IMPROVED PNLMS ALGORITHM EMPLOYING WAVELET TRANSFORM AND SPARSE FILTERS

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ABSTRACT

The proportionate normalized least mean-square algorithm (PNLMS) has been proposed with the objective of improving the adaptation convergence rate when modeling high-order sparse finite impulse response systems. Whereas fast initial adaptation convergence rate is obtained with the PNLMS algorithm for white-noise input, slow convergence is observed for colored input signals. In this paper, we derive a new proportionate-type NLMS algorithm which employs a wavelet transform and sparse adaptive subfilters, and results in better convergence rate than the PNLMS algorithm for colored input signals. Simulation results for the digital network echo canceler application illustrate the convergence improvement obtained with the proposed approach when compared to the NLMS, PNLMS and other recently proposed proportionate-type algorithms.

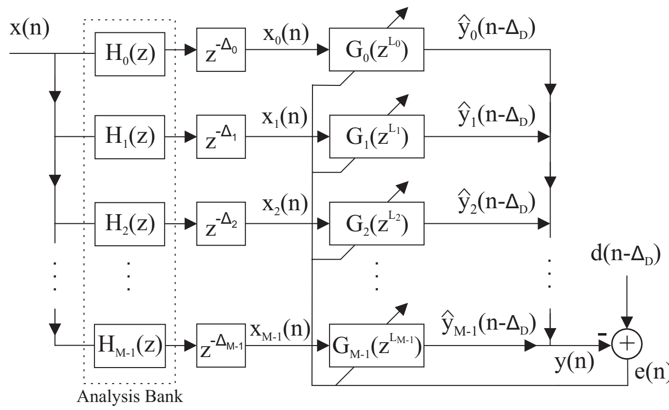

Figure 1: Adaptive subband structure composed of a wavelet transform and sparse subfilters.

1. INTRODUCTION

It is well known that the convergence of adaptive filtering algorithms becomes slow when the number of coefficients is very large. However, in many applications, such as digital networks and acoustical echo cancelers, the system being modeled has a sparse impulse response, that is, most of its coefficients have small magnitudes. The conventional adaptation techniques, such as the least-mean-square (LMS) and recursive least-squares (RLS) algorithms, do not take into account the sparseness characteristics of such systems. To improve the convergence for these applications, several methods have been proposed recently, which employ individual step-sizes for the updating of different coefficients. The adaptation step-sizes are made larger for the coefficients with larger magnitudes, resulting in a faster convergence for the most significant coefficients. This idea was first introduced in [1], resulting in the so-called proportionate NLMS (PNLMS) algorithm. Improved versions of such a procedure were presented in [2], [3]. Even though the initial convergence and the tracking abilities of such methods are improved when compared to the LMS and NLMS algorithms, the also well-known slow convergence of the gradient-based adaptation procedures for colored input signals is not eased by such approaches.

In this paper, we combine the ideas of the PNLMS algorithm, for improving the adaptation convergence rate when modeling sparse impulse responses, and of the transform-domain adaptive algorithm, for accelerating the convergence for colored input signals. The proposed method employs a wavelet transform to decompose the input signal and sparse adaptive subfilters. For the updating of the adaptive coefficients, the step-size normalization is performed separately for each subfilter, taking into account the magnitudes of its coefficients and the spectral characteristics of the input signal. A transform-domain PNLMS-type algorithm, which also applies a wavelet transform of the input signal, was independently proposed in [4]. However, the input signal decomposition as well as the step-size normalization strategy employed in our method are different from those of [4], as will be explained in this paper. The convergence improvement obtained with the proposed method for modeling sparse systems with colored input signals is verified through computer simulations for the identification of the channel impulse responses in the digital network echo cancellation application.

2. ADAPTIVE STRUCTURE WITH WAVELET AND SPARSE FILTERS

The algorithm proposed in this paper employs an adaptive structure composed of a wavelet transform and sparse adaptive filters [5]. Such a structure is illustrated in Fig. 1, where $x(n)$ is the input signal, $d(n)$ is the desired signal, and $e(n)$ is the error signal used in the adaptation algorithm. The wavelet transform is represented by the equivalent non-uniform filter bank with analysis filters $H_k(z)$, and $G_k(z^{L_k})$ are the sparse adaptive subfilters.

The development of the proposed adaptation algorithm will be carried out for octave-band wavelet decomposition, which are frequently employed in the analysis of sound signals. This choice was made due to the lowpass characteristics of the signals commonly encountered in the applications of echo cancellation and channel equalization. However, the results can be easily extended to other types of wavelet decompositions (wavelet packets). For an octave-band wavelet, the equivalent analysis filters of the M-channel filter bank are [6]

$$
\begin{aligned}
H_0(z) &= \prod_{j=0}^{M-2} H^0(z^{2^j}), \\
H_k(z) &= H^1(z^{2^{M-1-k}}) \prod_{j=0}^{M-k-2} H^0(z^{2^j}), \\
& \qquad k = 1, \cdots, M-1,
\end{aligned} \tag{1}
$$

where $H^0(z)$ and $H^1(z)$ are the lowpass and high-pass filters, respectively, associated with the wavelet functions [6]. The orders of the analysis filters are given by

