

ADAPTIVE DETECTION FOR A PERMUTATION-BASED MULTIPLE-ACCESS SYSTEM ON TIME-VARYING MULTIPATH CHANNELS WITH UNKNOWN DELAYS AND COEFFICIENTS

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ABSTRACT

In this paper, a multiple-access transmission scheme based on random permutations is studied. This scheme provides both a spectrum spreading and a time spreading, combined with a chip interleaving. One considers asynchronous transmissions on frequency-selective and time-varying channels. It is assumed that the channel coefficients are unknown by the receiver, and that only vague estimates of the minimum and maximum delays are available. In such a context, an LMS algorithm is used to solve the bit detection problem. Theoretical performance results are given. Moreover, this random permutation-based scheme is compared with the DS-CDMA system, for which an equivalent LMS detector is developed in the same context.

1. INTRODUCTION

Spectrum spreading is one of the main characteristics of the multiple access schemes used in 3G communications systems. In particular, most of these schemes employ techniques based on code division. However, it is possible to define other spectrum spreading multiple-access schemes, which present similar spreading capabilities. This paper addresses in particular a spectrum spreading technique based on random permutations, which will be named Random Permutation Multiple Access (RPMA). The basic idea of this scheme consists of randomly permuting the samples of an input data block in order to spread the data both in time and in frequency. The interest of such a technique is that the bit energy is spread in time (and not only in frequency), by using a kind of chip-interleaving; this allows to mitigate the effects of the channel fading, in particular when the channel is time-varying. The concepts of block-spreading and chip-interleaving are not new. For instance, in [1] is proposed a technique based on a row-column matrix interleaver combined with the use of Walsh-Hadamard codes. In [2] is investigated another technique which resorts to so-called shift-orthogonal codes. However, the methods proposed in these papers assume synchronous or quasi-synchronous transmission. Also, chip-interleaving is investigated for instance in [3] and related works, and in [4], where the detection is achieved using a turbo-decoding. It is assumed in these articles that the channel coefficients and/or delays are known by the receiver. However, the channel estimation in asynchronous multiple-access systems is a difficult problem. In the present paper, the case of asynchronous transmissions on time-varying frequency-selective channels is addressed, where channel coefficients and delays are unknown. Different papers have already been published on this topics (e.g. [5], [6]). In each of them, a comparison with the DS-CDMA system has been provided. However, in [5], the channel delays were assumed to be known by the receiver (while the channel coefficients were actually unknown). Here, it is only assumed that rough esti-

mates of the minimum and maximum delays are available for each user. Moreover, the matched-filter bank used in [5] is replaced by a classical uniform sampling, which dramatically reduces the dimension of the problem. As in [5], a Least-Mean-Squares (LMS) algorithm is used to manage the fact that the channels are unknown. Obviously, the resort to the classical LMS algorithm does not constitute the originality of the paper. This one rather relies on the use of a new spectrum/time spreading technique, which gives generally better detection performance results than those of the DS-CDMA system, for equivalent transmission characteristics.

Section 2 presents the signal modeling in RPMA systems. The detection problem is addressed in section 3 for the RPMA system, along with the similar detection problem derived for the DS-CDMA system. Theoretical performance results are given in section 4, and some simulation results are presented in section 5.

2. SIGNAL MODELING

2.1. The permutation process

Let $(b_n)_{n \in \mathbb{Z}}$ ($b_n \in \{-1; +1\}$) be a sequence of equiprobable bits. This sequence is modulated by an antipodal baseband code with duration T and waveform pattern $m(t)$. The modulated process $Z(t)$, defined by $Z(t) \triangleq \sum_{n \in \mathbb{Z}} b_n m(t - nT)$, is sampled with period T_s , such that $N_s \triangleq T/T_s$ is an integer number (i.e., N_s is the number of samples per bit). Let $(Z_n)_{n \in \mathbb{Z}}$ denote this sampled sequence. A new sequence $(U_n)_{n \in \mathbb{Z}}$ is formed from $(Z_n)_{n \in \mathbb{Z}}$ as follows: considering blocks of N_b consecutive bits, the $N_s N_b$ samples of Z_n corresponding to a given block are permuted using an uniformly distributed permutation of the set $\{1, \dots, N_s N_b\}$. The sequence $(U_n)_{n \in \mathbb{Z}}$ is defined as the resulting sequence of this block permutation. One can then show that the power spectral density of $(U_n)_{n \in \mathbb{Z}}$ is spread by a factor N_s with respect to the one of $(Z_n)_{n \in \mathbb{Z}}$. Consequently, this permutation procedure is a particular spread-spectrum technique. Moreover, if one regards the N_s samples per bit as chips, by comparison with the CDMA terminology, this method can also be considered as a chip-interleaving method. An example of this permutation procedure is given in fig. 1, for one block of $N_b = 4$ bits and $N_s = 8$ samples. Note that this chip interleaving can also be performed in addition to a classical bit interleaving.

