EFFECTS OF PROPAGATION DELAYS AND SAMPLING RATE ON FEED-BACK BSS AND COMPARATIVE STUDIES WITH FEED-FORWARD BSS

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ABSTRACT
Feed-forward (FF-) and Feed-back (FB-) structures have been proposed for Blind Source Separation (BSS) systems. The FF-BSS system has some degrees of freedom in the solution space, and signal distortion is likely to occur in convolutive mixtures. On the other hand, in the FB-BSS structure, the signal distortion is automatically suppressed after the sources are separated. However, it requires some condition on propagation delays in the mixing process. In this paper, source separation performance in the FB-BSS is analyzed taking the propagation delays into account. Based on this analysis, separation performances of the FF-BSS and the FB-BSS are compared. Furthermore, an over-sampling method is proposed for the FB-BSS in order to relax the constraint of the propagation delays. Simulation results support theoretical analysis and usefulness of the over sampling FB-BSS. Consequently, the FB-BSS expands its application fields.

1. INTRODUCTION
Two kinds of structures have been proposed for Blind Source Separation (BSS), which include a Feed-forward (FF-) BSS and a Feed-back (FB-) BSS [1],[2]. From design and application viewpoint, it is very important how to select the structure of BSS. This point has not been well discussed.

In the FF-BSS, there exist some degrees of freedom in determining a separation block, and signal distortion is likely caused. Several methods have been proposed to suppress the signal distortion [3],[5],[6],[7]. On the other hand, the FB-BSS has no degree of freedom, and the signal distortion is well suppressed after the sources are separated. Therefore, the FB-BSS can provide good performances in both source separation and signal distortion. However, it requires some condition on propagation delays in a convolutive mixture [4].

In this paper, first, we analyze source separation performance of the FB-BSS based on the propagation delays in the mixture. Furthermore, the FF-BSS and the FB-BSS are compared. Next, an over-sampling method is proposed for the FB-BSS, in order to relax the constraint on the propagation delays. Simulation results obtained by using white signals and speech signals will be shown.

2. TWO KINDS OF STRUCTURES FOR BSS
2.1. Feed-forward BSS
A block diagram of the FF-BSS and an FIR filter, used in the separation block, are shown in Figs.1(Upper) and 1(Lower), respectively.

\[
X(z) = H_1(z)S_1(z) + H_2(z)S_2(z) + \cdots + H_N(z)S_N(z)
\]

The sources \( s_i(n), i = 1, 2, \ldots, N \) are convolved with impulse responses of the mixing block \( h_{ji}(n) \), and are observed as \( x_j(n), j = 1, 2, \ldots, N \). The separation block outputs \( y_k(n) \) are convolution sums of \( w_{kj}(n) \) and \( x_j(n) \).

\[
x_j(n) = \sum_{i=1}^{N} \sum_{m=0}^{K_i-1} h_{ji}(m)s_i(n-m)
\]

\[
y_k(n) = \sum_{j=1}^{N} \sum_{l=0}^{K_j-1} w_{kj}(l)x_j(n-l)
\]

2.2. Feed-back BSS
A block diagram of the FB-BSS is shown in Fig.2(Upper) [1]. The separation block employs an FIR filter shown in Fig.2(Lower). Since a feedback loop in a discrete time system needs at least one sample delay, a direct path from the input to the output is not used in the FIR filter, that is \( c_{kj}(0) = 0 \). The separation block output is expressed by

\[
y_k(n) = x_k(n) - \sum_{j=1}^{N} \sum_{l=1}^{K_j-1} c_{kj}(l)y_j(n-l)
\]
3. RELATION BETWEEN PROPAGATION DELAYS AND LEARNING SEPARATION BLOCK IN FB-BSS

For simplicity, the FB-BSS having 2 sources, 2 sensors and 2 outputs, as shown in Fig.2(upper), is considered. It is assumed that the propagation delays of $H_{11}(z)$ and $H_{22}(z)$ are less than those of $H_{21}(z)$ and $H_{12}(z)$. This means that the sensors of $X_1(z)$ and $X_2(z)$ locate close to $S_1(z)$ and $S_2(z)$, respectively. Thus, this assumption is actually acceptable.

