OPTIMAL ZERO-FORCING TRANSCEIVER DESIGN FOR MAXIMIZING BIT RATE SUBJECT TO A TOTAL TRANSMIT POWER CONSTRAINT

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ABSTRACT

In this paper, we design the optimal zero-forcing transceiver that maximizes the transmission bit rate for multiple-input multiple-output (MIMO) channels. The transmission bit rate is maximized subject to a total power constraint for a given error rate. Instead of using the same input constellation size for all subchannels as in earlier designs, the bit allocation is also taken into consideration. The bit allocation and the zero-forcing transceiver are jointly designed for bit rate maximization. The optimal transceiver is obtained in a closed form. The bits are allocated according to the subchannel signal to noise ratios. The larger the signal to noise ratio is, the more the number of bits are allocated. In the simulation, we have demonstrated that a higher bit rate can be achieved compared to previously reported methods.

1. INTRODUCTION

Multiple-input multiple-output (MIMO) channels arise in applications such as wireless communication systems that use multiple antennas and also telephone cables that consist of many twisted wire pairs. Optimal transceivers of different design criteria for MIMO channels have been considered in the literature earlier, e.g., [1]-[8]. Optimal transceivers for two design criteria: maximum signal to noise ratio under zero-forcing constraint and minimum mean-square error (MMSE), are developed in [1]. For the same input constellation size, the two designs are optimized under a transmit power constraint. The solutions are given in closed forms. The optimal zero-forcing transceiver that minimizes the bit error rate (BER) is derived in [2]. Assuming all input symbols carry the same number of bits, the system is optimized for a given transmit power. It provides a simple analytic form of the minimum BER transceiver [2]. Minimum BER design with channel independent transmitter was considered in [3]. Zero-forcing solutions with the aim of minimizing the total transmit power for a given BER are developed in [4]. The transceivers with two design criteria: minimum mean-squared error and minimum error rate for a given power constraint, are proposed in [5]. The designs lead to simple closed-form solutions that convert an MIMO channel with memory into a set of parallel subchannels. To incorporate quality of service criterion in the design, a weighted minimum mean-squared error criterion subject to a transmit power constraint is proposed in [6]. In these works, the input symbols again are assumed to be of the same constellation size. A unified framework for designing MIMO systems with an MMSE receiver is proposed in [7]. A number of useful objective functions can be considered in this framework. For example, for a given bit allocation, the optimal MMSE system can be designed using this unified approach. As an extension of [7], the transmit power is minimized in [8] with different quality of service requirements such as mean-squared error, signal to interference ratio, and bit error rate.

In this paper, we design the zero-forcing transceiver for bit rate maximization over MIMO channels. We jointly optimize the transceiver and bit allocation for a given error rate. The solution is derived in two steps. First, for a given zero-forcing transceiver, we design the optimal bit and power allocation that maximizes the bit rate. Second, we design the zero-forcing transceiver that maximizes the bit rate based on the optimal bit and power allocation. Instead of using a given bit allocation as in earlier works, the transceiver and bit allocation are jointly designed. In the resulting optimal system, the subchannels with larger signal to noise ratios are allocated with more bits. As a result, a higher bit rate can be achieved. The rest of the paper is organized as follows. In section 2, we will give the system model of an MIMO zero-forcing transceiver and formulate the problem. In section 3, we will find the optimal power allocation for a given transceiver. In section 4, we will find the optimal zero-forcing transceiver. In section 5, we will evaluate the performance for the proposed optimal zero-forcing transceiver.

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2. SYSTEM MODEL

A generic MIMO communication system is shown in Fig. 1. The memoryless MIMO channel is modeled by a $P \times N$ matrix $\mathbf{H}$. The $P \times 1$ channel noise $\mathbf{q}$ is additive white gaussian with variance N_0. The transmitter matrix $\mathbf{F}$ is of size $N \times M$. The receiver matrix $\mathbf{G}$ is of size $M \times P$. The input of the transmitter is $\mathbf{s}$, an $M \times 1$ vector of modulation symbols. The autocorrelation matrix of the input symbols is given by

$$\mathbf{\Lambda}_s = \mathrm{E}[\mathbf{s}\mathbf{s}^\dagger], \tag{1}$$

where the notation $\mathbf{X}^\dagger$ denotes the transpose conjugate of $\mathbf{X}$. Assume that the input symbols are zero mean and uncorrelated; hence $\mathbf{\Lambda}_s$ is diagonal. The total transmit power is