Denote $\mathbf{b}_j = [b_{(j-1)N_b+1}, \dots, b_{jN_b}]^T$ as the j th block of bits, and $\mathbf{m} \triangleq [m_1, \dots, m_{N_s}]^T$ as the result of the sampling of the waveform pattern $m(t)$ with N_s samples. Then, the j th block of sequence $(Z_n)_{n \in \mathbb{Z}}$, i.e. vector $\mathbf{Z}_j = [Z_{(j-1)N_b+1}, \dots, Z_{jN_b}]^T$, can be expressed as: $\mathbf{Z}_j = \mathbf{M}^T \mathbf{b}_j$, where $\mathbf{M} = \mathbf{m}^T \otimes \mathbf{I}_{N_b}$ (\otimes is the Kronecker product, and \mathbf{I}_n is the identity matrix of order n). Let \mathbf{P} denote the $(N_s N_b) \times (N_s N_b)$ permutation matrix. The j th block of sequence $(U_n)_{n \in \mathbb{Z}}$, defined by $\mathbf{U}_j \triangleq [U_{(j-1)N_b+1}, \dots, U_{jN_b}]^T$, can be ex-

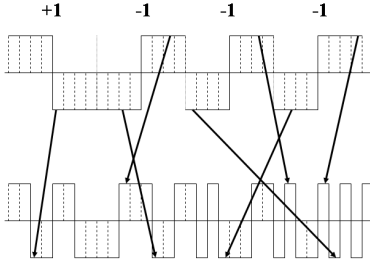


Fig. 1. Example of permutations for one block, using biphasic signaling, $N_b = 4$, and $N_s = 8$.

pressed as: $\mathbf{U}_j = \mathbf{P}\mathbf{Z}_j = \mathbf{P}\mathbf{M}^T \mathbf{b}_j$. The continuous-time process obtained from $(U_n)_{n \in \mathbb{Z}}$ using a rectangular waveform signaling is then given by

$$\sum_{j \in \mathbb{Z}} \sum_{i=1}^{N_s N_b} (\mathbf{P}\mathbf{M}^T \mathbf{b}_j)_i \rho(t - (i-1)T_s - jN_b T)$$

where $(\mathbf{v})_i$ denotes the i th component of any vector \mathbf{v} , and $\rho(t)$ is the indicator function on $[0; T_s]$. Note that if an adaptive detection based on a training sequence is used at the receiver, as it is proposed in section 3, the receiver does not need to know the random permutation, which can then change during the transmission, increasing the time-diversity (in particular if a bit interleaver is used before the permutation process).

2.2. The continuous received signal

Consider the asynchronous transmission of K users using the spread-spectrum technique presented above. Let $\mathbf{b}_{k,j}$ and \mathbf{P}_k denote respectively the j th block of bits and the permutation matrix associated to user k . The channel of the k th user is a time-varying frequency-selective channel, whose impulse response at time t is given by:

$$c_k(t, \tau) = \sum_{l=0}^{L_k-1} c_{k,l}(t) \delta(\tau - \tau_{k,l}).$$

where L_k is the number of paths, $c_{k,l}(t)$ is the time-varying gain of the l th path of the k th user, $\tau_{k,l}$ is the propagation delay, and δ is the Dirac function. The received signal is expressed as:

$$r(t) = \sum_j \sum_{k=1}^K \sum_{l=0}^{L_k-1} \sum_{i=1}^{N_s N_b} c_{k,l}(t) (\mathbf{P}_k \mathbf{M}^T \mathbf{b}_{k,j})_i \times \rho(t - (i-1)T_s - jN_b T - \tau_{k,l}) + n(t)$$

where $n(t)$ is an additive white Gaussian noise with variance σ^2 , independent of the transmitted signals.