The separation block outputs can be expressed by
\[
\begin{bmatrix}
Y_1(z) \\
Y_2(z)
\end{bmatrix} = \begin{bmatrix}
1 & 1 - C_{21}(z) \\
1 - C_{21}(z) & 1
\end{bmatrix} \begin{bmatrix}
H_{11}(z) & H_{12}(z) \\
H_{21}(z) & H_{22}(z)
\end{bmatrix} \begin{bmatrix}
S_1(z) \\
S_2(z)
\end{bmatrix}
\]

When source separation is complete, $C_{kj}(z)$ and $y_k(n)$ have the following two kinds of solutions (a) and (b) [4].

(a) Non-diagonal elements are zero.
\[
C_{12}(z) = \frac{H_{12}(z)}{H_{22}(z)}, \quad C_{21}(z) = \frac{H_{21}(z)}{H_{11}(z)} \quad (5)
\]
\[
Y_1(z) = H_{11}(z)S_1(z), \quad Y_2(z) = H_{22}(z)S_2(z) \quad (6)
\]

(b) Diagonal elements are zero.
\[
C_{12}(z) = \frac{H_{11}(z)}{H_{21}(z)}, \quad C_{21}(z) = \frac{H_{22}(z)}{H_{12}(z)} \quad (7)
\]
\[
Y_1(z) = H_{12}(z)S_2(z), \quad Y_2(z) = H_{21}(z)S_1(z) \quad (8)
\]

From the assumption regarding the delay of $H_{ji}(z)$, $C_{21}(z)$ and $C_{12}(z)$ in (a) have a positive delay, that is, they are causal systems. On the other hand, $C_{21}(z)$ and $C_{12}(z)$ in (b) have a negative delay, resulting in non-causal systems, which cannot be realized. For this reason, $S_1(z)$ cannot be cancelled at $X_1(z)$. In the same manner, $S_2(z)$ cannot be cancelled at $X_2(z)$. This means that the diagonal elements of Eq.(4) cannot be cancelled by adjusting $C_{jk}(z)$. On the other hand, $S_2(z)$ and $S_1(z)$ can be cancelled at $X_1(z)$ and $X_2(z)$, respectively. In other words, the non-diagonal elements of $C_{12}(z)$ can be cancelled. Based on these analysis, learning the separation block in the FB-BSS is equivalent to minimize the output powers [4].

From Eq.(6), $Y_i(z)$ are the same as the $S_i(z)$ components observed at $X_i(z)$. This means no signal distortion occur in the separation block [5], [6].

4. ANALYSIS OF SOURCE SEPARATION IN FB-BSS BASED ON PROPAGATION DELAY

4.1. Effects of Propagation Delay on Learning FB-BSS

It is also assumed that, in the FB-BSS shown in Fig.2, $S_1(z)$ and $S_2(z)$ are separated at $Y_1(z)$ and $Y_2(z)$, respectively. This does not lose generality. Source separation performance of the FB-BSS is determined by the following conditions.

1. $S_2(z)$ and $S_1(z)$ should be cancelled at $X_1(z)$ and $X_2(z)$, respectively.
2. $S_1(z)$ and $S_2(z)$ should be preserved at $X_1(z)$ and $X_2(z)$, respectively.

As discussed in Sec.3, the learning of the FB-BSS is equivalent to minimize the output powers. If the delay of $-C_{12}(z)H_{21}(z)$ is large enough compared to that of $H_{11}(z)$, the $S_1(z)$ components, transmitted through these functions, cannot be synchronized at $X_1(z)$, then they cannot be cancelled at $X_1(z)$. On the other hand, if the delay of $H_{12}(z)$ is large enough compared to that of $H_{22}(z)$, then $S_2(z)$ can be cancelled at $X_1(z)$. This means that the power of $Y_1(z)$ can be minimized by cancelling the $S_2(z)$ components at $X_1(z)$. Situation is the same as in minimizing the power of $Y_2(z)$. In these cases, $C_{kj}(z)$ converge to the optimal solutions given by Eq.(5).