$$
\begin{aligned}
N_{H_0} &= \sum_{j=0}^{M-2} 2^j N_{H^0}, \\
N_{H_k} &= \sum_{j=0}^{M-k-2} 2^j N_{H^0} + 2^{M-k-1} N_{H^1}, \\
& \qquad k = 1, \cdots, M-1,
\end{aligned} \tag{2}
$$

where N_{H^0} and N_{H^1} are the orders of $H^0(z)$ and $H^1(z)$, respectively. The sparsity factors are

$$L_0 = 2^{M-1}, \quad L_k = 2^{M-k}, \quad k = 1, \cdots, M-1, \tag{3}$$

and the delays Δ_k in Fig. 1, introduced for the purpose of matching the delays of the different length analysis filters, are given by

$$\Delta_k = N_{H_0} - N_{H_k}. \tag{4}$$

This structure results in an additional system delay (compared to a direct-form FIR structure) equal to

$$\Delta_D = N_{H_0}. \tag{5}$$

It has been shown in [5] that the wavelet structure of Fig. 1 can exactly model any FIR system. For the modeling of a length N_S FIR system, the number of adaptive coefficients of the subfilters $G_k(z)$ (non-zero coefficients of $G_k(z^{L_k})$) should be at least

$$N_k = \left\lfloor \frac{N_S + N_{F_k}}{L_k} \right\rfloor + 1 \tag{6}$$

where N_{F_k} are the orders of the corresponding synthesis filters which, when associated to the analysis filters $H_k(z)$, yield perfect reconstruction.

3. WAVELET-BASED PNLMS ALGORITHM

For an adaptive filter with coefficients $w_i(n)$, for $1 \leq i \leq N-1$, the proportionate normalized least mean-square (PNLMS) algorithm is given in Table 1. The PNLMS algorithm employs a different step-size for each coefficient such that larger adjustments are applied to the larger coefficients (or active

Table 1: PNLMS Algorithm

$$
\begin{aligned}
&\mathbf{x}(n) = [\; x(n) \quad x(n-1) \quad \cdots \quad x(n-N+1) \;]^T \\
&\mathbf{w}(n) = [\; w_0(n) \quad w_1(n) \quad \cdots \quad w_{N-1}(n) \;]^T \\
&y(n) = \mathbf{x}^T(n)\mathbf{w}(n) \\
&e(n) = d(n) - y(n) \\
&\gamma_{\min}(n+1) = \rho \max\{\delta_p, |w_0(n)|, \cdots, |w_{N-1}(n)|\} \\
&\text{For } i = 0, 1, \cdots, N-1 \\
&\quad \gamma_i(n+1) = \max\{\gamma_{\min}(n+1), |w_i(n)|\} \\
&\text{End} \\
&\text{For } i = 0, 1, \cdots, N-1 \\
&\quad g_i(n+1) = \frac{\gamma_i(n+1)}{\frac{1}{N}\sum_{j=0}^{N-1} \gamma_i(n+1)} \\
&\text{End} \\
&\mathbf{\Gamma}(n+1) = \operatorname{diag}\{g_0(n+1), \cdots, g_{N-1}(n+1)\} \\
&\mathbf{w}(n+1) = \mathbf{w}(n) + \frac{\beta \mathbf{\Gamma}(n+1)\mathbf{x}(n)e(n)}{\mathbf{x}^T(n)\mathbf{\Gamma}(n+1)\mathbf{x}(n) + \delta}
\end{aligned}
$$

coefficients), resulting in faster convergence rate when modeling systems with sparse impulse responses [1]. In Table 1, the parameter β is a fixed step-size factor, δ is a small constant needed in order to avoid division by zero, and δ_p and ρ are small positive constants which are important when all the coefficients are zero (such as in the beginning of the adaptation process) or when a coefficient is much smaller than the largest one. Typical values of these constants are $\delta_p = 0.01$ and $\rho = 0.01$.

Although the PNLMS algorithm presents better convergence than the NLMS algorithm for sparse impulse responses, its performance is degraded when the excitation signal is colored. In order to improve the adaptation speed of the PNLMS algorithm for colored input signals, we employ the adaptive wavelet structure of Fig. 1 to derive the wavelet-based proportionate normalized least mean square (WPNLMS) algorithm presented in Table 2. In this table, $x_k(n)$ is the input signal at k-th subband ($x(n)$ filtered by $H_k(z)$) and $w_{k,i}$ is the i-th coefficient of $G_k(z)$. The step-size normalization of the coefficients of a given subfilter is performed separately from the other subfilters coefficients, taking into account the coefficients values and the input signal power of that subfilter. For colored input signals, the proposed algorithm presents faster convergence than the NLMS and PNLMS algorithms, since its step-size normalization strategy takes into account the input power at the different frequency bands.