2.3. Discretizing of the continuous received signal

In this paper, it is assumed that the channel delays $\tau_{k,l}$ are unknown to the receiver. It is only assumed that estimates $\hat{\tau}_{k,\min}$ and $\hat{\tau}_{k,\max}$ of $\min_l \{\tau_{k,l}\}$ and $\max_l \{\tau_{k,l}\}$, respectively, are available for all users. More generally, one can define $\hat{\tau}_{k,\min}$ and $\hat{\tau}_{k,\max}$ such that the main amount of energy transmitted by user k for the j th bit is (assumed to be) received between samples $\hat{\tau}_{k,\min} + jN_s + 1$ and $\hat{\tau}_{k,\max} + (j+1)N_s$. These estimates could be obtained for instance from the position of the user in the cell. However, this problem is still open and is beyond the scope of this paper. We propose to obtain a discrete signal by a uniform sampling of the process $r(t)$ based

on these estimates. More precisely, for a given sampling frequency f_0 (with $f_0 \geq 2/T_s$), define $\hat{\tau}_{k,\min}^s$ and $\hat{\tau}_{k,\max}^s$ as the entire parts of $\hat{\tau}_{k,\min} \times f_0$ and $\hat{\tau}_{k,\max} \times f_0$, respectively, and N_{samp} as the number of samples during a bit period. Denote then

$$S_k^j \triangleq \{\hat{\tau}_{k,\min}^s + jN_b N_{\text{samp}} + 1, \dots, \hat{\tau}_{k,\max}^s + (j+1)N_b N_{\text{samp}}\},$$

i.e., S_k^j is the set of sample indexes which corresponds to the time interval during which the j th block of the k th user is (assumed to be) received. Thus, the sampled vector for the detection of this later block is defined by:

$$\mathbf{r}_k^j \triangleq \{r(n/f_0) | n \in S_k^j\}$$

The length of S_k^j will be denoted by γ_k (it does not depend on j). If the estimates $\hat{\tau}_{k,\min}$ and $\hat{\tau}_{k,\max}$ are accurate enough, then \mathbf{r}_k^j contains almost all the samples corresponding to the bits transmitted during this block (i.e., the bits in $\mathbf{b}_{k,j}$). Now, vector \mathbf{r}_k^j contains also many interference terms: inter-symbol interference due to other bits of other blocks of user k , and multiple-access interference due to the bits of other users. This is particularly true for samples which do not correspond to any bit of $\mathbf{b}_{k,j}$; but only a complete knowledge of the channel delays would allow one to not take these samples into account, which is not assumed here.

3. LMS DETECTOR

3.1. The RPMA case

The problem consists now of detecting the bit vector $\mathbf{b}_{k,j}$ from the data \mathbf{r}_k^j . This must be done by mitigating both the interferences and the additive Gaussian noise. To this purpose, a classical strategy consists of defining the estimated bit vector by

$$\hat{\mathbf{b}}_{k,j} \triangleq \text{sign}(\mathbf{H}_{k,j}^T \mathbf{r}_k^j) \quad (1)$$

where $\mathbf{H}_{k,j}$ is the $\gamma_k \times N_b$ matrix which minimizes the mean-square error (MSE) $E[\|\mathbf{b}_{k,j} - \mathbf{H}^T \mathbf{r}_k^j\|^2]$ with respect to matrix \mathbf{H} (the matrix norm is defined by $\|\mathbf{A}\| \triangleq (\text{trace}(\mathbf{A}\mathbf{A}^T))^{1/2}$, and \cdot^T denotes transposition). This detector acts as a combination of matched-filters, a Rake receiver, and a multi-user detector, where these operations are not separated. The optimal matrix $\mathbf{H}_{k,j}$ is given by:

$$\mathbf{H}_{k,j} = \Sigma_{k,j}^{-1} \mathbf{R}_{k,j} \quad (2)$$

Analytical expressions of $\Sigma_{k,j}$ and $\mathbf{R}_{k,j}$ can be obtained as a function of the user's and the channel's characteristics. For clarity, the expressions of $\Sigma_{k,j}$ and $\mathbf{R}_{k,j}$ are reported in appendix 8.1. Now, it is assumed that the channel coefficients and delays are unknown, so that the quantities in (2) are not available to the receiver. Moreover, they are also variable since the channels are time-varying. Thus, one resorts to the LMS algorithm, which allows to adaptively converge to the optimal solution, and to track its time variations. The update equations (for learning and decision-directed sequences) of this well-known algorithm are not recalled here and can be found for instance in [7]¹.

¹Note however that the LMS algorithm operates here on matrices, and not on vectors, which is more usual. The up-date equations actually remain unchanged.

