,5}(\tau,\tau,i,i) = \mu_2 C_{s,u,3}(\tau,\tau) + \mu_2 C_{s,u,3}(i,i) + 6a^2 h_4(\tau,\tau,i,i).$$

From this relation, it is straightforward to deduce $h_4(\tau,\tau,i,i)$ and then $G_{2,2}(\tau,\tau)$ as expressed in (21).

Now, let us determine the non-diagonal coefficients of the second order Volterra kernel. From definition (20) and relation (42), we obtain:

$$\begin{aligned} D_{s,u,5}(\tau_1,\tau_2) &= 4\mu_2\mu_4 h_2(\tau_1,\tau_2) + 4dg_4(\tau_1,\tau_2) \\ &\quad + 24\mu_2^2\mu_4 G_{2,2}(\tau_1,\tau_2), \end{aligned} \quad (23)$$

where $g_4(\tau_1,\tau_2)$ is defined in Appendix A. Solving (23) and (40) leads to:

$$h_2(\tau_1,\tau_2) = \frac{\mu_2\mu_4 D_{s,u,5}(\tau_1,\tau_2) - dC_{s,u,3}(\tau_1,\tau_2)}{2e\mu_2^2} - 6\mu_2 G_{2,2}(\tau_1,\tau_2)$$

In order to complete the computation of the non-diagonal coefficients of the second-order Volterra kernel, we determine the expression of $G_{2,2}(\tau_1,\tau_2) = \sum_{i\neq\tau_1, i\neq\tau_2} h_4(\tau_1,\tau_2,i,i)$. From (40) and (44), we get

$$C_{s,u,5}(\tau_1,\tau_2,i,i) = 12a\mu_2^2 h_4(\tau_1,\tau_2,i,i) + \mu_2 C_{s,u,3}(\tau_1,\tau_2).$$

From this relation, we deduce $h_4(\tau_1,\tau_2,i,i)$ and then expression (22) of $G_{2,2}(\tau_1,\tau_2)$. ■

4. ESTIMATION OF WIENER NONLINEAR CHANNELS

As stated in Section 2, once the second order kernel of the Volterra model estimated by using the closed form expressions (18) and (19), the linear subsystem is estimated by computing the eigenvalue decomposition of the matrix $\mathbf{H}_2$. Now we focus on the estimation of the polynomial subsystem. For this purpose, we reconstruct the intermediate signal $\hat{z}(.)$ as the output of the estimated linear subsystem $\hat{g}(.)$ excited by the same input signal $u(.)$ that was used to generate the measured output $s(.)$: $\hat{z}(n) = \sum_{i=0}^{M-1} \hat{g}(i)u(n-i)$.

Then, the reconstructed output signal is given by $\hat{y}(n) = \hat{\mathbf{z}}(n)^T \boldsymbol{\alpha}$, with $\hat{\mathbf{z}}(n) = \left(\hat{z}(n) \cdots \hat{z}^P(n)\right)^T$. By concatenating these signals for $n = 1, \cdots, N$, we get:

$$\hat{\mathbf{y}} = \begin{pmatrix} \hat{y}(1) \\ \vdots \\ \hat{y}(N) \end{pmatrix} = \begin{pmatrix} \hat{\mathbf{z}}^T(1) \\ \vdots \\ \hat{\mathbf{z}}^T(N) \end{pmatrix} \boldsymbol{\alpha} = \hat{\mathbf{Z}}\boldsymbol{\alpha}.$$

By minimizing the least square criterion $\|\mathbf{s} - \hat{\mathbf{y}}\|^2$, $\mathbf{s}$ containing a block of N measured output signals, we can estimate the parameters of the polynomial subsystem as:

$$\hat{\boldsymbol{\alpha}} = \hat{\mathbf{Z}}^\dagger \mathbf{s}, \quad (24)$$

where † denotes the matrix pseudo-inverse.

