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AN ANALYTICAL MODEL FOR THE CONSTRAINED STOCHASTIC GRADIENT ALGORITHM

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ABSTRACT

This paper presents an analytical model for the constrained stochastic gradient (CSG) algorithm. This algorithm is used to obtain the weights of antenna arrays for mobile communications, aiming to maximize the signal-to-interference-plus-noise ratio (SINR) of such systems. For the algorithm performance evaluation, several operating conditions are considered by combining a wide variety of angles-of-arrival for both desired and interfering signals. To have an analytical model describing the system behavior is very useful, since it permits efficient evaluation of the algorithm performance under different working conditions. Thus, analytical expressions for the first moment of the weight vector and the SINR characteristic are derived here. The proposed model is obtained under a small step-size condition, resulting in smooth behavior for both the weight and SINR characteristics. The accuracy of the proposed model is assessed through numerical simulations.

1. INTRODUCTION

Over the last few years, wireless communication networks have become more and more popular. Thus, the increasing number of users is generating serious problems in densely urbanized areas since the frequency spectrum is approaching the limit of its capacity [1]. The limited spectrum problem can be overcome by using sectorized cells and frequency reuse along with a reduction of the cell size. For instance, with a layout of seven cells/cluster, having three sectors of 120° per cell, the number of carrier frequencies available in each sector is 1/21 of the total number of available frequencies. A layout of three cells per cluster increases the system capacity by a factor of approximately 7/3. However, the distance between co-channel cells decreases by a factor of $\sqrt{21/9}$, thus increasing the level of co-channel interfering signals in the system [2]. A strategy to reduce such interference is the use of antenna arrays in the base stations, so that the array radiation pattern is directed to the position of the mobile terminals inside the proper cell and canceling the co-channel interference to other cells.

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Ideally, each cell must have knowledge of the position of all mobile terminals in its area, thus determining an optimum global solution. Unfortunately, such an approach is not practical. Currently, all information about the mobile terminals is obtained through the uplink channel. With this information available, an approach in which the neighboring cells reduce the transmitted signal power in the direction of the co-channel mobile terminals can be used. In this way, the interference level is globally decreased. By considering that the signal propagation of the uplink and downlink channels is similar, it can be assumed that the covariance matrices of the channels are similar [1]. For instance, in [3]-[5] such an assumption is used for controlling the adaptive array by using the downlink and interference covariance matrices. Both estimates are obtained from uplink measurements. Such a procedure is used in [1] to define a local objective function, thereby obtaining an approximate optimum global solution in a cooperative network, without the need of having communication between cells. From that objective function, the constrained stochastic gradient (CSG) algorithm is derived. The CSG algorithm is interesting because of its low computational complexity and very good convergence properties in comparison with other algorithms [1].

The performance of an adaptive array can be assessed through the signal-to-interference-plus-noise ratio (SINR) [1]. To obtain adequate knowledge of such a function, it is necessary to consider a wide range of operating conditions. Specifically, several combinations of angles-of-arrival (AoA) for the in-cell and co-channel interfering uplink signals must be considered. In this way, the performance assessment of the algorithm requires a large number of simulations to determine the mean behavior for each combination. In this context, the possibility of having an algorithm model becomes important since much simulation time is saved. The model allows an exhaustive study of the algorithm performance in different working conditions, also providing useful insights on the algorithm behavior. Thus, this work aims to develop model expressions for the mean weight behavior of the adaptive filter weights as well as for the SINR characteristics. The simplifying assumption considered to derive the analytical model is a small step-size condition.

2. CSG ALGORITHM

In adaptive antenna array systems, the adaptive algorithm aims to maximize the irradiated power to the mobile terminal (MT) inside the cell P_{IC} and to minimize that power to the co-channel MT in other cells P_{CC}. These powers are expressed, respectively, as

$$P_{\mathrm{IC}} = \mathbf{w}^{\mathrm{H}}\mathbf{R}_{\mathrm{IC}}\mathbf{w} \tag{1}$$

and

$$P_{\mathrm{CC}} = \mathbf{w}^{\mathrm{H}}\mathbf{R}_{\mathrm{CC}}\mathbf{w} \tag{2}$$

