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STOCHASTIC MODEL FOR THE MODIFIED FILTERED-ERROR LMS ALGORITHM

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ABSTRACT

This paper proposes an improved stochastic model for the first and second moments of the modified filtered-error least-mean-square (MFELMS) algorithm. The proposed model is obtained not invoking the classic Independence Theory (IT), allowing for a slow adaptation assumption and Gaussian input signal. Numerical simulations corroborate the good agreement between the results obtained with the Monte Carlo (MC) method and through the proposed model for both white and colored inputs.

1. INTRODUCTION

LMS adaptive filters have been used successfully in a wide variety of applications, such as communications, radar, seismology, biomedical electronics, and active noise control and vibrations. In some of these applications, a direct implementation of the adaptive structure is not always possible. A common problem is the fact that, in some situations, the error signal used to update the coefficients of the adaptive filter is not readily available or is inaccessible. In these cases, it is only possible to obtain a filtered version of the error signal, as occurs in active noise/vibrations control applications, acoustic echo canceling, among others. In these applications, the family of filtered-error least-mean-square (FELMS) algorithms should then be used. However, such algorithms present a poor behavior regarding convergence speed and stability issues. To circumvent such problems, a modification in the adaptive algorithm has been proposed [1], leading the modified algorithm to have a behavior equivalent to the standard LMS one. That is, the characteristics of convergence and stability of the modified algorithm become the same as the standard LMS one. Such an algorithm is termed modified FELMS (MFELMS) algorithm [1], [2].

In the open literature, there are three types of adaptive algorithms belonging to the family of filtered-error algorithms, namely the filtered-x LMS (FxLMS) [3], [4]; modified FxLMS (MFxLMS) [2]; and FELMS algorithms [5], [6]. The former is widely used in active noise control applications. The second one is a modified version of the standard FxLMS algorithm in order to increase the convergence rate, also used in active noise control. The latter is an alternative form of implementing the FxLMS algorithm (aiming to improve the convergence properties) used in echo canceling, adaptive equalization [5], [6], among other applications.

Several statistical analyses of the different versions of the LMS algorithm have been accomplished under the light of the Independence Theory (IT) [7], [8]. However, there are certain situations in which such an analysis assumption can no longer be applied. One of them is in the modeling of the FxLMS algorithm. Stochastic models for the first and second moments of the FxLMS algorithm without invoking the IT are derived in [9] and [10], respectively.

To the best of our knowledge, in the open literature there is no available analytical model for the modified filtered-error LMS (MFELMS) algorithm. Such a model is the main contribution of this research work. Since the use of the independence assumption has proved to be no longer adequate for this algorithm class (as shown in [9] and [10]), such an assumption is not invoked here. In this way, the proposed models for the mean weight behavior and learning curve are more accurate than those using IT. Through numerical simulations, very good agreement between the results obtained with the Monte Carlo (MC) method and the proposed model is verified for both white and colored input signals.

2. STANDARD FELMS ALGORITHM

2.1. Algorithm Description

Figure 1 shows the block diagram of the FELMS algorithm in which the following notation is used: $\mathbf{w}_\mathrm{o} = [w_{\mathrm{o},0}\ w_{\mathrm{o},1} \cdots w_{\mathrm{o},N-1}]^\mathrm{T}$ represents the plant impulse response, $\mathbf{w}(n) = [w_0(n)\ w_1(n) \cdots w_{N-1}(n)]^\mathrm{T}$ denotes the adaptive weight vector, $\mathbf{s} = [s_0\ s_1 \cdots s_{M-1}]^\mathrm{T}$ and $\hat{\mathbf{s}} = [\hat{s}_0\ \hat{s}_1 \cdots \hat{s}_{\hat{M}-1}]^\mathrm{T}$ are, respectively, the error filter impulse response and its estimate, and $d(n)$ and $z(n)$ denote the desired signal and the process measurement noise, respectively. The latter is