In summary, the proposed procedure for estimating a fifth-order Wiener nonlinear channel is composed of the following steps:

1. Estimate the needed input-output cross-moments.
2. Estimate the matrix $\mathbf{H}_2$ associated with the second-order Volterra kernel by using the closed-form expressions (18) and (19);
3. Estimate the linear subsystem parameter vector $\hat{\mathbf{g}}$ as the eigenvector associated with the greatest eigenvalue of $\mathbf{H}_2$.
4. Generate the reconstructed intermediate signal $\hat{z}(n)$, with $n = 1, \cdots, N$, and construct the matrix $\hat{\mathbf{Z}}$.
5. Estimate the polynomial coefficient vector $\hat{\boldsymbol{\alpha}}$ using (24).

Due to cross-moments estimation, obtaining accurate estimates with the proposed estimation method may require a huge number of data. To reduce this data number, we apply the input design scheme proposed in [14]. In a noiseless case, such a scheme guarantees that the estimated moments are exactly the same as the analytical ones without requiring a huge amount of data.

Consider a finite alphabet Λ with cardinality Q. Each output data $s(n)$ depends on a sequence $\mathscr{U}(n) = \{u(n-M+1), ..., u(n-1), u(n)\}$ of M input data belonging to Λ, each element of Λ having an equal probability of occurrence, and M being the channel memory. As a consequence, we can generate Q^M sequences $\mathscr{U}(.)$. By concatenating these Q^M sequences, we get an MQ^M-length i.i.d. input sequence. The output sequence is collected every M signaling periods to obtain Q^M output values, each output resulting from a particular input combination. The cross-moment estimator is then given by:

$$C_{s,u,q+1}(\tau_1,\cdots,\tau_q) = \frac{1}{Q^M} \sum_{n=0}^{Q^M-1} s(1+nM) \prod_{j=1}^{q} u(1+nM-\tau_j).$$

We can generalize this estimator as follows. Let $\mathscr{W}$ be a sequence where each sequence $\mathscr{U}(n)$, $n = 1, \cdots, Q^M$ randomly appears F times. $\mathscr{W}$ gives rise to an FMQ^M-length i.i.d. input sequence. The cross-moments are estimated as follows:

$$C_{s,u,q+1}(\tau_1,\cdots,\tau_q) = \frac{1}{FQ^M} \sum_{n=0}^{FQ^M-1} s(1+nM) \prod_{j=1}^{q} u(1+nM-\tau_j).$$

5. SIMULATION RESULTS

In this section, we present some simulation results for illustrating the performance of the proposed identification method. An additive, zero-mean, white Gaussian noise was added to the channel output. The simulation results were averaged over 100 independent Monte Carlo runs. The performances are evaluated in terms of the output Normalized Mean Square Error (NMSE). We assume that $g(0) = 1$.

5.1 Example 1

We first consider a fifth-order nonlinear channel, with memory $M = 3$, structured as a Wiener system. The input sequence was a 8-PAM one. The channel parameters and their estimated values are given in tables 1 and 2 for two different SNR values and three different data numbers N.

Table 1: Estimates of the channel parameters (SNR=10 *dB*)

Actual	N= 512	N=1024	N=2048
Parameters	mean± St. dev.	mean± St. dev.	mean± St. dev.
$g(1) = 0.2$	0.197 ± 0.064	0.210 ± 0.054	0.196 ± 0.023
$g(2) = -0.3$	-0.286 ± 0.059	-0.310 ± 0.053	-0.290 ± 0.029
$\alpha_1 = 1$	1.012 ± 0.064	1.003 ± 0.046	0.996 ± 0.033
$\alpha_2 = 0.7$	0.683 ± 0.033	0.695 ± 0.021	0.700 ± 0.017
$\alpha_3 = -0.1$	-0.107 ± 0.073	-0.107 ± 0.055	-0.098 ± 0.038
$\alpha_4 = 0.01$	0.019 ± 0.029	0.007 ± 0.022	0.013 ± 0.014
$\alpha_5 = 0.02$	0.021 ± 0.017	0.020 ± 0.012	0.021 ± 0.010

Table 2: Estimates of the channel parameters (SNR=30 dB)