where $\mathbf{w} = [w_1\ w_2 \cdots w_M]^{\mathrm{T}}$ denotes the adaptive complex weight vector, and $\mathbf{R}_{\mathrm{IC}}$ and $\mathbf{R}_{\mathrm{CC}}$ are the in-cell and co-channel interference downlink covariance matrices, respectively. Then, the goal of the adaptive algorithm is to maximize over $\mathbf{w}$ the following expression [1]:

$$\xi = \frac{\mathbf{w}^{\mathrm{H}}\mathbf{R}_{\mathrm{IC}}\mathbf{w}}{\mathbf{w}^{\mathrm{H}}(\mathbf{R}_{\mathrm{CC}}+\mathbf{I})\mathbf{w}} \tag{3}$$

where $\mathbf{I}$ is the identity matrix, representing normalized additive noise power. The complex weight vector in (3) is considered fixed. In [1], the CSG algorithm is derived from an intuitive interpretation of (3). In this way, to maximize (3), the steepest descent algorithm is used for the co-channel interference power along with the steepest ascent algorithm for the in-cell signal power. Since two optimization processes are simultaneously running, such a strategy may result in slow algorithm convergence. A heuristic solution to circumvent the convergence problem is to use a two-step update scheme. Firstly, the numerator of (3) is adapted keeping the denominator fixed; next, the denominator is adapted, considering the numerator fixed. Mathematically, such a scheme can be implemented by incorporating a projection matrix into the updating expression. As such, the CSG algorithm uses both the in-cell uplink signal $s(n)$ and the co-channel uplink signal $u(n)$ to estimate the downlink covariance matrices. For the case of two interferers, the updating equations are given by [1]

$$\mathbf{v}_1(n) = \mathbf{w}(n) + \mu_{\mathrm{s}}\left[\mathbf{I} - \frac{\hat{\mathbf{R}}_{\mathrm{u}_1}(n)}{\|\mathbf{u}_1(n)\|^2} - \frac{\hat{\mathbf{R}}_{\mathrm{u}_2}(n)}{\|\mathbf{u}_2(n)\|^2}\right]\hat{\mathbf{R}}_{\mathrm{s}}(n)\mathbf{w}(n) \tag{4}$$

and

$$\mathbf{v}_2(n) = \mathbf{v}_1(n) - \mu_{\mathrm{u}}\left[\mathbf{I} - \frac{\hat{\mathbf{R}}_{\mathrm{s}}(n)}{\|\mathbf{s}(n)\|^2}\right][\hat{\mathbf{R}}_{\mathrm{u}_1}(n) + \hat{\mathbf{R}}_{\mathrm{u}_2}(n)]\mathbf{w}(n) \tag{5}$$

with

$$\mathbf{w}(n+1) = \frac{\mathbf{v}_2(n)}{\|\mathbf{v}_2(n)\|} \tag{6}$$

where $\hat{\mathbf{R}}_{\mathrm{s}}(n) = \mathbf{s}(n)\mathbf{s}^{\mathrm{H}}(n)$ and $\hat{\mathbf{R}}_{\mathrm{u}_k}(n) = \mathbf{u}_k(n)\mathbf{u}_k^{\mathrm{H}}(n)$. Vectors $\mathbf{s}(n)$ and $\mathbf{u}_k(n)$ are the in-cell uplink and the kth co-channel interference vectors, respectively. Parameters μ_{s} and μ_{u} are the corresponding algorithm step sizes. The weight vector is normalized at each iteration, maintaining the term $\mathbf{w}^{\mathrm{H}}\mathbf{I}\mathbf{w}$ constant in (3).

3. STOCHASTIC MODEL OF THE CSG ALGORITHM

3.1. Mean Weight Behavior

The first step to obtain a model describing the mean weight behavior of the adaptive vector is to determine the expected value of both sides of (6). Thus,

$$E[\mathbf{w}(n+1)] = E\left[\frac{\mathbf{v}_2(n)}{\|\mathbf{v}_2(n)\|}\right]. \tag{7}$$

Since obtaining such an expected value is not a trivial task, by considering that the evolution of $\mathbf{v}_2(n)$ is smooth (small dispersion condition), the following approximation can be used [6]:

$$E[\mathbf{w}(n+1)] \approx \frac{E[\mathbf{v}_2(n)]}{\|E[\mathbf{v}_2(n)]\|}. \tag{8}$$