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i.i.d, zero-mean and uncorrelated with any other signal in the system. In this analysis, the input vector is $\mathbf{x}(n) = [x(n)\ x(n-1) \cdots x(n-N+1)]^{\mathrm{T}}$, with $\{x(n)\}$ being a zero-mean Gaussian process with variance σ_x^2. The filtered input vector is given by $\mathbf{x}_{\mathrm{f}}(n) = [x_{\mathrm{f}}(n)\ x_{\mathrm{f}}(n-1) \cdots x_{\mathrm{f}}(n-N+1)]^{\mathrm{T}}$, where

$$\mathbf{x}_{\mathrm{f}}(n) = \sum_{i=0}^{\hat{M}-1} \hat{s}_i \mathbf{x}(n-i). \tag{1}$$

In this work, we consider that vectors $\mathbf{w}_{\mathrm{o}}$ and $\mathbf{w}(n)$ have the same dimension, while the dimensions of vectors $\mathbf{s}$ and $\hat{\mathbf{s}}$ may be different; usually, in practical applications one has $M > \hat{M}$.

From Figure 1, the error signal is given by

$$e(n) = d(n) - y(n) + z(n) \tag{2}$$

where $d(n)$ and $y(n)$ are obtained as follows:

$$d(n) = \mathbf{w}_{\mathrm{o}}^{\mathrm{T}} \mathbf{x}(n) = \mathbf{x}^{\mathrm{T}}(n) \mathbf{w}_{\mathrm{o}} \tag{3}$$

and

$$y(n) = \mathbf{w}^{\mathrm{T}}(n) \mathbf{x}(n) = \mathbf{x}^{\mathrm{T}}(n) \mathbf{w}(n). \tag{4}$$

Then, substituting (4) into (2), we get

$$e(n) = d(n) - \mathbf{w}^{\mathrm{T}}(n) \mathbf{x}(n) + z(n). \tag{5}$$

The filtered error signal, used in the weight-updating rule of the adaptive algorithm, is given by

$$e_{\mathrm{f}}(n) = \sum_{i=0}^{M-1} s_i e(n-i). \tag{6}$$

Now, substituting (5) into (6), we obtain

$$e_{\mathrm{f}}(n) = \sum_{i=0}^{M-1} s_i d(n-i) - \sum_{i=0}^{M-1} s_i \mathbf{w}^{\mathrm{T}}(n-i) \mathbf{x}(n-i) + \sum_{i=0}^{M-1} s_i z(n-i). \tag{7}$$

Thus, weight update equation of the FELMS algorithm is finally given by [11]

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e_{\mathrm{f}}(n) \mathbf{x}_{\mathrm{f}}(n). \tag{8}$$

2.2. Modified FELMS Algorithm

The MFELMS algorithm is obtained by compensating the filtering operation that takes place at the error path, represented by (6). Such compensation is achieved by including the term $-\Lambda_{\mathrm{f}}(n)$ (named compensation term) into (7) [1]. In this way, the weight update and compensated error $\hat{e}(n)$ expressions are given by

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu \hat{e}_{\mathrm{f}}(n) \mathbf{x}_{\mathrm{f}}(n) \tag{9}$$

and

$$\begin{aligned} \hat{e}_{\mathrm{f}}(n) = {} & \sum_{i=0}^{M-1} s_i d(n-i) - \sum_{i=0}^{M-1} s_i \mathbf{w}^{\mathrm{T}}(n-i) \mathbf{x}(n-i) \\ & + \sum_{i=0}^{M-1} s_i z(n-i) - \Lambda_{\mathrm{f}}(n). \end{aligned} \tag{10}$$

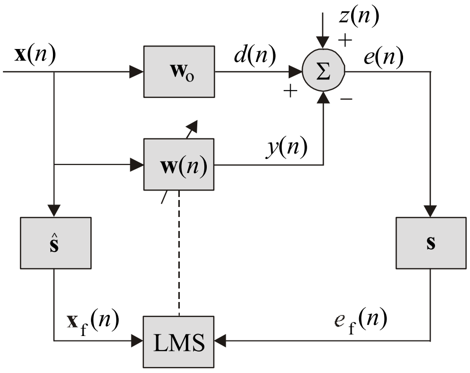

Figure 1 - Block diagram of the FELMS algorithm.