Actual Parameters	N= 512 mean± St. dev.	N=1024 mean± St. dev.	N=2048 mean± St. dev.
$g(1) = 0.2$	0.199 ± 0.005	0.200 ± 0.004	0.199 ± 0.003
$g(2) = -0.3$	-0.300 ± 0.007	-0.300 ± 0.005	-0.300 ± 0.004
$\alpha_1 = 1$	0.999 ± 0.005	1.001 ± 0.004	1.000 ± 0.003
$\alpha_2 = 0.7$	0.700 ± 0.003	0.700 ± 0.002	0.700 ± 0.002
$\alpha_3 = -0.1$	-0.099 ± 0.006	-0.101 ± 0.005	-0.100 ± 0.003
$\alpha_4 = 0.01$	0.010 ± 0.003	0.010 ± 0.002	0.010 ± 0.002
$\alpha_5 = 0.02$	0.020 ± 0.002	0.020 ± 0.001	0.020 ± 0.001

One can note that the mean value of the estimated parameters are close to the actual parameters and don't significantly vary when the data number N is increased, unlike standard deviation that decreases when N increases. In Figure 1, we can see that the output NMSE is approximately equal to the SNR. The performances are slightly improved when increasing the data number.

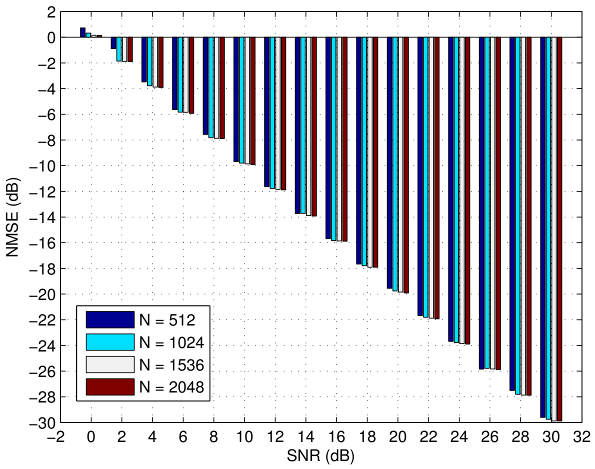

Figure 1: Output NMSE for different data sizes N

5.2 Example 2

In this example, we compare the proposed estimation method with that of [6] which consists in first estimating the overall Volterra model and then the linear subsystem parameters are deduced from the SVD of a matrix constituted with all the kernels. Such a method is much more demanding in terms of computational resources. The simulated system is given by: $y(n) = z(n) + 10z^2(n) + z^3(n)$, $z(n) = u(n) + 0.5u(n-1)$. The input signal was a 6-PAM. Fig. 2 depicts the $NMSE_1$ and $NMSE_2$ respectively obtained with the method in [6] and that proposed in this paper. Table 3 gives the performance gap between the two considered methods, i.e. $NMSE_1 - NMSE_2$. From these simulation results, we can conclude that both methods give very close performances, our method allowing to significantly reduce the computational cost due to the estimation of only the second-order Volterra kernel instead of the three ones, as it is the case with the method of [6].

6. CONCLUSION

In this paper, we have proposed a new method for estimating the parameters of a nonlinear Wiener channel with a static nonlinearity represented by a fifth degree polynomial. The estimation is carried out in three steps: estimation of the second order kernel of the associated Volterra model using third- and fifth-order input-output crossmoments, estimation of the linear subsystem through an EVD of the estimated Volterra kernel, estimation of the polynomial coefficients in the least squares sense using the intermediate output signal reconstructed from the estimated linear subsystem. For estimating the second order Volterra kernel without requiring the estimation of the overall Volterra model, we have derived closed-form expressions based on the i.i.d. assumption on the input signal. We have shown by means of simulations that the proposed estimation method provides performances very close to the MMSE bound.