$$\begin{aligned} \mathrm{E}\{\mathbf{x}^\dagger\mathbf{x}\} &= \mathrm{E}\{(\mathbf{F}\mathbf{s})^\dagger(\mathbf{F}\mathbf{s})\} \\ &= \mathrm{Tr}(\mathbf{F}\mathbf{\Lambda}_s\mathbf{F}^\dagger), \end{aligned} \tag{2}$$

where $\mathbf{x}$ is the transmitter output. The output of the receiver is

$$\hat{\mathbf{s}} = \mathbf{G}\mathbf{H}\mathbf{F}\mathbf{s} + \mathbf{e}, \tag{3}$$

where $\mathbf{e} = \mathbf{G}\mathbf{q}$. To satisfy the zero-forcing condition, the transceiver pair ($\mathbf{F}$,$\mathbf{G}$) needs to satisfy

$$\mathbf{G}\mathbf{H}\mathbf{F} = \mathbf{I}_M, \tag{4}$$

where $\mathbf{I}_M$ denotes the $M \times M$ identity matrix. In this case, the k-th receiver output $\hat{s}_k$ is

$$\hat{s}_k = s_k + e_k, \quad k = 0, 1, \cdots, M-1. \tag{5}$$

The output noise variance at k-th subchannel is given by

$$\begin{aligned} \sigma_{e_k}^2 &= [\mathrm{E}\{\mathbf{e}\mathbf{e}^\dagger\}]_{kk} \\ &= N_0[\mathbf{G}\mathbf{G}^\dagger]_{kk}, \end{aligned} \tag{6}$$

where the notation $[\mathbf{X}]_{kk}$ denotes the k-th diagonal element of $\mathbf{X}$. Assume the symbol error (SER) rates are the same for all the subchannels. Let b_k be the number of bits carried by the k-th symbol. Let $\sigma_{s_k}^2$ be the k-th diagonal element of $\mathbf{\Lambda}_s$. For QAM modulation,

$$b_k = \log_2\left(1 + \frac{\sigma_{s_k}^2}{\sigma_{e_k}^2\Gamma}\right), \tag{7}$$

where $\Gamma = \frac{1}{3}[Q^{-1}(SER/4)]^2$ is a parameter determined by the given symbol error rate [9]. The function $Q(x)$ is the area under a Gaussian tail, i.e., $Q(x) = \int_x^\infty e^{-u^2/2}du$. The derivation in this paper is given for the QAM case. The results for the PAM case can be obtained in a similar way. (For the PAM case, there is an additional scalar of $\frac{1}{2}$). When b_k is large enough so that $2^{b_k} \gg 1$, we have

$$b_k \approx \log_2\left(\frac{\sigma_{s_k}^2}{\sigma_{e_k}^2\Gamma}\right). \tag{8}$$

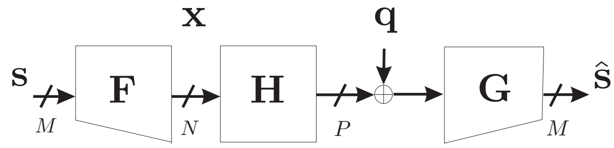

Figure 1: MIMO communication system.

The total number of bits that can be transmitted is $\sum_{k=0}^{M-1} b_k$. Therefore, the problem of maximizing bit rate subject to a total transmit power constraint P_0 can be formulated as

$$\begin{array}{ll} \mathrm{maximize} & b = \sum_{k=0}^{M-1} \log_2\left(\frac{\sigma_{s_k}^2}{\sigma_{e_k}^2\Gamma}\right) \\ \mathrm{subjectto} & \mathrm{Tr}(\mathbf{F}\mathbf{\Lambda}_s\mathbf{F}^\dagger) = P_0 \end{array}. \tag{9}$$

For a given transceiver, we will first find the optimal power allocation, i.e., optimal $\sigma_{s_k}^2$, under the transmit power constraint P_0 (section 3). Based on the optimal power allocation, we will continue to derive the optimal transceiver for maximizing the bit rate (section 4).