The compensation term $\Lambda_{\mathrm{f}}(n)$ is then obtained by enforcing (10) to be equal to the error signal of the standard LMS algorithm [1]. Thus,

$$\begin{aligned} e(n)\big|_{\mathrm{LMS}} &= d_{\mathrm{f}}(n) - \mathbf{w}^{\mathrm{T}}(n) \mathbf{x}_{\mathrm{f}}(n) + z_{\mathrm{f}}(n) \\ &= \sum_{i=0}^{M-1} s_i d(n-i) - \sum_{i=0}^{M-1} s_i \mathbf{w}^{\mathrm{T}}(n) \mathbf{x}(n-i) \\ &\quad + \sum_{i=0}^{M-1} s_i z(n-i). \end{aligned} \tag{11}$$

Note that in (10) the error signal now depends on the current value of the adaptive weight vector. From (9) and (10) the required compensation term is

$$\Lambda_{\mathrm{f}}(n) = \sum_{i=0}^{M-1} s_i [\mathbf{w}^{\mathrm{T}}(n) - \mathbf{w}^{\mathrm{T}}(n-i)] \mathbf{x}(n-i). \tag{12}$$

In (12), the difference $[\mathbf{w}^{\mathrm{T}}(n) - \mathbf{w}^{\mathrm{T}}(n-i)] = 0$ for $i = 0$; otherwise, $[\mathbf{w}^{\mathrm{T}}(n) - \mathbf{w}^{\mathrm{T}}(n-i)]$ is obtained from (9). Thus,

$$\Lambda_{\mathrm{f}}(n) = \mu \sum_{i=1}^{M-1} \sum_{j=1}^{i} s_i \hat{e}_{\mathrm{f}}(n-j) \mathbf{x}_{\mathrm{f}}^{\mathrm{T}}(n-j) \mathbf{x}(n-i). \tag{13}$$

Now, by substituting (13) into (10), we get

$$\begin{aligned} \hat{e}_{\mathrm{f}}(n) = {} & \sum_{i=0}^{M-1} s_i d(n-i) - \sum_{i=0}^{M-1} s_i \mathbf{w}^{\mathrm{T}}(n-i) \mathbf{x}(n-i) \\ & - \mu \sum_{i=1}^{M-1} \sum_{j=1}^{i} s_i \hat{e}_{\mathrm{f}}(n-j) \mathbf{x}_{\mathrm{f}}^{\mathrm{T}}(n-j) \mathbf{x}(n-i) + \sum_{i=0}^{M-1} s_i z(n-i). \end{aligned} \tag{14}$$

Note that in (14) $\hat{e}_{\mathrm{f}}(n)$ depends on both current and past error values; the latter is not found in the case of the standard FELMS algorithm. However, such a feature leads the modified algorithm to have a similar convergence behavior as the standard LMS one.

3. ANALYSIS

3.1. Analysis Assumptions

To determine the model expressions, we assume the following assumptions:

i) The correlations between input vectors at different lags are much more important than the correlations between the input and weight vectors.

ii) The proposed model is derived by considering slow adaptation; thus, the terms affected by μ^{β} for $\beta \geq 4$ can be disregarded.

iii) Since the input signal is Gaussian, the fourth and sixth order moments of the input signal are obtained by using the Moment Factoring Theorem [12].

3.2. Mean Weight Behavior

In this section, a model expression for the first moment of the adaptive weight vector is derived. By substituting (1) and (14) into (9) and taking the expected value of both sides of the resulting expression, we obtain

$$
\begin{aligned}
E[\mathbf{w}(n+1)] &= E[\mathbf{w}(n)] + \mu \sum_{i=0}^{M-1} \sum_{k=0}^{\hat{M}-1} s_i \hat{s}_k E[\mathbf{x}(n-k) d(n-i)] \\
&- \mu \sum_{i=0}^{M-1} \sum_{k=0}^{\hat{M}-1} s_i \hat{s}_k E[\mathbf{x}(n-k)\mathbf{w}^{\mathrm{T}}(n-i)\mathbf{x}(n-i)] \\
&- \mu^2 \sum_{i=1}^{M-1} \sum_{j=1}^{i} \sum_{k=0}^{\hat{M}-1} s_i \hat{s}_k E[\mathbf{x}(n-k)\hat{e}_{\mathrm{f}}(n-j)\mathbf{x}_{\mathrm{f}}^{\mathrm{T}}(n-j)\mathbf{x}(n-i)] \\
&+ \mu \sum_{i=0}^{M-1} \sum_{k=0}^{\hat{M}-1} s_i \hat{s}_k E[\mathbf{x}(n-k) z(n-i)].
\end{aligned} \tag{15}
$$