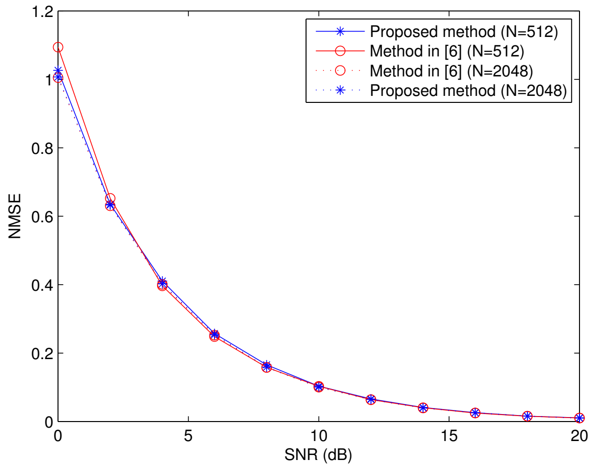

Figure 2: Comparison of the proposed method with that of [6]

Table 3: Gap in dB between NMSE obtained with the two compared methods

	SNR 0 dB	SNR 10 dB	SNR 30 dB
$N = 512$	-0.276	0.020	0.088
$N = 1024$	0.014	0.081	0.047
$N = 1536$	0.015	0.036	0.027
$N = 2048$	0.007	0.025	0.026

Appendix A: Cross-moments computation

Let us define:

$$H_2(\tau) = \sum_{i \neq \tau} h_2(i,i), \qquad H_{2,2}(\tau) = \sum_{i \neq \tau} \sum_{j \neq \tau, j > i} h_4(i,i,j,j),$$
$$H_4(\tau) = \sum_{i \neq \tau} h_4(i,i,i,i), \quad G_{2,2}(\tau_1,\tau_2) = \sum_{i \neq \tau_1, i \neq \tau_2} h_4(\tau_1,\tau_2,i,i),$$
$$g_4(\tau_1,\tau_2) = h_4(\tau_1,\tau_1,\tau_1,\tau_2) + h_4(\tau_1,\tau_2,\tau_2,\tau_2).$$

Taking the i.i.d. assumption on the input signal and the symmetry of the Volterra kernels into account, we get the following expressions for the cross-moments $C_{y_p,u,q+1}(\tau_1,\cdots,\tau_q)$, with $q = 0,2,4$, and $p = 2,4$, involved in (12):

$$\begin{aligned} C_{y_2,1} &= \mu_2 \sum_i h_2(i,i) \\ &= \mu_2 h_2(\tau,\tau) + \mu_2 H_2(\tau) && (25) \\ C_{y_2,u,3}(\tau,\tau) &= \mu_4 h_2(\tau,\tau) + \mu_2^2 H_2(\tau) && (26) \\ C_{y_2,u,3}(\tau_1,\tau_2) &= 2\mu_2^2 h_2(\tau_1,\tau_2) && (27) \\ C_{y_2,u,5}(\tau,\tau,\tau,\tau) &= \mu_6 h_2(\tau,\tau) + \mu_2\mu_4 H_2(\tau) && (28) \\ C_{y_2,u,5}(\tau_1,\tau_1,\tau_1,\tau_2) &= 2\mu_2\mu_4 h_2(\tau_1,\tau_2) && (29) \end{aligned}$$

$$C_{y_2,u,5}(\tau_1,\tau_1,\tau_2,\tau_2) = \mu_2\mu_4 h_2(\tau_1,\tau_1) + a\mu_2 h_2(\tau_2,\tau_2) + \mu_2^3 H_2(\tau_1) \quad (30)$$

$$C_{y_2,u,5}(\tau_1,\tau_2,\tau,\tau) = 2\mu_2^3 h_2(\tau_1,\tau_2) \quad (31)$$

$$C_{y_4,1} = \mu_4 \sum_i h_4(i,i,i,i) + 6\mu_2^2 \sum_{i\neq j}\sum_j h_4(i,i,j,j) = \mu_4 h_4(\tau,\tau,\tau,\tau) + 6\mu_2^2 G_{2,2}(\tau,\tau) + \mu_4 H_4(\tau) + 6\mu_2^2 H_{2,2}(\tau), \quad (32)$$

$$C_{y_4,u,3}(\tau,\tau) = \mu_6 h_4(\tau,\tau,\tau,\tau) + 6\mu_2\mu_4 G_{2,2}(\tau,\tau) + \mu_2\mu_4 H_4(\tau) + 6\mu_2^3 H_{2,2}(\tau) \quad (33)$$