When difference between the delays of $-C_{12}(z)H_{21}(z)$ and $H_{11}(z)$ is not large enough, correlation between $-C_{12}(z)H_{21}(z)S_1(z)$ and $H_{11}(z)S_1(z)$ will be generated. As a result, some cancellation between them can be possible at $X_1(z)$. In other words, the power of $Y_1(z)$ can be minimized by reducing not only the $S_2(z)$ component but also the $S_1(z)$ component. In this situation, $C_{kj}(z)$ are shifted from the optimal solutions given by Eq.(5) toward the undesirable solutions given by Eq.(7), resulting in poor source separation performances.

First, Condition 1 is considered. As shown in Fig.2, the transfer functions $C_{kj}(z)$ includes one sample delay, corresponding to $z^{-1}$. Therefore, in order to cancel $S_2(z)$ at $X_1(z)$, the difference between the delays of $H_{12}(z)$ and $H_{22}(z)$ should be equal to or larger than one sample delay. If this condition is not satisfied, $S_2(z)$ cannot be well cancelled. The same situation is held in $X_2(z)$.

Next, Condition 2 is considered. Preserving the $S_1(z)$ component in $X_1(z)$ is highly dependent on the difference between the delays of $-C_{12}(z)H_{21}(z)$ and $H_{11}(z)$. This point will be more analyzed in the next section.

4.2. Cancelation of $S_1(z)$ at $X_1(z)$

Here, the signal is simply denoted $u(n)$ for convenience. The sampling frequency of $u(n)$ is denoted $f_s$. In order to consider the delay difference less than the sampling period $T = 1/f_s$, the sampling frequency $f_s$ is converted to...
\( K f_s, K > 1 \). This up-sampling is carried out by inserting zero samples and band limitation. Let \( u_z(n) \) be the up-sampled signal by inserting \( K - 1 \) zero samples between the \( u(n) \) samples. The frequency component of \( u(n) \) is assumed to be distributed in \( 0 \sim f_s/m \). \( u_z(n) \) is band limited by using the ideal filter, whose pass band is \( 0 \sim f_s/m \) and the sampling frequency is \( K f_s \). An impulse response of this ideal filter is given by

\[
\phi(n) = \frac{2}{Km} \sin \left( \frac{n \pi}{Km} \right) \tag{9}
\]

\( \phi(n) \) is regarded as an interpolation function. Let the band limited signal be \( u_K(n) \), which is a convolution sum of \( u_z(n) \) and \( \phi(n) \) as shown below.

\[
u_K(n) = \sum_{k=0}^{n} \phi(n-k)u_z(k) \tag{10}
\]

Next, canceling \( u_K(n) \) by using \( u_K(n-l) \) will be discussed. In this case, the delay difference is \( l \) samples under the sampling frequency of \( K f_s \). We will consider cancelation of \( S_1(z) \) at \( X_1(z) \) depending on delay difference. Since amplitude and phase responses of the signals can be adjusted by \( C_{jk}(z) \), an effect of the delay difference is only considered in this section. For this reason, \( u_K(n) \) and \( u_K(n-l) \) can be regarded as \( h_{11}(n) * s_1(n) \) and \( c_{12}(n) * h_{21}(n) * s_1(n) \), respectively. The operation \(*\) is a convolution sum. Cancelation of \( u_K(n) \) by using \( u_K(n-l) \) can be expressed by

\[
u_K(n) = u_K(n-l) \]

\[
= \sum_{k=0}^{n} \phi(n-k)u_z(k) - \sum_{k=0}^{n} \phi(n-k-l)u_z(k) \]

\[
= \sum_{k=0}^{n} [\phi(n-k) - \phi(n-k-l)]u_z(k) \tag{11}
\]