By determining $d(n-i)$, $\mathbf{x}_{\mathrm{f}}^{\mathrm{T}}(n-j)$, and $\hat{e}_{\mathrm{f}}(n-j)$ from (3), (1), and (14), respectively, and substituting these results into (15), by using (i)-(iii) and after some algebra, we get

$$
\begin{aligned}
E[\mathbf{w}(n+1)] &= E[\mathbf{w}(n)] + \mu \sum_{i=0}^{M-1} \sum_{k=0}^{\hat{M}-1} s_i \hat{s}_k \mathbf{R}_{i-k} \{\mathbf{w}_{\mathrm{o}} - E[\mathbf{w}(n-i)]\} \\
&- \mu^2 \sum_{i=1}^{M-1} \sum_{j=1}^{i} \sum_{k=0}^{\hat{M}-1} \sum_{l=0}^{M-1} \sum_{p=0}^{\hat{M}-1} s_i \hat{s}_k s_l \hat{s}_p \big\{\mathbf{R}_{j+p-k}\mathbf{R}_{j+l-i} + \mathbf{R}_{i-k} \\
&\times \mathbf{R}_{l-p} + \mathbf{R}_{j+l-k}\,\mathrm{tr}[\mathbf{R}_{j+p-i}^{\mathrm{T}}]\big\}\{\mathbf{w}_{\mathrm{o}} - E[\mathbf{w}(n-j-l)]\} \\
&+ \mu^3 \sum_{i=1}^{M-1} \sum_{j=1}^{i} \sum_{k=0}^{\hat{M}-1} \sum_{l=1}^{M-1} \sum_{m=1}^{l} \sum_{p=0}^{\hat{M}-1} \sum_{q=0}^{M-1} \sum_{t=0}^{\hat{M}-1} s_i \hat{s}_k s_l \hat{s}_p s_q \hat{s}_t \\
&\times \{\mathbf{R}_{j+m+t-k}\mathbf{R}_{p-l}\mathbf{R}_{j+m+q-i} + \mathbf{R}_{j+m+t-k}\mathbf{R}_{i-j-l}\mathbf{R}_{m+q-p} \\
&+ \mathbf{R}_{j+m+t-k}\mathbf{R}_{m+q-l}\mathrm{tr}[\mathbf{R}_{i-j-p}] + \mathbf{R}_{j+l-k}\mathbf{R}_{p-m-t}\mathbf{R}_{j+m+q-i} \\
&+ \mathbf{R}_{j+l-k}\mathbf{R}_{i-j-m-t}\mathbf{R}_{m+q-p} + \mathbf{R}_{j+l-k}\mathbf{R}_{q-t}\mathrm{tr}[\mathbf{R}_{i-j-p}] \\
&+ \mathbf{R}_{j+p-k}\mathrm{tr}[\mathbf{R}_{l-m-t}]\mathbf{R}_{j+m+q-i} + \mathbf{R}_{j+p-k}\mathbf{R}_{i-j-m-t}^{\mathrm{T}}\mathbf{R}_{m+q-l} \\
&+ \mathbf{R}_{j+p-k}\mathbf{R}_{i-j-l}^{\mathrm{T}}\mathbf{R}_{q-t} + \mathbf{R}_{i-k}\mathbf{R}_{p-l}^{\mathrm{T}}\mathbf{R}_{q-t} \\
&+ \mathbf{R}_{i-k}\mathrm{tr}[\mathbf{R}_{m+t-l}]\mathbf{R}_{m+q-p} + \mathbf{R}_{i-k}\mathbf{R}_{m+t-p}\mathbf{R}_{m+q-l} \\
&+ \mathbf{R}_{j+m+q-k}\mathrm{tr}[\mathbf{R}_{p-l}\mathbf{R}_{j+m+t-i}] + \mathbf{R}_{j+m+q-k}\mathrm{tr}[\mathbf{R}_{i-j-l}\mathbf{R}^{\mathrm{T}}{}_{m+t-p}] \\
&+ \mathbf{R}_{j+m+q-k}\mathrm{tr}[\mathbf{R}_{m+t-l}]\mathrm{tr}[\mathbf{R}_{i-j-p}^{\mathrm{T}}]\}\{\mathbf{w}_o - E[\mathbf{w}(n-j-m-q)]\}.
\end{aligned} \tag{16}
$$