$$C_{y_4,u,3}(\tau_1,\tau_2) = 4\mu_2\mu_4 g_4(\tau_1,\tau_2) + 12\mu_2^3 G_{2,2}(\tau_1,\tau_2) \quad (34)$$

$$C_{y_4,u,5}(\tau,\tau,\tau,\tau) = \mu_8 h_4(\tau,\tau,\tau,\tau) + 6\mu_2\mu_6 G_{2,2}(\tau,\tau) + \mu_4^2 H_4(\tau) + 6\mu_2^2\mu_4 H_{2,2}(\tau) \quad (35)$$

$$C_{y_4,u,5}(\tau_1,\tau_1,\tau_1,\tau_2) = 4\mu_4^2 h_4(\tau_1,\tau_2,\tau_2,\tau_2) + 4\mu_2\mu_6 h_4(\tau_1,\tau_1,\tau_1,\tau_2) + 12\mu_2^2\mu_4 G_{2,2}(\tau_1,\tau_2) \quad (36)$$

$$C_{y_4,u,5}(\tau_1,\tau_1,\tau_2,\tau_2) = \mu_2\mu_6 h_4(\tau_1,\tau_1,\tau_1,\tau_1) + b\mu_2 h_4(\tau_2,\tau_2,\tau_2,\tau_2) + 6a^2 h_4(\tau_1,\tau_1,\tau_2,\tau_2) + 6\mu_2^2\mu_4 G_{2,2}(\tau_1,\tau_1) + 6\mu_2^4 H_{2,2}(\tau_1) + 6a\mu_2^2 G_{2,2}(\tau_2,\tau_2) + \mu_2^2\mu_4 H_4(\tau_1) \quad (37)$$

$$C_{y_4,u,5}(\tau_1,\tau_2,\tau,\tau) = 12a\mu_2^2 h_4(\tau_1,\tau_2,\tau,\tau) + 4\mu_2^2\mu_4 g_4(\tau_1,\tau_2) + 12\mu_2^4 G_{2,2}(\tau_1,\tau_2) \quad (38)$$

Appendix B: Basic relationships

For $q=0$, equation (12) can be written as $C_{s,1} = h_0 + C_{y_2,1} + C_{y_4,1}$. Since $s(.)$ is centered, then $h_0 = -C_{y_2,1} - C_{y_4,1}$, with $C_{y_2,1}$ and $C_{y_4,1}$ respectively given in (25) and (32).

For q=2, using the definitions (11), (13), and (14), the i.i.d. property of the input signal, the above expression of h_0, and equations (26), (27), (33) and (34), equation (12) becomes:

$$C_{s,u,3}(\tau,\tau) = a h_2(\tau,\tau) + b h_4(\tau,\tau,\tau,\tau) + 6a\mu_2 G_{2,2}(\tau,\tau) \quad (39)$$

$$C_{s,u,3}(\tau_1,\tau_2) = 2\mu_2^2 h_2(\tau_1,\tau_2) + 4\mu_2\mu_4 g_4(\tau_1,\tau_2) + 12\mu_2^3 G_{2,2}(\tau_1,\tau_2). \quad (40)$$

Similarly, for $q=4$, by using definitions (14) and (15), equations (28)-(31), (35)-(38), and the expression of h_0, equation (12) yields:

$$C_{s,u,5}(\tau,\tau,\tau,\tau) = b h_2(\tau,\tau) + c h_4(\tau,\tau,\tau,\tau) + 6b\mu_2 G_{2,2}(\tau,\tau) \quad (41)$$

$$C_{s,u,5}(\tau_1,\tau_1,\tau_1,\tau_2) = 2\mu_2\mu_4 h_2(\tau_1,\tau_2) + 4\mu_4^2 h_4(\tau_1,\tau_2,\tau_2,\tau_2) + 4\mu_2\mu_6 h_4(\tau_1,\tau_1,\tau_1,\tau_2) + 12\mu_2^2\mu_4 G_{2,2}(\tau_1,\tau_2) \quad (42)$$