3.2. The DS-CDMA case

Consider now the DS-CDMA system. For such a system, the bits $b_k(j)$ of the k th user are modulated by a signature waveform (code) $s_k(t)$, which is assumed to be zero outside the interval $[0, T]$. The signal transmitted by user k is then

$$\sum_{j \in \mathbb{Z}} b_k(j) s_k(t - jT),$$

and the received signal can be expressed as:

$$\tilde{r}(t) = \sum_j \sum_{k=1}^K \sum_{l=0}^{L_k-1} c_{k,l}(t) b_k(j) s_k(t - (j-1)T - \tau_{k,l}) + n(t)$$

Define

$$\tilde{S}_k^j \triangleq \{\hat{\tau}_{k,\min}^s + jN_{\text{samp}} + 1, \dots, \hat{\tau}_{k,\max}^s + (j+1)N_{\text{samp}}\}.$$

Denote $\tilde{\gamma}_k^j$ as the length of \tilde{S}_k^j . The vector used for the detection of the j th bit of the k th user (i.e. $b_k(j)$) is then defined by

$$\tilde{\mathbf{r}}_k^j \triangleq \{\tilde{r}(n/f_0) | n \in \tilde{S}_k^j\}.$$

The optimal detection (in the sense, as previously, of the Linear Minimum MSE (LMMSE)) is given by

$$\hat{b}_k(j) \triangleq \text{sign}(\tilde{\mathbf{H}}_{k,j}^T \tilde{\mathbf{r}}_k^j) \text{ with } \tilde{\mathbf{H}}_{k,j} = \tilde{\Sigma}_{k,j}^{-1} \rho_{k,j} \quad (3)$$

where $\tilde{\Sigma}_{k,j}$ is the covariance matrix of $\tilde{\mathbf{r}}_k^j$, and $\rho_{k,j} = E[b_k(j) \tilde{\mathbf{r}}_k^j]$ (these terms are given in appendix 8.2). An LMS algorithm can then be performed, similarly as for the RPMA case, to converge to this optimal solution and track its time-variations.

3.3. Complexity issue

As mentioned in the introduction, a similar problem has been addressed in [5], for RPMA and DS-CDMA systems, where the sampling of the received signal was achieved using a bank of matched filters. A filter was needed for each user, each path, each block of bits, and each chip, resulting in high-dimension detection matrices. Moreover, the knowledge of the all channel delays was necessary for this matched-filtering. In the present paper, this knowledge is not required (see section 2.3). Moreover, thanks to the sampling strategy used in this paper, the dimensions of the matrices involved in the detection are dramatically reduced. For instance, in the simulation results presented section 5, the dimensions of matrices $\mathbf{H}_{1,j}$ are 67×4 ($\gamma_1 = 67$ and $N_b = 4$), i.e. 268 elements; using the matched filtering proposed in [5], the dimensions of the matrices would be 384×16 , i.e. 6144 elements. Moreover, with this latter method, these dimensions linearly increase with the number of paths, while with the present method, the dimensions are independent of the number of paths (furthermore, the knowledge of this number is not required here).

4. THEORETICAL PERFORMANCE

4.1. LMMSE performance

In this section, the instantaneous performance of the LMMSE detector given by eq. (1) (2), and eq. (3), respectively, is given, i.e. we derive the optimal performance for a particular block of bits (or, for a particular bit in the case of the DS-CDMA). This performance must be considered as a lower bound for the performance of the LMS algorithm. For the RPMA system, the estimate of the m th bit of $\mathbf{b}_{k,j}$ can be written as

$$\hat{\mathbf{b}}_{k,j}(m) = \text{sign}(\theta_{k,j}^m),$$

with:

$$\theta_{k,j}^m \triangleq \mathbf{H}_{k,j}^m \mathbf{r}_k^j = A_{k,j}^m \mathbf{b}_{k,j}(m) + \mathbf{H}_{k,j}^m \mathbf{r}_{k,m}^{j,\text{interf}} + \mathbf{H}_{k,j}^m \mathbf{n}_k^j$$