The autocorrelation matrices in (16) are obtained according to the general form $\mathbf{R}_{\beta-\alpha} = E[\mathbf{x}(n-\alpha)\mathbf{x}^{\mathrm{T}}(n-\beta)]$.

3.3. Steady-State Value of $\mathbf{w}(n)$

By assuming algorithm convergence, the steady-state value of the weight vector is obtained from the following condition:

$$
\begin{aligned}
\lim_{n\to\infty} E[\mathbf{w}(n+1)] &= \lim_{n\to\infty} E[\mathbf{w}(n-i)] = \lim_{n\to\infty} E[\mathbf{w}(n-j-l)] \\
&= \lim_{n\to\infty} E[\mathbf{w}(n-j-m-q)] = \lim_{n\to\infty} E[\mathbf{w}(n)] = \mathbf{w}_{\infty}.
\end{aligned} \tag{17}
$$

By using (17) in (16), and considering a system identification problem as illustrated in Figure 1, it follows that:

$$\mathbf{w}_{\infty} = \mathbf{w}_{\mathrm{o}}. \tag{18}$$

From (18), we notice that the MFELMS algorithm has the same steady-state value than the FELMS algorithm. It is also important to point out that the compensation term only has effect during the transient phase of the algorithm. Such a fact is observed in (12), where at convergence the difference $[\mathbf{w}^{\mathrm{T}}(n) - \mathbf{w}^{\mathrm{T}}(n-i)]$ is equal to zero, canceling thus the effect of the compensating term $\Lambda_{\mathrm{f}}(n)$.

3.4. Learning Curve

To determine the model expression for the learning curve of the MFELMS algorithm, we use (12) in (10), and after a simple mathematical manipulation one has

$$\hat{e}_{\mathrm{f}}(n) = \sum_{i=0}^{M-1} s_i d(n-i) - \sum_{i=0}^{M-1} s_i \mathbf{w}^{\mathrm{T}}(n)\mathbf{x}(n-i) + \sum_{i=0}^{M-1} s_i z(n-i). \tag{19}$$

By defining the weight-error vector as $\mathbf{v}(n) = \mathbf{w}(n) - \mathbf{w}_{\infty}$, where $\mathbf{w}_{\infty}$ is the steady-state value given by (18), expressing (19) as a function of $\mathbf{v}(n)$, squaring and taking the expected value of both sides of the resulting expression, and according to the analysis assumptions, we obtain

$$
\begin{aligned}
E[\hat{e}_f^2(n)] &= E[e_{\infty}^2(n)] - 2\sum_{i=0}^{M-1} s_i E[\hat{e}_{\infty}(n)\mathbf{v}^{\mathrm{T}}(n)\mathbf{x}(n-i)] \\
&+ \sum_{i=0}^{M-1}\sum_{j=0}^{M-1} s_i s_j E\{[\mathbf{v}^{\mathrm{T}}(n)\mathbf{x}(n-i)]^2\}
\end{aligned} \tag{20}
$$

where the error expression

$$\hat{e}_{\infty}(n) = \sum_{i=0}^{M-1} s_i d(n-i) - \sum_{i=0}^{M-1} s_i \mathbf{w}_{\infty}^{\mathrm{T}}\mathbf{x}(n-i) + \sum_{i=0}^{M-1} s_i z(n-i) \tag{21}$$