From the above equation, the cancelation of \( u_K(n) \) can be evaluated by \( [\phi(n-k) - \phi(n-k-l)] \), which is equivalent to \( \phi(l) \). \( \phi(l) \) becomes zero at \( l = Km/2 \), and takes small value after that. Therefore, as a rule of thumb, the delay difference, with which \( u_K(n) \) can be cancelled by using \( u_K(n-l) \), is less than a \( Km/2 \) sample delay under the sampling frequency of \( K f_s \), which is equivalent to an \( m/2 \) sample delay with the \( f_s \) sampling frequency.

An example of \( \phi(n) \) is shown in Fig.3, in which \( m = 4 \) and \( K = 4 \). In this figure, one scale on the horizontal axis indicates \( T/K \) sec and \( 4 (= K) \) scales corresponds to \( T \) sec. \( \phi(n) \) becomes zero at 8 samples (sampled by \( K f_s \)), that is 2 samples (sampled by \( f_s \)), and takes small values after this point.

### 4.3. Effects of Fractional Propagation Delays

The propagation delay in the mixing process is not always an integer times of the sampling period \( T = 1/f_s \). It may be sometimes a fractional period. This means the transfer functions \( H_{ji}(z) \) in the mixing process cannot be expressed by a rational function of \( z^{-1} \). On the other hand, the FIR filters used in the separation block are implemented with the sampling frequency of \( f_s = 1/T \), and their transfer function are expressed by a power series of \( z^{-1} \). Therefore, if the propagation delay is not an integer times of \( T \), the FIR filter cannot realize the inverse function of the mixing process, as a result, the source separation performance is degraded. This point is also investigated through simulation.

### 5. OVERSAMPLING FEED-BACK BSS

#### 5.1. Relations between Conditions 1 and 2, and Propagation Delay

It is assumed that the sources are sampled by 8kHz. Thus, one sample delay is \( 1/8 \times 125 = 125 \mu \text{sec} \).

**Case 1:** <Delay of \( H_{ji}(z) \), \( i \neq j \) is equal to delay of \( H_{ii}(z) + 125/2 \mu \text{sec} \).> Condition 1 is satisfied, that is the \( S_2(z) \) components transmitted through two paths can be synchronized at \( X_i(z) \). Condition 2 for \( S_1(z) \) is also satisfied by the delay difference of at least two sample delay, that is \( 125 \mu \text{sec} \times 2 \) samples = 256sec. Correlation between \( s_1(n) \) and \( s_1(n-2) \) is small as shown in Fig.3.

**Case 2:** <Delay of \( H_{ji}(z) \), \( i \neq j \) is equal to delay of \( H_{ii}(z) + 125/2 \mu \text{sec} \).> Condition 1 is not satisfied. Because, the \( S_2(z) \) component transmitted through \( H_{ii}(z) \) is 1/2 sample ahead of the \( S_2(z) \) component transmitted through \( -C_{i2}(z) H_{ii}(z) \). They are not synchronized. On the other hand, Condition 2 is satisfied, because 1.5 sample delay 125+125/2sec for the adjacent samples is guaranteed.

**Case 3:** <Delay of \( H_{ji}(z) \), \( i \neq j \) is equal to delay of \( H_{ii}(z) \).> Condition 1 is not satisfied. The \( S_2(z) \) components cannot be synchronized at \( X_i(z) \). Condition 2 is still satisfied, because 1 sample delay can be used. However, for a narrow band signal, Condition 2 is weakly satisfied due to some correlation between the adjacent samples.

The above discussions are summarized in Table 1. In this table, ‘Delay differ’ means the delay of \( H_{ji}(z) \), \( i \neq j \) - the delay of \( H_{ii}(z) \). In the BSS, Condition 1 is more important, because the \( S_2(z) \) and \( S_1(z) \) components should be cancelled at \( X_1(z) \) and \( X_2(z) \), respectively.