Small dispersion is ensured by using a small-step-size assumption, so the above approximation is valid. Also (8) guarantees the condition $\|E[\mathbf{w}(n)]\| = 1$, which is a characteristic of the CSG algorithm ($\|\mathbf{w}(n)\| = 1$ is enforced at each iteration). Then, by taking the expected value of both sides of (4) and (5), we obtain, as explained below,

$$E[\mathbf{v}_1(n)] = E[\mathbf{w}(n)] + \mu_{\mathrm{s}}(\mathbf{I} - \mathbf{R}_{\mathrm{u}_1}^{\mathrm{N}} - \mathbf{R}_{\mathrm{u}_2}^{\mathrm{N}})\mathbf{R}_{\mathrm{s}}E[\mathbf{w}(n)] \tag{9}$$

and

$$E[\mathbf{v}_2(n)] = E[\mathbf{v}_1(n)] - \mu_{\mathrm{u}}(\mathbf{I} - \mathbf{R}_{\mathrm{s}}^{\mathrm{N}})(\mathbf{R}_{\mathrm{u}_1} + \mathbf{R}_{\mathrm{u}_2})E[\mathbf{w}(n)] \tag{10}$$

where $\mathbf{R}_{\mathrm{s}}^{\mathbf{N}} = E\left[\frac{\mathbf{s}(n)\mathbf{s}^{\mathrm{H}}(n)}{\mathbf{s}^{\mathrm{H}}(n)\mathbf{s}(n)}\right]$ and $\mathbf{R}_{\mathrm{u}_k}^{\mathbf{N}} = E\left[\frac{\mathbf{u}_k(n)\mathbf{u}_k{}^{\mathrm{H}}(n)}{\mathbf{u}_k^{\mathrm{H}}(n)\mathbf{u}_k(n)}\right]$ are the normalized sample covariance matrices. To obtain such matrices, some useful simplifying assumptions can be used [7]. However, in this work the expected values of such matrices are calculated exactly, thus keeping very good accuracy for the resulting model (see Appendix).

To obtain (9) and (10), the weight vector and the arrived signal vectors are assumed statistically independent. This consideration is justifiable from the small-step-size condition [8]. By substituting (9) into (10) and the resulting expression into (8), the expression for mean weight behavior is obtained. Thus,

$$E[\mathbf{w}(n+1)] = \frac{\mathbf{A}E[\mathbf{w}(n)]}{\sqrt{E[\mathbf{w}^{\mathrm{H}}(n)]\mathbf{A}^{\mathrm{H}}\mathbf{A}\,E[\mathbf{w}(n)]}} \tag{11}$$

where $\mathbf{A} = \mathbf{I} + \mu_{\mathrm{s}}(\mathbf{I} - \mathbf{R}_{\mathrm{u}_1}^{\mathrm{N}} - \mathbf{R}_{\mathrm{u}_2}^{\mathrm{N}})\mathbf{R}_{\mathrm{s}} - \mu_{\mathrm{u}}(\mathbf{I} - \mathbf{R}_{\mathrm{s}}^{\mathrm{N}})(\mathbf{R}_{\mathrm{u}_1} + \mathbf{R}_{\mathrm{u}_2})$.

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3.2. SINR Expression

The basic objective of the adaptive array is to maximize the downlink SINR. Therefore, the SINR behavior is an important parameter governing the array performance. By definition, the SINR is given by

$$\gamma = \frac{E[P_{\mathrm{IC}}]}{E[P_{\mathrm{CC}}] + E[P_\eta]} \tag{12}$$

where P_η is the noise power. Considering that the weight vector and the downlink signals are independent, the evolution of the SINR can be written as

$$\gamma(n) = \frac{E[\mathbf{w}^{\mathrm{H}}(n)\mathbf{R}_{\mathrm{IC}}\mathbf{w}(n)]}{E[\mathbf{w}^{\mathrm{H}}(n)\mathbf{R}_{\mathrm{CC}}\mathbf{w}(n)] + E[P_\eta]}. \tag{13}$$