represents the steady-state error. After taking its expected value, by considering $E[\hat{e}_{\infty}(n)\mathbf{x}^{\mathrm{T}}(n-i)] = 0$ (Orthogonality Principle [7]), and defining $\xi_{\mathrm{f}}(n) = E[\hat{e}_{\mathrm{f}}^2(n)]$ and $\xi_{\min} = E[\hat{e}_{\infty}^2(n)]$, (20) can then be rewritten as follows:

$$\xi_{\mathrm{f}}(n) = \xi_{\min} + \sum_{i=0}^{M-1}\sum_{j=0}^{M-1} s_i s_j \mathrm{tr}\{E[\mathbf{v}(n)\mathbf{v}^{\mathrm{T}}(n)]\mathbf{R}_{j-i}\}. \tag{22}$$

To complete the derivation of (22), we should obtain the second moment of $\mathbf{v}(n)$. For such, allowing for the slow adaptation condition, the following approximation can be considered $E[\mathbf{v}(n)\mathbf{v}^{\mathrm{T}}(n)] \cong E[\mathbf{v}(n)]E[\mathbf{v}^{\mathrm{T}}(n)]$ [13].

3.5. Model Using IT

The stochastic model for the mean weight behavior under IT is obtained from (16) by using the condition $\mathbf{R}_{\beta-\alpha} = E[\mathbf{x}(n-\alpha)\mathbf{x}^{\mathrm{T}}(n-\beta)] = 0$ if $\alpha \neq \beta$. Regarding the leaning curve, the corresponding IT-based model is given by

$$\xi_{\mathrm{f}}(n) = \xi_{\min} + \sum_{i=0}^{M-1} s_i^2 \mathrm{tr}\{E[\mathbf{v}(n)\mathbf{v}^{\mathrm{T}}(n)]\mathbf{R}\} \qquad (23)$$

Since matrix $\mathbf{R}$ is determined as $\mathbf{R} = E[\mathbf{x}(n)\mathbf{x}^{\mathrm{T}}(n)]$, under IT, some terms in (22) are disregarded, making the model inaccurate.

4. SIMULATION RESULTS

In this section, two examples are presented in order to verify the accuracy of the proposed analytical model for both white and colored Gaussian inputs, considering a system identification problem. For comparison purposes the model results obtained by using the classical independence assumption are also shown.

4.1. Example 1

For this example, the plant is the length-17 vector $\mathbf{w}_{\mathrm{o}} = [-0.047\ -0.08\ -0.10\ -0.07\ 0.10\ 0.23\ 0.34\ 0.42\ 0.45\ 0.42\ 0.34\ 0.23\ 0.10\ -0.07\ -0.09\ -0.08\ -0.04]^{\mathrm{T}}$, the error-path filter is modeled by an IIR filter given by

$$s(z) = \frac{0.0675 + 0.1349z^{-1} + 0.0675z^{-2}}{1 - 1.1430z^{-1} + 0.4128z^{-2}} \qquad (24)$$

and its estimate is obtained by a FIR filter $\hat{\mathbf{s}} = [0.131\ 0.503\ 0.670\ 0.557\ 0.295\ 0.182\ 0.059\ -0.007]^{\mathrm{T}}$. The input signal is white with variance $\sigma_x^2 = 1$. The maximum step-size value (experimentally determined) for which the algorithm converges is $\mu_{\max} = 0.04$. The presented results are obtained by using $\mu = 0.2\mu_{\max}$. The Monte Carlo (MC) simulations are obtained from averaging 100 independent runs. Figures 2 and 3 show, respectively, the mean weight behavior and learning curve obtained from MC simulations, the proposed model [(16) and (22)] as well as the model derived under IT, used for comparison purposes. In this case, we observe that the proposed model improves the prediction accuracy for both the mean weight behavior and learning curve of the MFELMS algorithm; in contrast, the IT-based one does not model well the transient behavior of the algorithm.