3. OPTIMAL POWER ALLOCATION

In this section, we will find the optimal power allocation that maximizes the bit rate under the constraint on total transmit power for a given zero-forcing transceiver pair. Unlike earlier works [1]-[8] that use the same constellation size for all subchannels, the bits are assigned according to subchannel signal to noise ratios in (7). To obtain the optimal power allocation $\sigma_{s_k}^2$, we can use the method of the Lagrange multiplier [10]. Let the Lagrangian function be

$$L = \sum_{k=0}^{M-1} \log_2\left(\frac{\sigma_{s_k}^2}{\sigma_{e_k}^2\Gamma}\right) + \alpha(\mathrm{Tr}(\mathbf{F}\mathbf{\Lambda}_s\mathbf{F}^\dagger) - P_0), \tag{10}$$

where α is the Lagrange multiplier. By solving $\frac{\partial L}{\partial \sigma_{s_k}^2} = 0$, we have

$$\sigma_{s_k}^2 = \frac{-1}{\alpha[\mathbf{F}^\dagger\mathbf{F}]_{kk}\log_e 2}. \tag{11}$$

Using the fact that $\mathbf{\Lambda}_s$ is diagonal, the total transmit power constraint becomes

$$\mathrm{Tr}(\mathbf{F}\mathbf{\Lambda}_s\mathbf{F}^\dagger) = \sum_{l=0}^{M-1} \sigma_{s_k}^2[\mathbf{F}^\dagger\mathbf{F}]_{kk} = P_0 \tag{12}$$

To find the Lagrange multiplier, we can substitute (11) into the total transmit power constraint in (12). Then the Lagrange multiplier is given by

$$\alpha = \frac{-M}{P_0\log_e 2}. \tag{13}$$

Therefore, the optimal power allocation is

$$\sigma_{s_k}^2 = \frac{P_0}{M[\mathbf{F}^\dagger \mathbf{F}]_{kk}}. \quad (14)$$

From (14), we can see that the optimal power allocation depends only on the transmitter for the given P_0 and M. Using the optimal power allocation obtained in (14), the bit rate b in (9) is given by

$$b = \sum_{k=0}^{M-1} \log_2 \left(\frac{P_0}{M\Gamma[\mathbf{F}^\dagger \mathbf{F}]_{kk}\sigma_{e_k}^2} \right). \quad (15)$$

In the next section, we will design the optimal zero-forcing transceiver that maximizes the bit rate in (15).

4. OPTIMAL ZERO-FORCING TRANSCEIVER

Suppose the $P \times N$ channel matrix $\mathbf{H}$ has rank K. Let the singular value decomposition of $\mathbf{H}$ be

$$\mathbf{H} = \mathbf{U} \begin{bmatrix} \mathbf{\Lambda} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \mathbf{V}^\dagger, \quad (16)$$

where the $K \times K$ diagonal matrix $\mathbf{\Lambda}$ contains the nonzero singular values of $\mathbf{H}$. The $P \times P$ matrix $\mathbf{U}$ and the $N \times N$ matrix $\mathbf{V}$ are unitary that correspond respectively to the eigenvectors of $\mathbf{H}\mathbf{H}^\dagger$ and $\mathbf{H}^\dagger\mathbf{H}$. We assume that the elements of $\mathbf{\Lambda}$ are in nonincreasing order and $K \geq M$, so that solutions of zero-forcing transceivers exist.

Lemma 1 *Without loss of generality, we can express* $\mathbf{F}$ *to be of the following form:*

$$\mathbf{F} = \mathbf{V} \begin{bmatrix} \mathbf{A} \\ \mathbf{0} \end{bmatrix}, \quad (17)$$

for appropriate $K \times M$ *matrix* $\mathbf{A}$ *of rank* M.

Proof: Suppose $(\mathbf{G}, \mathbf{F})$ is a transceiver pair that satisfies the zero-forcing condition. Because $\mathbf{V}$ is an $N \times N$ unitary matrix, the columns of $\mathbf{F}$ can be represented as linear combinations of the columns of $\mathbf{V}$. Then we have

$$\mathbf{F} = \mathbf{V} \begin{bmatrix} \mathbf{A} \\ \mathbf{A}_1 \end{bmatrix}, \quad (18)$$

where $\mathbf{A}$ is a $K \times M$ matrix and $\mathbf{A}_1$ is an $(N-K) \times M$ matrix. Define a new transceiver $\mathbf{F}'$ as

$$\mathbf{F}' = \mathbf{V} \begin{bmatrix} \mathbf{A} \\ \mathbf{0} \end{bmatrix}. \quad (19)$$

The transfer matrix when we use $\mathbf{F}'$ is given by

$$\mathbf{GHF}' = \mathbf{GHF} = \mathbf{I}_M. \quad (20)$$

Therefore, when we replace the transmitter by $\mathbf{F}'$, the new system still satisfies the zero-forcing condition $\mathbf{GHF} = \mathbf{I}_M$. Because the receiver is not changed, the new system has the same subchannel noise variances. The subchannel signal to noise ratios are the same when we use $\mathbf{F}'$ and hence the bit rate performance is the same.