where $A_{k,j}^m \triangleq \mathbf{R}_{k,j}^m \Sigma_{k,j}^{-1} \mathbf{R}_{k,j}^m$; $\mathbf{H}_{k,j}^m$ and $\mathbf{R}_{k,j}^m$ denote the m th row and the m th column of $\mathbf{H}_{k,j}$ and $\mathbf{R}_{k,j}$, respectively; $\mathbf{r}_{k,m}^{j,\text{interf}}$ is the unnoisy part of $\mathbf{r}_{k,m}^j$ which does not contain $\mathbf{b}_{k,j}(m)$; and \mathbf{n}_k^j is the noise term. Now, $\mathbf{r}_{k,m}^{j,\text{interf}}$ is the sum of many independent random variables, and can then be approximated by a Gaussian variable, according to the central limit theorem. Denote σ_{interf}^2 as the variance of this variable. We have: $\sigma_{\text{interf}}^2 = E[(\mathbf{H}_{k,j}^m)^T \mathbf{r}_{k,m}^{j,\text{interf}}]^2 = (\mathbf{H}_{k,j}^m)^T \text{cov}(\mathbf{r}_{k,m}^{j,\text{interf}}) (\mathbf{H}_{k,j}^m)$ where $\text{cov}(\mathbf{r}_{k,m}^{j,\text{interf}}) = \text{cov}(\mathbf{r}_{k,m}^j) - (\mathbf{R}_{k,j}^m)(\mathbf{R}_{k,j}^m)^T - \sigma^2 \mathbf{I}_{\gamma_k}$, and $\text{cov}(\mathbf{r}_{k,m}^j)$ is the m th diagonal block of $\Sigma_{k,j}$. Thus, the bit error rate (BER) for $\mathbf{b}_{k,j}(m)$ is given by:

$$\text{BER}_{k,j}(m) = Q\left(A_{k,j}^m / \sqrt{\sigma_{\text{interf}}^2 + \sigma_{\mathbf{n}}^2}\right)$$

where $Q(x) \triangleq \int_x^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$, and $\sigma_{\mathbf{n}}^2 \triangleq \sigma^2 \|\mathbf{H}_{k,j}^m\|^2$ is the variance of the noise term. A very similar expression of the BER can be obtained for the DS-CDMA system.

4.2. LMS performance

The LMMSE performance results constitute a lower bound for the LMS performance. Now, the LMS coefficients are never equal to the LMMSE coefficients, but rather randomly oscillate around these latter. It is then interesting to take this random behavior into account in order to derive more accurate performance of the LMS estimation. Denote $\mathbf{K}_{k,m}^j$ as the covariance matrix of the error between the LMS coefficient vector for the m th bit of the j th iteration of the algorithm, and the optimal LMMSE coefficient vector, It is known that $\mathbf{K}_{k,m}^{j+1}$ is linked to $\mathbf{K}_{k,m}^j$ by a recursive equation (see [7] p.395). To the best of our knowledge, there is no expression for the limit of $\mathbf{K}_{k,m}^j$. However, for fixed channels, this limit can be numerically computed by iterating the recursive equation mentioned above. For all computations performed in that way, it always appeared that the limit covariance matrix, denoted by $\mathbf{K}_{k,m}^\infty$, is a diagonal matrix of the form $\mathbf{K}_{k,m}^\infty = \sigma_{\text{LMS},k}^{m,2} \mathbf{I}_{\gamma_k}$. Note that for time-varying channels, such a computation should be achieved at each iteration j of the LMS algorithm, yielding to a covariance matrix of the form $\sigma_{\text{LMS},k,j}^{m,2} \mathbf{I}_{\gamma_k}$. The form of the limit covariance matrix shows that the errors obtained on different coefficients are uncorrelated. Moreover, it also appeared from the simulations of the LMS algorithm that the coefficients are approximately normally distributed. One can then conclude that the errors corresponding to different coefficients are statistically independent. Define the matrix $\Sigma_{k,j}^m \triangleq \Sigma_{k,j} - \mathbf{R}_{k,j}^m (\mathbf{R}_{k,j}^m)^T$. Then it can be shown that the BER for the k th user and the m th bit of the j th block can be approximated by (still using the Central Limit theorem)

$$\text{BER}_{k,j}^{\text{lms}}(m) = Q\left(A_{k,j}^m / \sqrt{\sigma_{k,j}^{m,2}}\right)$$

with $\sigma_{k,j}^{m,2} \triangleq (\mathbf{H}_{k,j}^m)^T \Sigma_{k,j}^m \mathbf{R}_{k,j}^m + \sigma_{\text{LMS},k,j}^{m,2} (\|\mathbf{R}_{k,j}^m\|^2 + \text{trace} \Sigma_{k,j}^m)$ (for brevity reasons, details of this computation cannot be given here). A similar result can be obtained for the DS-CDMA system.