4.2. Example 2

In this case, a correlated input signal is used, which is obtained from an AR(2) process given by

$$x(n) = a_1 x(n-1) + a_2 x(n-2) + u(n) \qquad (25)$$

where $u(n)$ is white noise with unit variance. The coefficients of the AR process are $a_1 = 0.5833$ and $a_2 = -0.75$ with an eigenvalue spread of the input autocorrelation matrix equal to 63.1. The plant vector and the error-path filter are the same as in the previous example. The maximum step size for this case is $\mu_{\max} = 0.08$ (experimentally determined). For this example, $\mu = 0.1\mu_{\max}$ is used. Figures 4 and 5 show the mean weight behavior and learning curves obtained from MC simulations and theoretical models (using IT and the proposed model), respectively. From these figures, a good agreement of the proposed model is verified. Figures 6 and 7 depict the results for $\mu = 0.5\mu_{\max}$, pointing out that the acurracy of the proposed model is maintained. Again, the model derived under IT (not shown here) does not match the MC simulation.

5. CONCLUSIONS

In this paper, an improved model for the modified FELMS (MFELMS) algorithm has been derived considering a slow adaptation condition and without invoking IT. Comparisons with standard modeling assumptions have shown the validity of the proposed modeling approach. From numerical simulations, the accuracy of the proposed model for both white and colored signals is confirmed.

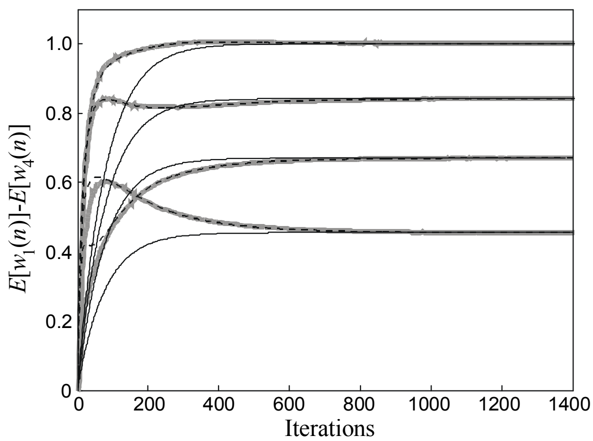

Figure 2 – Example 1. Mean weight behavior curves $E[w_1(n)]$ to $E[w_4(n)]$ using $\mu = 0.2\mu_{\max}$ for white input signal: (gray lines) MC simulations, (solid-black lines) model considering IT, and (dashed-black lines) proposed model (16).

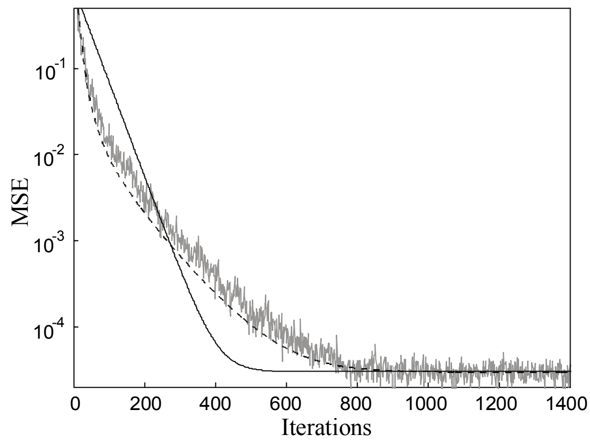

Figure 3 – Example 1. MSE curves using $\mu = 0.2\mu_{\max}$ for white input signal: (ragged-gray line) MC simulations, (solid-black line) model using IT (23), and proposed model (22).

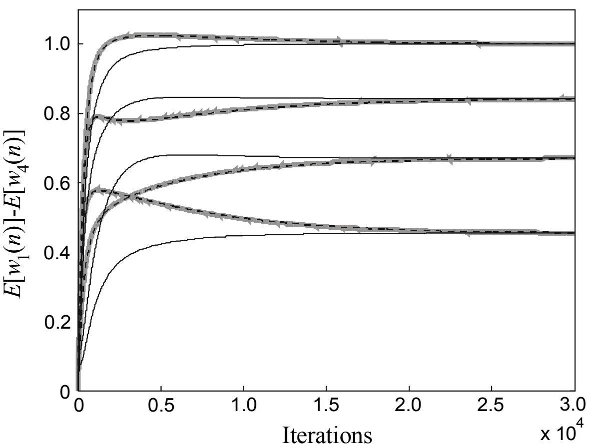

Figure 4 – Example 2. Mean weight behavior curves $E[w_1(n)]$ to $E[w_4(n)]$ using $\mu = 0.1\mu_{\max}$ for correlated input signal: (gray lines) MC simulations, (solid-black lines) model using IT, and (dashed-black lines) proposed model (16).