Alternatively, (13) can also be expressed as

$$\gamma(n) = \frac{\mathrm{tr}[\mathbf{R}_{\mathrm{IC}}\mathbf{K}(n)]}{\mathrm{tr}[\mathbf{R}_{\mathrm{CC}}\mathbf{K}(n)] + E[P_\eta]} \tag{14}$$

where $\mathbf{K}(n) = E[\mathbf{w}(n)\mathbf{w}^{\mathrm{H}}(n)]$ is the second moment of the adaptive weight vector. Then, normalizing (14) with respect to the mean noise power, results in [1]

$$\gamma(n) = \frac{\rho_{\mathrm{IC}}\,\mathrm{tr}[\tilde{\mathbf{R}}_{\mathrm{IC}}\mathbf{K}(n)]}{\rho_{\mathrm{CC}}\,\mathrm{tr}[\tilde{\mathbf{R}}_{\mathrm{CC}}\mathbf{K}(n)] + 1} \tag{15}$$

where ρ_{IC} and ρ_{CC} denote the signal-to-noise ratio and the interference-to-noise ratio, respectively. Note that from (15), the second moment must be known. Matrices $\tilde{\mathbf{R}}_{\mathrm{IC}}$ and $\tilde{\mathbf{R}}_{\mathrm{CC}}$ are normalized with respect to the noise power. Since the weight evolution of the CSG algorithm is smooth, the variance is small compared to the mean, so that for the sake of mathematical simplicity, the following approximation can be considered [9]:

$$\mathbf{K}(n) = E[\mathbf{w}(n)\mathbf{w}^{\mathrm{H}}(n)] \approx E[\mathbf{w}(n)]E[\mathbf{w}^{\mathrm{H}}(n)]. \tag{16}$$

Thus, (13) can now be computed from the mean weight expression using

$$\gamma(n) = \frac{\rho_{\mathrm{IC}} E[\mathbf{w}^{\mathrm{H}}(n)]\mathbf{R}_{\mathrm{IC}} E[\mathbf{w}(n)]}{\rho_{\mathrm{CC}} E[\mathbf{w}^{\mathrm{H}}(n)\mathbf{R}_{\mathrm{CC}} E[\mathbf{w}(n)] + 1}. \tag{17}$$

4. SIMULATION RESULTS

In this section, results comparing Monte Carlo (MC) simulations and the proposed model are presented for the two-interferer case. In the examples presented, the covariance matrices are determined as in [1], considering twelve independent fading paths ($I = 12$). For the uplink signal, the in-cell signal angle-of-arrival $\theta_{\mathrm{s}} = 60^\circ$ and the interference angles-of-arrival $\theta_{\mathrm{u}_1} = -16^\circ$ and $\theta_{\mathrm{u}_2} = 36^\circ$ are considered. For the downlink signals, the normalizing power factors $\rho_{\mathrm{IC}} = 36$ dB and $\rho_{\mathrm{CC}} = 15$ dB are used. For the MC simulations, 100 independent realizations are used.

4.1. Behavior of SINR

To assess the proposed model with respect to the behavior of the SINR $\gamma(n)$, expression (15) is used. Matrix $\mathbf{K}(n)$ is determined from both MC simulations and the proposed analytical model. Four situations are considered (see Table 1).

Table 1
Parameter values for the numerical simulations

Case	AoA spread	M	$\mu_{\mathrm{s}} = \mu_{\mathrm{u}}$
1	30°	4	0.01
2	30°	8	0.007
3	3°	4	0.05
4	3°	8	0.01

Figure 1 shows the resulting curves for each case, exhibiting very good agreement between the simulation results and the proposed model.

4.2. Mean Weight Behavior

To evaluate the mean weight behavior of the adaptive weight vector, Cases 3 and 4 are considered. In Figure 2 both cases are shown, presenting the results obtained from the MC simulations and the proposed model. Again, very good model accuracy is observed.

5. CONCLUDING REMARKS

In this work, analytic models for the first moment of the weight vector and the SINR characteristics were presented. The models are obtained assuming a slow adaptation condition. A very good match between MC simulations and the prediction model is obtained for both transient and steady-state behavior. In addition, by using the proposed model, the behavior of the adaptive array can be easily assessed for a wide range of working conditions.