5. SIMULATION RESULTS

This section presents some simulation results. For these simulations, the characteristics of the users (i.e., parameters K , N_b , N_s , and the permutations) are defined, which determines matrices \mathbf{P}_k . The multipath time-varying channels are generated from the Jake's model. More precisely, The channel coefficients have been drawn independently according a Rayleigh distribution with parameter 0.5. We first considered static channels. Indeed, in that case, it is possible to confirm the theoretical BERs by the estimated BERs computed during one simulation. For time-varying channels, the BERs are also time-varying, and many simulations would be necessary at each instant to estimate the instantaneous BER, which is not computationally feasible. For the simulations with static channels, the parameters are as follows: $T_b = 4.88 \cdot 10^{-7} s$, $K = 4$, $N_b = 4$, $N_s = 8$, $L_k = 3$ for all k . For DS-CDMA, one uses Gold codes with 7 chips per bit (thus, with an equivalent spectrum spreading factor). The channel delays are drawn uniformly in the set $[0, 8T_b]$, and the estimates $\hat{\tau}_{k,\min}$ and $\hat{\tau}_{k,\max}$ are uniformly drawn in a window of length T_b centered on $\tau_{k,\min}$ and $\tau_{k,\max}$, respectively. The results presented in this section are those obtained for user 1 (similar results have been obtained for the other users). Fig. 2 presents the simulated and theoretical BERs for both the RPMA and the DS-CDMA systems. The BER is given as a function of the signal-to-noise ratio (SNR), defined as the ratio between the power of the un-noisy multi-user received signal and the power of the additive Gaussian noise, and not as a function of the E_b/N_0 ratio, which is more standard in communications systems. Indeed, the SNR takes into account the coefficients of all channels. At the contrary, the E_b/N_0 ratio only concerns the user of interest: thus, for a same E_b/N_0 ratio, the performance results can be poor or good, depending on the strength of the interferer signals. Hence, the SNR is a better measure of the importance of the noise with respect to the information-bearing signal at the receiver. It would also be interesting to some results obtained as a function of the E_b -over-interference ratio, or the E_b -over-interference-plus-noise ratio, as it is more usual in the communication community. This would help to make a quantitative assessment of the resistance of the scheme to multi-user interference. However, for brevity reasons, this analysis cannot be conducted here. For static channels, the variance of the noise can then be obtained from the SNR and the expression of the power of the unnoisy received signal given in [6]. Clearly, the theoretical LMS results are confirmed by the simulation results. Moreover, these results are very close to the LMMSE results (for DS-CDMA, they are quasi indistinguishable). One can also note that the RPMA and DS-CDMA performance are quite similar (the RPMA BER is just a bit lower than the DS-CDMA BER). Indeed, for static channels, one cannot benefit from the chip interleaving proposed in RPMA. Time-varying channels are next considered. The parameters of the simulations are the same as previously. In addition, the speeds of the users are between 20 and 30 km/h, and the carrier frequencies (which are required for the computation of the Doppler spreads used in the Jake's channel model) are around 830MHz. Figures 3 and 4 illustrate the behavior of the LMS algorithm for the first user, by plotting the time variations of the computed LMS coefficients, along with the variations of the optimal LMMSE coefficients, in the RPMA and DS-CDMA cases, respectively (for clarity, only 5 coefficients are considered in the figures). It can be observed that the LMS algorithm globally succeeds in tracking these variations, as long as these ones are not too fast (which occurs for instance around instant 0.07s). Figure 5 shows the time variations of the theoretical LMMSE and LMS BERs for the first user. For RPMA, each curve corresponds to the mean curve obtained for the 4 bits of the block. One can observe that the LMS performance follow the LMMSE performance, except around instant 3.5e-5s, where the LMMSE BERs are much better. Moreover, one can see that the theoretical LMS results are better for RPMA than for DS-CDMA. This observation is confirmed in table 1, which gives the estimated LMS BER computed

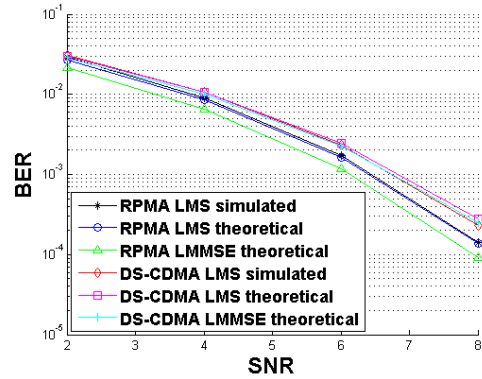


Fig. 2. Simulated and theoretical BERs for static channels.

during the whole simulation for both systems and different SNRs. One can note that the RPMA performance are much better, which is mainly due to the chip-interleaving. For instance, for SNR= 14dB and SNR= 16dB (which corresponds to cases where the channels and the choice of the transmission system have an important impact on the performance), the BER is 14 and 27 times smaller for RPMA than for DS-CDMA, respectively.