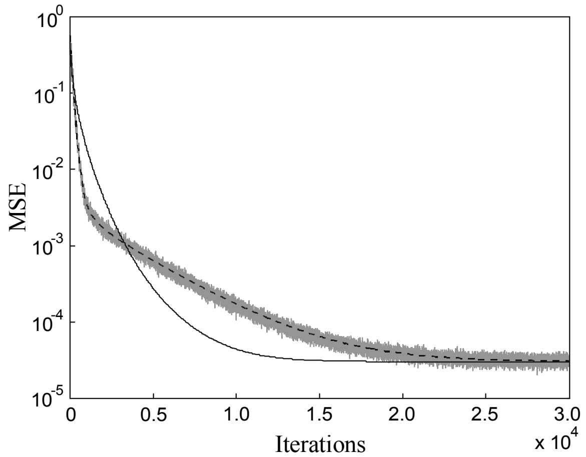

Figure 5 - Example 2. MSE curves using $\mu = 0.1\mu_{\max}$ for correlated input signal: (ragged-gray line) MC simulations, (solid-black line) model considering IT (23), and (dashed-black line) proposed model (22).

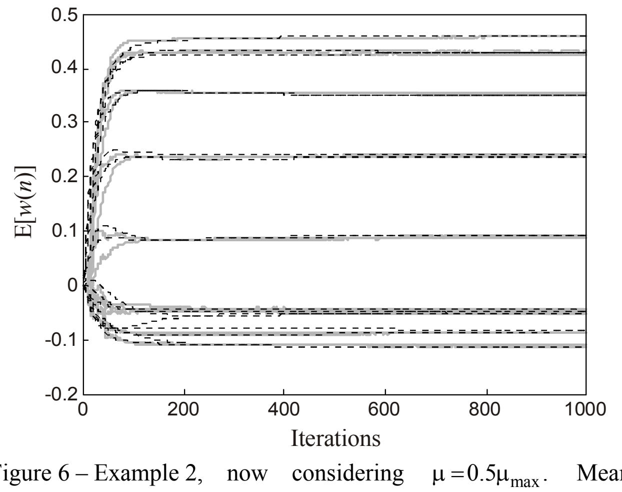

Figure 6 – Example 2, now considering $\mu = 0.5\mu_{\max}$. Mean weight behavior curves $E[w(n)]$ for correlated input signal: (gray lines) MC simulations and (dashed-black lines) proposed model (15).

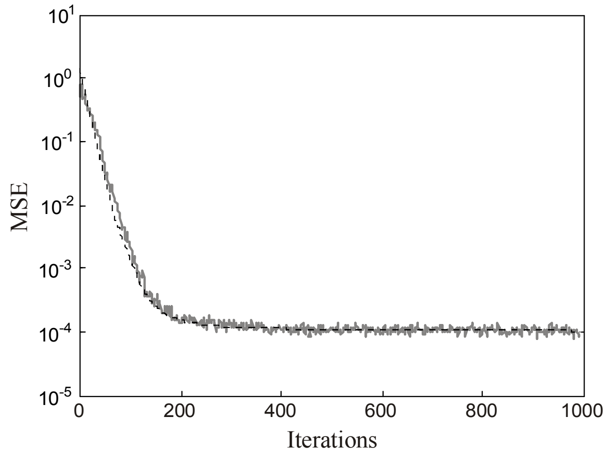

Figure 7 – Example 2, now considering $\mu = 0.5\mu_{\max}$. MSE curves for correlated input signal: (ragged-gray line) MC simulations and (dashed-black line) proposed model (22).

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