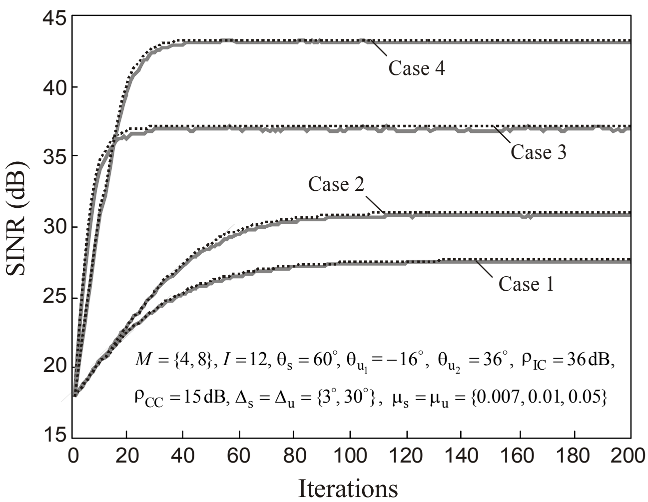
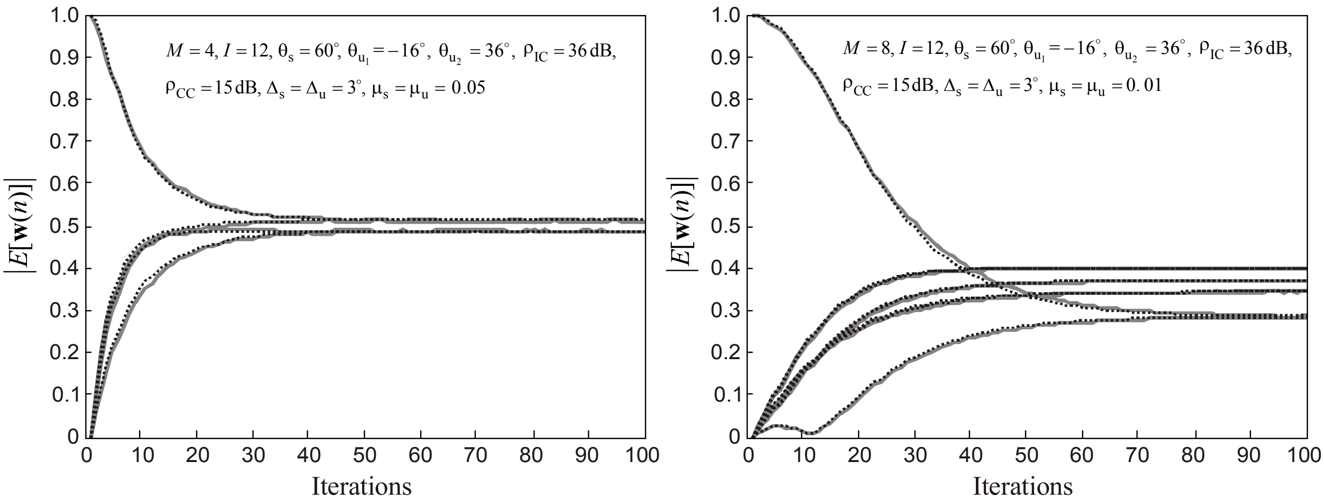


Figure 1 - Behavior of SINR $\gamma(n)$, showing MC simulations (gray lines) and proposed model (dotted black lines).



(a) (b)

Figure 2 - Mean weight behavior, showing MC simulations (gray lines) and proposed model (dotted black lines). (a) Case 3. (b) Case 4.

6. APPENDIX

DETERMINATION OF THE NORMALIZED SAMPLE COVARIANCE MATRIX

In this section, an approach for determining the normalized sample covariance matrix $\mathbf{R}_{\mathbf{x}}^{\mathrm{N}}$ is derived. A complex Gaussian random vector $\mathbf{x}(n)$, having a covariance matrix $\mathbf{R}$, is considered. Thus,

$$\mathbf{R}_{\mathbf{x}}^{\mathrm{N}} = E\left[\frac{\mathbf{x}(n)\mathbf{x}^{\mathrm{H}}(n)}{\mathbf{x}^{\mathrm{H}}(n)\mathbf{x}(n)}\right]. \tag{18}$$