#### 5.2. Effects of Oversampling Separation Block

By oversampling the separation block, the delay included in \( -C_{ij}(z) \), \( i \neq j \) can be decreased. For example, by increasing the sampling frequency to 16kHz, the delay is decreased to \( T/2 \) sec, that is \( 62.5 \mu \text{sec} \). Thus, satisfaction of Conditions 1 and 2 can be summarized in Table 2.
Table 1. Satisfaction of Condition 1 and 2 for Case 1, 2 and 3. T is one sample delay, that is 125µsec.

<table>
<thead>
<tr>
<th>Case</th>
<th>Delay differ</th>
<th>Condition 1</th>
<th>Condition 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>T</td>
<td>Satisfied</td>
<td>Satisfied</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.5T</td>
<td>Not satisfied Synchronized</td>
<td>Satisfied 1.5T delay</td>
</tr>
<tr>
<td>Case 3</td>
<td>0</td>
<td>Not satisfied T ahead</td>
<td>Satisfied T delay</td>
</tr>
</tbody>
</table>

Table 2. Satisfaction of Condition 1 and 2 for Case 1, 2 and 3. Separation block is oversampled by 16kHz. T=125µsec.

<table>
<thead>
<tr>
<th>Case</th>
<th>Delay differ</th>
<th>Condition 1</th>
<th>Condition 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>T</td>
<td>Satisfied</td>
<td>Satisfied</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.5T</td>
<td>Satisfied</td>
<td>Satisfied 1.5T delay</td>
</tr>
<tr>
<td>Case 3</td>
<td>0</td>
<td>Not satisfied T ahead</td>
<td>Weakly satisfied 0.5T delay</td>
</tr>
</tbody>
</table>

In Case 2, Condition 1 is satisfied, because the \( S_2(z) \) components transmitted through two paths can be synchronized at \( X_1(z) \). Condition 2 is still satisfied except for Case 3. Therefore, the source separation performance can be improved for short delay difference in the convolutive mixing process.

In Case 3, Condition 2 is weakly satisfied. Since the distance between the adjacent two samples is reduced to 0.5 sample delay, the correlation between the adjacent samples is increased. This makes cancellation of \( S_1(z) \) at \( X_1(z) \) easy.

6. SIMULATIONS AND DISCUSSIONS

6.1. Simulation Setup

2-channel and 3-channel models are used. Simulation results only for the 2-channel model are demonstrated due to page limitation. The sampling frequency is \( f_s = 8kHz \). White signals and speech signals are used as the sources. Figure 4 shows a 2-channel mixing process.

![Fig. 4. Mixing process with delay difference.](image)

The delay difference between \( H_{ij}(z) \) and \( H_{ji}(z), j \neq i \) is realized by \( \tau \). Two kinds models for \( H_{ji}(z) \) are used. One of them is an instantaneous mixing process shown in Eq.(12) and the other is an approximately actual room environment. The source separation is evaluated by the Signal-to-Interference Ratio (SIR) given by Eq.(15). \( A_{ki}(z) \) is a transfer function from the \( i \)th signal source to the \( k \)th output of the separation block.

\[
H(z) = \begin{bmatrix} 1 & 0.9 \\ 0 & 1 \end{bmatrix}
\]
\[
\sigma^2_s = \frac{1}{2\pi} \sum_{n=1}^{N} \int_{-\pi}^{\pi} |A_{ki}(e^{j\omega})S_i(e^{j\omega})|^2 d\omega
\]
\[
\sigma^2_i = \frac{1}{2\pi} \sum_{n=1}^{N} \sum_{n=1}^{N} \int_{-\pi}^{\pi} |A_{ki}(e^{j\omega})S_i(e^{j\omega})|^2 d\omega
\]
\[
SIR = 10 \log_{10} \frac{\sigma^2_i}{\sigma^2_s} [dB]
\]

The learning algorithms proposed for the FF-BSS [2],[6],[7], and for the FB-BSS [1],[4] were applied. Both the FF-BSS and the FB-BSS are implemented in the time domain.