Now, let's compare the transmit power of $\mathbf{F}$ and $\mathbf{F}'$ for the same input autocorrelation matrix $\mathbf{\Lambda}_s$. The transmit power when we use $\mathbf{F}$ is

$$\mathrm{Tr}(\mathbf{F}\mathbf{\Lambda}_s\mathbf{F}^\dagger) = \mathrm{Tr}(\mathbf{A}\mathbf{\Lambda}_s\mathbf{A}^\dagger) + \mathrm{Tr}(\mathbf{A}_1\mathbf{\Lambda}_s\mathbf{A}_1^\dagger). \quad (21)$$

The transmit power with $\mathbf{F}'$ is

$$\begin{aligned} \mathrm{Tr}(\mathbf{F}'\mathbf{\Lambda}_s\mathbf{F}'^\dagger) &= \mathrm{Tr}(\mathbf{A}\mathbf{\Lambda}_s\mathbf{A}^\dagger) \\ &\leq \mathrm{Tr}(\mathbf{F}\mathbf{\Lambda}_s\mathbf{F}^\dagger). \end{aligned} \quad (22)$$

As a result, we will have a smaller transmit power when we use $\mathbf{F}'$. For the same power constraint and error rate, we can transmit more bits when we use $\mathbf{F}'$. This means a transmitter of the form in (17) is no loss of generality.

△△△

Lemma 2 *It is no loss of generality to choose* $\mathbf{G}$ *as the pseudo inverse of* $\mathbf{HF}$. *That is,*

$$\mathbf{G} = (\mathbf{A}^\dagger\mathbf{\Lambda}^2\mathbf{A})^{-1}[\ \mathbf{A}^\dagger\mathbf{\Lambda} \quad \mathbf{0}\ ]\mathbf{U}^\dagger, \quad (23)$$

where $\mathbf{A}$ *is the matrix given in Lemma 1. In this case, the noise variance at the* k*-th subchannel is given by*

$$\sigma_{e_k}^2 = N_0[(\mathbf{A}^\dagger\mathbf{\Lambda}^2\mathbf{A})^{-1}]_{kk}. \quad (24)$$

Proof: Suppose $(\mathbf{G}, \mathbf{F})$ ia a transceiver pair that satisfies the zero forcing condition, and $\mathbf{F}$ is of the form in (17). Let $\mathbf{G}'$ be the pseudo inverse of $\mathbf{HF}$, i.e.,

$$\begin{aligned} \mathbf{G}' &= (\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{H}\mathbf{F})^{-1}\mathbf{F}^\dagger\mathbf{H}^\dagger \\ &= (\mathbf{A}^\dagger\mathbf{\Lambda}^2\mathbf{A})^{-1}[\ \mathbf{A}^\dagger\mathbf{\Lambda} \quad \mathbf{0}\ ]\mathbf{U}^\dagger \end{aligned}. \quad (25)$$

Note that the system still satisfies zero-forcing condition when we use $\mathbf{G}'$. Define $\mathbf{\Delta} = \mathbf{G} - \mathbf{G}'$. It follows that

$$\begin{aligned} \mathbf{\Delta}\mathbf{G}'^\dagger &= \mathbf{\Delta}\mathbf{HF}(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{HF})^{-1} \\ &= (\mathbf{GHF} - \mathbf{G}'\mathbf{HF})(\mathbf{F}^\dagger\mathbf{H}^\dagger\mathbf{HF})^{-1} \\ &= \mathbf{0}. \end{aligned} \quad (26)$$