	SNR=8dB	SNR=10dB	SNR=12dB
RPMA	0.008280	0.002150	0.000470
DS-CDMA	0.018135	0.008380	0.002985
	SNR=14dB	SNR=16dB	
RPMA	0.000055	0.000005	
DS-CDMA	0.000790	0.000135	

Table 1: mean estimated BERs for RPMA and DS-CDMA systems.

6. CONCLUSION

In this paper, a multiple-access system based on random permutations is studied for an asynchronous transmission on frequency-selective time-varying channels. This study assumes that the channel coefficients and delays are unknown by the receiver. Instead, only rough estimates of the minimum and maximum delays are required for each user. In that context, the bit detection is achieved using an LMS algorithm, which is able to track the variations of the theoretical optimal coefficients. Theoretical performance results are provided, both for the LMMSE and the LMS detectors. A comparison with the DS-CDMA system is presented for an equivalent transmission context. This comparison, achieved both theoretically and practically, shows that the RPMA performs (in general) better than the DS-CDMA. This is mainly due to the fact that the bit energy is spread in time, and that the chips are interleaved, which gives more robustness regarding the variations of the fading channels. One of the important remaining issue is the estimation of the minimum and maximum delays, and the analysis of the performance regarding the quality of these estimates. This issue is however beyond the scope of this article, and is currently under study. Moreover, it would be also interesting to incorporate the coding and the decoding in the modulation/demodulation schemes.

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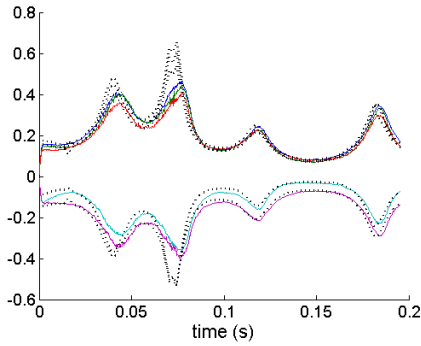


Fig. 3. Evolution of the LMS coefficients for time-varying channels for RPMA. solid: estimated - dotted: theoretical.

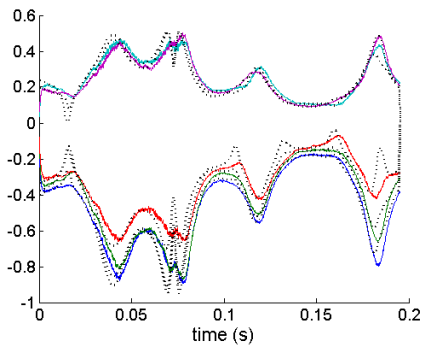


Fig. 4. Evolution of the LMS coefficients for time-varying channels for DS-CDMA. solid: estimated - dotted: theoretical.

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8. APPENDIX

8.1. Matrices $\mathbf{R}_{k,j}$ and $\Sigma_{k,j}$

Denote $\mathbf{R}_{k,j}^m$ as the m th column of the $\gamma_k \times N_b$ matrix $\mathbf{R}_{k,j}$. We have: $\mathbf{R}_{k,j}^m = E[\mathbf{b}_{k,j}(m)\mathbf{r}_{k,m}^j]$. It can be shown that the p th element of

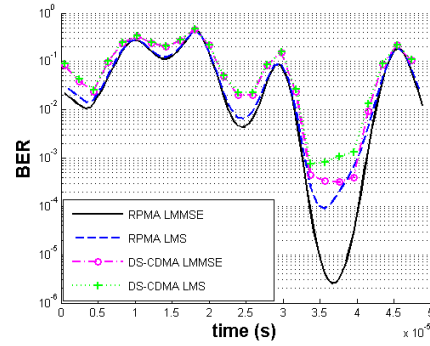


Fig. 5. Time variations of the theoretical BERs for time-varying channels.