6.2. Source Separation for 8kHz Sampling

Figure 5 shows the SIR for the white signal sources and the speech signal sources. The sampling frequency of the separation block is 8kHz. "Delay difference" means \( \tau \) in Fig.4.

![Fig. 5. SIR with respect to \( \tau \) for (Upper) white signal sources with instantaneous and delay mixture and (Lower) speech signal sources with room acoustic mixture.](image)

\( C_{12}(z) \) and \( C_{21}(z) \) have one sample delay as shown in Fig.2.

In Fig.5(Upper), SIR has peaks at every \( \tau = 125\mu s \). At the first 125µs, Condition 1 is satisfied, and good separation is obtained. At the following every 125µs, the mixing process can be expressed as a rational function of \( z^{-1} \) \( (f_s = 8kHz) \), whose inverse function can be approximated by the separation block, which is implemented with the 8kHz sampling frequency. In Fig.5(Lower), the peaks do not appear, because SIR is lower compared with the white source case. SIR is also going up as the delay difference is increased. In both cases, the cross point of SIRs by FF-BSS and FB-BSS is almost 100µs, which is theoretically analyzed in Sec.5.1 and is shown in Table 1. As \( \tau \) decreases under \( T \), Condition 1 is not gradually satisfied.
6.3. Separation Performance by Oversampling FB-BSS
The sampling frequency is increased to 16kHz. The white signal sources and the instantaneous and delay mixture are used. Figure 6 shows SIR, where $\times 1$ and $\times 2$ mean that the sampling frequency is 8kHz and 16kHz, respectively. From the upper figure, SIR can be drastically increased for $\tau = 30 \sim 100\mu\text{s}$. This means the oversampling FB-BSS can be applied to small delay difference in the mixing process. It can relax limitation on location of the sources and the sensors. Figure 6(Lower) shows SIR during $\tau = 625 \sim 735\mu\text{s}$. SIR can be also improved. This means the transfer function, which is a power series of $z^{-1/2}$, can approximate $(nT + \text{fractional})$ delay in the mixing process more accurately.

![Fig. 6. SIR with respect to $\tau$ for oversampling FB-BSS. (Upper) $\tau = 0 \sim 110\mu\text{s}$. (Lower) $\tau = 625 \sim 735\mu\text{s}$.](image)

6.4. Comparison between FF-BSS and FB-BSS
The FF-BSS is not affected by the delay difference $\tau$, and SIR is almost constant. The frequency domain FF-BSS has been simulated by using the distortion free learning algorithm [6,[7]. SIR for the white signal sources is almost the same as in the time domain shown in Fig.5(Upper). In the case of speech signal sources and the room acoustic mixture, SIR is $12 \sim 14\text{dB}$ for $\tau = 0 \sim 125\mu\text{s}$. Although SIR can be improved from that of the time domain FF-BSS as shown in Fig.5(Lower), still it is lower than that of the FB-BSS for $\tau > 160\mu\text{s}$. Furthermore, the oversampling FB-BSS can provide higher SIR for small $\tau$. The same property as shown in Fig.6 can be realized for the speech sources and the room acoustic mixture.

The simulation results for the 3-channel model are similar to those of the 2-channel model.

6.5. Constraint on Location of Signal Sources and Sensors.
A relation between the delay difference and location of the sources and the sensors is investigated based on sound propagation time. A layout of them shown in Fig.7(Left) is used. Relations of the delay differences and the layouts are shown in Fig.7(Right).

Fig. 7. (Left) Layout of signal sources and sensors. (Right) Delay difference $\tau$ depending on layout of signal sources and sensors.

7. CONCLUSIONS
The source separation performance of the FB-BSS is theoretically analyzed based on the propagation delay difference in the mixing process. Based on this analysis, an oversampling FB-BSS is proposed. Simulation results demonstrate the proposed method can drastically improve SIR.

8. REFERENCES