When we use $\mathbf{G}$, the noise variance at the k-th subchannel is given by

$$\begin{aligned} \sigma_{e_k}^2 &= N_0[\mathbf{G}\mathbf{G}^\dagger]_{kk} \\ &= N_0[(\mathbf{G}' + \mathbf{\Delta})(\mathbf{G}' + \mathbf{\Delta})^\dagger]_{kk} \\ &= N_0[\mathbf{G}'\mathbf{G}'^\dagger + \mathbf{\Delta}\mathbf{\Delta}^\dagger]_{kk} \\ &\geq N_0[\mathbf{G}'\mathbf{G}'^\dagger]_{kk}, \end{aligned} \quad (27)$$

where we have used $\mathbf{\Delta}\mathbf{G}'^\dagger = \mathbf{0}$ in the third equality. Therefore, we will have smaller subchannel noise variances when we replace $\mathbf{G}$ with $\mathbf{G}'$. For the same input autocorrelation

$\mathbf{\Lambda}_s$, the subchannel signal to noise ratios are larger when we use $\mathbf{G}^{'}$. Hence a higher bit rate can be achieved. Using the expression of the receiver in (25), we have the noise variance $\sigma^2_{e_k}$ as in (24). $\triangle\triangle\triangle$

Lemma 1 and Lemma 2 lead us to conclude that the matrix $\mathbf{A}$ is the only part of the transceiver left to be designed. In Lemma 3, we will find the matrix $\mathbf{A}$ that maximizes the bit rate.

Lemma 3 *Without loss of generality, the matrix* $\mathbf{A}$ *in Lemma 1 and Lemma 2 that maximizes the transmission rate subject to a total power constraint is given by*

$$\mathbf{A} = \left[\begin{array}{c} \mathbf{\Lambda}_M^{-1} \\ \mathbf{0} \end{array} \right]. \tag{28}$$

In this case, the optimal transceiver is

$$\mathbf{F} = \mathbf{V} \left[\begin{array}{c} \mathbf{\Lambda}_M^{-1} \\ \mathbf{0} \end{array} \right], \quad \mathbf{G} = [\ \mathbf{I}_M \quad \mathbf{0}\ ]\mathbf{U}^\dagger \ . \tag{29}$$

Proof: The derivation for the matrix $\mathbf{A}$ will be given in [12]. Substituting the matrix $\mathbf{A}$ into (17) and (23), we obtain the solution in (29). $\triangle\triangle\triangle$

Using the optimal transceiver in (29), the bit rate in (15) has the maximal value

$$b = \log_2 \left[\left(\frac{P_0}{MN_0\Gamma} \right)^M \det(\mathbf{\Lambda}_M^2) \right]. \tag{30}$$

Substituting (14) and (29) into (7), the bit allocation for the k-th subchannel is

$$b_k = \log_2 \left(1 + \frac{P_0[\mathbf{\Lambda}_M^2]_{kk}}{MN_0\Gamma} \right). \tag{31}$$

We can see that more bits are assigned to subchannels that correspond to larger singular values of the channel, unlike [1]-[8] that use the same constellation size for all subchannels.

Remark: Application standards such as IEEE P802.11n [11], the number of subchannels M can be chosen arbitrarily. For a given $P \times N$ channel matrix $\mathbf{H}$, increasing M does not necessarily achieve a higher bit rate. As an example, consider the case $\frac{P_0}{N_0\Gamma} \leq 1$ and $[\mathbf{\Lambda}]_{ll} < 1$ for $l = 0, \cdots, K-1$. In this case we can verify that the bit rate in (30) becomes a decreasing function of M. As a result, increasing M is not necessarily better.

5. SIMULATION

In the simulation, we evaluate the performance of the proposed method. The channel used is a 4×4 MIMO channel ($P = N = 4$). The elements of $\mathbf{H}$ are complex Gaussian random variables with zero mean and unit variance. The symbol error rates are 10^{-5} for all the subchannels. The transmission rate is evaluated for 10^4 channel realizations. QAM modulation is used for the input symbols. In the following examples, we will use the optimal zero-forcing transceiver in Lemma 3 for the proposed method. Although the high bit rate assumption ($b_k \gg 1$) is used in the derivation of the optimal transceiver, the assumption is not used in the computation of transmission bit rate in the simulations.