To determine the expectation (18), an auxiliary matrix function $\mathbf{F}(\omega)$ is defined with elements given by [10]

$$f_{i,j}(\omega) = \frac{1}{\pi^M \det(\mathbf{R})} \underbrace{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty}}_{M \text{ fold}} \frac{x_i x_j^*}{\mathbf{x}^{\mathrm{H}}\mathbf{x}} \mathrm{e}^{-\mathbf{x}^{\mathrm{H}}\mathbf{L}^{-1}(\omega)\mathbf{x}} d\mathbf{x} \tag{19}$$

where $\mathbf{L}(\omega) = (\mathbf{R}^{-1} + \omega\mathbf{I})^{-1}$. Note that for $\omega = 0$, (19) is by definition the desired expectation, i.e.,

$$E\left[\frac{\mathbf{x}(n)\mathbf{x}^{\mathrm{H}}(n)}{\mathbf{x}^{\mathrm{H}}(n)\mathbf{x}(n)}\right] = \mathbf{F}(0). \tag{20}$$

Applying partial differentiation in (19) with respect to ω, the term $\mathbf{x}^{\mathrm{H}}\mathbf{x}$ in the denominator of its integrand is eliminated, resulting in the expression

$$\frac{\partial f_{i,j}(\omega)}{\partial \omega} = \frac{-\det[\mathbf{L}(\omega)]}{\det(\mathbf{R})} A(\omega) \tag{21}$$

where

$$A(\omega) = \frac{1}{\pi^M \det[\mathbf{L}(\omega)]} \underbrace{\int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty}}_{M \text{ fold}} x_i x_j^* \, \mathrm{e}^{-\mathbf{x}^{\mathrm{H}}\mathbf{L}^{-1}(\omega)\mathbf{x}} d\mathbf{x}$$

corresponds to the cross-correlation between x_i and x_j when x_i and x_j are jointly Gaussian complex random variables with covariance matrix $\mathbf{L}(\omega)$. Therefore, the factor $A(\omega) = [\mathbf{L}(\omega)]_{i,j}$, and integrating (21) with respect to ω, considering that $\lim_{\omega\to\infty} f_{i,j}(\omega) = 0$, obtains a simpler expression for the elements of (20), given by

$$\left\{E\left[\frac{\mathbf{x}(n)\mathbf{x}^{\mathrm{H}}(n)}{\mathbf{x}^{\mathrm{H}}(n)\mathbf{x}(n)}\right]\right\}_{i,j} = \int_0^{\infty} \frac{\{\mathbf{R}[\mathbf{I} + \omega\mathbf{R}]^{-1}\}_{i,j}}{\det(\mathbf{I} + \omega\mathbf{R})} d\omega. \tag{22}$$

By expressing $\mathbf{R} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^{\mathrm{H}}$ where $\mathbf{Q}$ is the eigenvector matrix of $\mathbf{R}$ and $\mathbf{\Lambda}$ is a diagonal matrix containing the eigenvalues λ_i of $\mathbf{R}$ for $i = 1, 2, \ldots, M$, we can write

$$E\left[\frac{\mathbf{x}(n)\mathbf{x}^{\mathrm{H}}(n)}{\mathbf{x}^{\mathrm{H}}(n)\mathbf{x}(n)}\right] = \mathbf{Q}\mathbf{H}\mathbf{Q}^{\mathrm{H}} \tag{23}$$

where $\mathbf{H}$ is a diagonal matrix with elements

$$h_k = \int_0^{\infty} \frac{\lambda_k}{(1 + \omega\lambda_k)\det(\mathbf{I} + \omega\Lambda)} d\omega. \tag{24}$$

Now, (24) can be calculated by using the partial fraction expansion technique. For instance, by considering the eigenvalues distinct, the elements of matrix $\mathbf{H}$ are given by

$$h_k = A_{1,k}\lambda_k + A_{2,k}\ln(\lambda_k) + \sum_{\substack{i=1 \\ i\neq k}}^{M} B_{i,k}\ln(\lambda_i) \tag{25}$$

where

$$A_{1,k} = \frac{\lambda_k^{M-2}}{\prod_{\substack{i=1 \\ i\neq k}}^{M} (\lambda_k - \lambda_i)} \tag{26}$$

and

$$B_{i,k} = \frac{\lambda_k \lambda_i^{M-1}}{(\lambda_i - \lambda_k)\prod_{\substack{j=1 \\ j\neq i}}^{M} (\lambda_i - \lambda_j)} \tag{27}$$

with

$$A_{2,k} = -A_{1,k}\lambda_k \sum_{\substack{i=1 \\ i\neq k}}^{M} \frac{\lambda_i}{\lambda_k - \lambda_i}. \tag{28}$$

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