$$C_{s,u,5}(\tau_1,\tau_1,\tau_2,\tau_2) = a\mu_2 h_2(\tau_1,\tau_1) + b\mu_2 h_4(\tau_1,\tau_1,\tau_1,\tau_1) + a\mu_2 h_2(\tau_2,\tau_2) + b\mu_2 h_4(\tau_2,\tau_2,\tau_2,\tau_2) + 6a\mu_2^2 G_{2,2}(\tau_1,\tau_1) + 6a\mu_2^2 G_{2,2}(\tau_2,\tau_2) + 6a^2 h_4(\tau_1,\tau_1,\tau_2,\tau_2) \quad (43)$$

$$C_{s,u,5}(\tau_1,\tau_2,\tau,\tau) = 2\mu_2^3 h_2(\tau_1,\tau_2) + 12a\mu_2^2 h_4(\tau_1,\tau_2,\tau,\tau) + 4\mu_2^2\mu_4 g_4(\tau_1,\tau_2) + 12\mu_2^4 G_{2,2}(\tau_1,\tau_2) \quad (44)$$

REFERENCES

[1] G.B. Giannakis and E. Serpedin, "A bibliography on nonlinear system identification," *Signal Processing*, vol. 81, no. 3, pp. 533–580, March 2001.

[2] R. Lopez-Valcarce and Dasgupta, "Second-order statistical properties of nonlinearly distorted phase-shift keyed (PSK) signals," *IEEE Communications Letters*, vol. 7, no. 7, pp. 323–325, July 2003.

[3] X.N. Fernando and A.B. Sesay, "Fiber wireless channel estimation using correlation properties of PN sequences," *Canadian Journal of Electrical and Computer Engineering*, vol. 26, no. 2, pp. 43–44, April 2001.

[4] W. Greblicki, "Nonparametric identification of Wiener systems by orthogonal series," *IEEE Trans. Automatic Control*, vol. 39, no. 10, pp. 2077–2086, October 1994.

[5] R. Raich, G. Tong Zhou, and M. Viberg, "Subspace based approaches for Wiener system identification," *IEEE Trans. Automatic Control*, vol. 50, no. 10, pp. 1629–1634, 2005.

[6] S. Lacy and D. Bernstein, "Identification of FIR Wiener systems with unknown, noninvertible, polynomial nonlinearities," in *Proc. of the American Control Conference*, Anchorage, AK, May 8-10 2002, pp. 893–898.

[7] A.Y. Kibangou and G. Favier, "Wiener-Hammerstein systems modelling using diagonal Volterra kernels coefficients," *IEEE Signal Proc. Letters*, vol. 13, no. 6, pp. 381–384, June 2006.

[8] Y.W. Lee and M. Schetzen, "Measurement of the Wiener kernels of a nonlinear system by crosscorrelation," *Int. Journal of Control*, vol. 2, no. 3, pp. 237–254, 1965.

[9] M. Pirani, S. Orcioni, and C. Turchetti, "Diagonal kernel point estimation of nth-order discrete Volterra-Wiener systems," *EURASIP Journal on App. Signal. Proc.*, vol. 12, pp. 1807–1816, 2004.

[10] D.R. Brillinger, "The identification of polynomial systems by means of higher-order spectra," *Journal of Sound and Vibration*, vol. 12, pp. 301–313, 1970.

[11] L.J. Tick, "The estimation of transfer function of quadratic systems," *Technometr.*, vol. 3, no. 4, pp. 562–567, Nov. 1961.

[12] C.-H. Tseng and E.J. Powers, "Identification of cubic systems using higher order moments of I.I.D. signals," *IEEE Trans. on Signal Processing*, vol. 43, no. 7, pp. 1733–1735, 1995.

[13] C.-H. Cheng and E.J. Powers, "Optimal Volterra kernel estimation algorithms for a nonlinear communication system for PSK and QAM inputs," *IEEE Trans. Signal Processing*, vol. 49, no. 1, pp. 147–163, 2001.

[14] C.-H. Tseng and E.J. Powers, "Identification of nonlinear channels in digital transmission systems," in *Proc. of IEEE Signal Processing Workshop on Higher-order Statistics*, South Lake Tahoe, CA, June 1993, pp. 42–45.