Example 1. Fig. 2 shows the transmission rates for different transmit power to noise ratio (P_0/N_0). The number of subchannels M is 4. For comparison, we have also shown the results of two zero-forcing systems: the zero-forcing maximum signal to noise ratio (MSNR) transceiver in [1], and the zero-forcing unit noise variance (UNV) transceiver in [4]. In each of these two designs, the transmission rates are computed by (7). The result shows that the proposed method can achieve a higher bit rate. This is because the bit allocation is taken into consideration in the optimization. The subchannels with higher signal to noise ratios are assigned with more bits. For the same bit error rate constraint, we can transmit more bits.

In (7) (or equivalently (31)), the bits are not integers in general. We can use rounding,

$$b_k = \left\lfloor \log_2 \left(1 + \frac{\sigma^2_{s_k}}{\sigma^2_{e_k}\Gamma} \right) \right\rfloor, \tag{32}$$

where the notation $\lfloor z \rfloor$ denotes the largest integer that is less than or equal to z. Fig. 3 shows the transmission bit rate after rounding. The gaps between the proposed method, the MSNR design, and UNV design are similar to that in Fig. 2.

Example 2. In Fig. 4, we examine the transmission bit rate for $M = 1$, $M = 2$, and $M = 4$. Integer bit allocation in (32) is used. For $P_0/N_0 \geq 15$ dB, we see that the case $M = 4$ achieves a higher transmission rate. For $P_0/N_0 \leq 15$ dB, using $M = 2$ is better. This examples shows that increasing M does not necessarily achieve a higher transmission rate.

6. CONCLUSION

In this paper, we consider the optimal zero-forcing transceiver design for bit rate maximization over MIMO channels. Unlike earlier designs that use the same constellation size for all subchannels, bit allocation is also taken into consideration. For a given error rate, the transceiver is designed when the bits are optimally allocated. The solution is given in a closed form. In the simulations, we have demonstrated that the proposed method can achieve a higher transmission bit rate.

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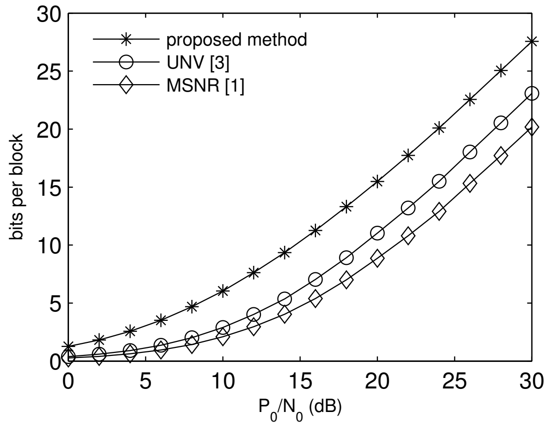

Figure 2: Transmission bit rates computed using (7) for a fixed error rate.

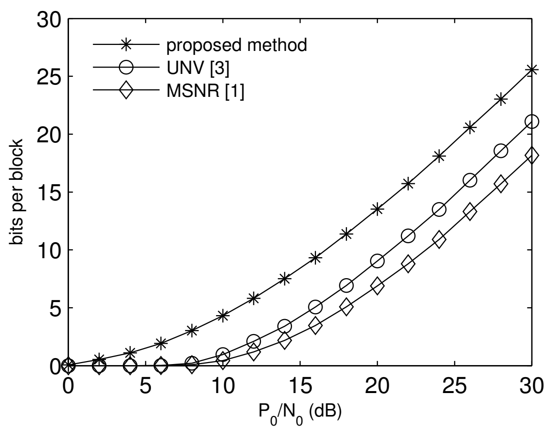

Figure 3: Transmission bit rates computed using the integer bit allocation in (32) for a fixed error rate.

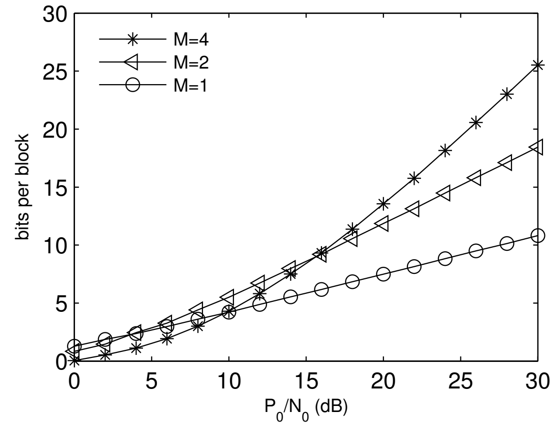

Figure 4: Transmission bit rates for different number of subchannels M.

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