$\mathbf{R}_{k,j}^m$ is expressed as:

$$\sum_{(j,l,i) \in \Omega_p^k} c_{k,l}(i_p/f_0)(P_k M^T)_{i,m}$$

where $\Omega_p^k \triangleq \{(l,i) \in \{0, \dots, L-1\} \times \{1, \dots, N_s N_b\} \text{ such that } 0 \leq p/f_0 - \tau_{k,l} - (j-1)N_b T - (i-1)T_s \leq T_s\}$, and $i_p \triangleq S_k^j(p)$. Using the same notations for $(p_1, p_2) \in \{1, \dots, \gamma_k\}^2$, it can be shown that the covariance matrix $\Sigma_{k,j}$ is given by

$$\Sigma_{k,j}(p_1, p_2) = \sum_{k'=1}^K \sum_{l_1, l_2=0}^{L-1} c_{k',l_1}(i_{p_1}/f_0) c_{k',l_2}(i_{p_2}/f_0) \sum_{i_1, i_2=1}^{N_s N_b} (P_{k'} M^T)_{i_1, \cdot} (P_{k'} M^T)_{i_2, \cdot}^T C_{i_{p_1}, l_1, i_1}^{i_{p_2}, l_2, i_2}(k') + \sigma^2 \quad (4)$$

where $(P_{k'} M^T)_{i, \cdot}$ denotes the i th row of $P_{k'} M^T$, and $C_{i_{p_1}, l_1, i_1}^{i_{p_2}, l_2, i_2}(k')$ is the cardinal of the set $\{j | (j, l_1, i_1) \in \Omega_{p_1}^{k'}, (j, l_2, i_2) \in \Omega_{p_2}^{k'}\}$. Note that since sets Ω_p^k depend on the delays, $\Sigma_{k,j}(p_1, p_2)$ also depends on the delays, although these latter do not appear explicitly in (4).

8.2. Vector $\tilde{\mathbf{R}}_{k,j}$ and Matrix $\tilde{\Sigma}_{k,j}$

By definition, $\tilde{\mathbf{R}}_{k,j} = E[b_k(j)\tilde{\mathbf{r}}_k^j]$. It is not difficult to prove that $\tilde{\mathbf{R}}_{k,j}$ is the vector whose p -th component is given by ($p \in \{1, \dots, \tilde{L}_k\}$):

$$\tilde{\mathbf{R}}_{k,j}(p) = \sum_{l \in \Theta_{k,j,p}} c_{k,l}(\tilde{i}_p/f_0) s_k(\tilde{i}_p/f_0 - \tau_{k,l} - jT)$$

where: $\tilde{i}_p \triangleq \tilde{S}_k^j(p)$ and $\Theta_{k,j,p} \triangleq \{l \in \{0, \dots, L-1\} | (i_p/f_0 - \tau_{k,l} - jT) \in [0; T]\}$.

Moreover, $\tilde{\Sigma}_{k,j}$ is the covariance matrix of $\tilde{\mathbf{r}}_k^j$. It can be shown that, for $(p_1, p_2) \in \{1, \dots, \tilde{\gamma}_k\}^2$:

$$\begin{aligned} \tilde{\Sigma}_{k,j}(p_1, p_2) &= \sum_{k'=1}^K \sum_{i \in \tilde{\Omega}_{p_1}^{k'} \cap \tilde{\Omega}_{p_2}^{k'}} \sum_{l \in T_{k',i}^{p_1, p_2}} c_{k',l}(\tilde{i}_{p_1}/f_0) \\ & c_{k',l}(\tilde{i}_{p_2}/f_0) s_{k'}(\tilde{i}_{p_1}/f_0 - \tau_{k',l} - iT) s_{k'}(\tilde{i}_{p_2}/f_0 - \tau_{k',l} - iT) \\ & + \sigma^2 \bar{\delta}_{p_1, p_2} \end{aligned}$$

where: $\tilde{i}_{p_1} \triangleq \tilde{S}_k^j(p_1)$, $\tilde{i}_{p_2} \triangleq \tilde{S}_k^j(p_2)$, $T_{k',i}^{p_1, p_2} \triangleq \Theta_{k',i,p_1} \cap \Theta_{k',i,p_2}$ and $\bar{\delta}$ is the Kronecker function, i.e. $\bar{\delta}_{m,m} = 1$, $\bar{\delta}_{m,n} = 0$ for $m \neq n$.