A feasible choice for the function $\mathrm{F}(\cdot)$ in Table 2 is $\mathrm{F}(x) = x$ for the PNLMS algorithm, $\mathrm{F}(x) = \ln(1+\mu x)$ for the μ-law PNLMS algorithm (MPNLMS) [3], or $\mathrm{F}(x) = 600x$ if $x < 0.005$ and $\mathrm{F}(x) = 3$ if $x \geq 0.005$ for the segmented PNLS (SPNLMS) algorithm [3]. The MPNLMS algorithm has been derived in order to achieve faster overall convergence through the use of specially chosen step-size control factors, while the SPNLMS algorithm is a simplified version of the MPNLMS algorithm where the logarithm function is approximated by linear segments.

Table 2: Generalized Wavelet-based PNLMS Algorithm

For $k=0,1,\cdots,M-1$

$\mathbf{x}_k(n)=[\ x_k(n)\ \ x_k(n-L_k)\ \ \cdots\ \ x_k(n-(N_k-1)L_k)\ ]^T$

$\mathbf{w}_k(n)=[\ w_{k,0}(n)\ \ w_{k,1}(n)\ \ \cdots\ \ w_{k,N_k-1}(n)\ ]^T$

$\hat{y}_k(n-\Delta_D)=\mathbf{x}_k^T(n)\mathbf{w}_k(n)$

End

$y(n)=\sum_{k=0}^{M-1}\hat{y}_k(n-\Delta_D)$

$e(n)=d(n-\Delta_D)-y(n)$

For $k=0,1,\cdots,M-1$

$\gamma_{\min,k}(n+1)=\rho\max\{\delta_p,\mathrm{F}(|w_0(n)|),\cdots,\mathrm{F}\left(|w_{N_k-1}(n)|\right)\}$

For $i=0,1,\cdots,N_k-1$

$\gamma_{k,i}(n+1)=\max\{\gamma_{\min,k}(n+1),\mathrm{F}(|w_i(n)|)\}$

End

For $i=0,1,\cdots,N_k-1$

$$g_{k,i}(n+1)=\frac{\gamma_{k,i}(n+1)}{\frac{1}{N_k}\sum_{j=0}^{N_k-1}\gamma_{k,j}(n+1)}$$

End

$\mathbf{\Gamma}_k(n+1)=\mathrm{diag}\{g_{k,0}(n+1),\cdots,g_{k,N_k-1}(n+1)\}$

$$\mathbf{w}_k(n+1)=\mathbf{w}_k(n)+\frac{\beta\mathbf{\Gamma}_k(n+1)\mathbf{x}_k(n)e(n)}{\mathbf{x}_k^T(n)\mathbf{\Gamma}_k(n+1)\mathbf{x}_k(n)+\delta}$$

End

It should be observed that the proposed step-size normalization strategy takes into account the value of each coefficient when compared to the values of the corresponding subfilter coefficients (and not of all coefficients, as is done in [4]). In the next section it will be seen that the proposed step-size normalization method results in improved convergence when compared to that of [4]. Another difference between the proposed algorithm and the transform-domain wavelet algorithm of [4] is in the application of the wavelet transform to the input signal. In [4], the input vector $\mathbf{x}(n)$ is transformed to $\mathbf{x}_T(n)=\mathbf{T}\mathbf{x}(n)$, with $\mathbf{T}$ formed according to the selected wavelet functions. Efficient implementations of such vector transformation are not straightforward. In the method proposed here, however, a tree-structure analysis filter bank can be used to decompose the input signal, requiring $(M-1)(N_{H^0}+N_{H^1})$ multiplications. For example, for a two-level decomposition ($M=3$) and daubechies 2 (Db2) wavelet functions ($N_{H^0}=N_{H^1}=4$), 16 multiplications (per each new input sample) would be required for the input signal decomposition. One good thing when using [4] is that it does not introduce delay.

4. EXPERIMENTAL RESULTS

Computer simulations are presented to illustrate the convergence behavior of the WPNLMS algorithm investigated in this paper for different wavelets, and to compare the results with the ones produced by the NLMS and PNLMS algorithms.

The identification of systems with sparse impulse responses, corresponding to the digital network channels of [7], using an adaptive filter with 512 coefficients was considered. The input signal was generated by passing a white gaussian noise with zero-mean and unit variance through the filter with transfer function

$$H(z)=\frac{0.25\sqrt{3}}{1-1.5z^{-1}+z^{-2}-0.25z^{-3}} \tag{7}$$

which results in a signal with power spectrum similar to speech signal [8]. White noise with variance $\sigma_v^2=10^{-6}$ was added to the desired signal. In all simulations, the parameters were set to $\rho=0.01$, $\delta_p=0.01$, $\delta=0.01$, while β was selected (empirically) in order to obtain the fastest convergence for each algorithm.

Figure 2 presents the experimental MSE evolution of the WPNLMS algorithm (with $\mathrm{F}(x)=x$) for the wavelet Biorthogonal 4.4 having 1, 2 and 3 levels of decomposition (which correspond to $M=2$, 3 and 4 subbands, respectively), with the channel model *gm*7 from [7]. Also shown in Fig. 2 are the MSE evolutions of the NLMS and PNLMS algorithms. As can be observed, the WPNLMS algorithm has a much better performance than the NLMS and PNLMS ones. The use of $M=3$ subbands is sufficient, in practice, to decorrelate the colored input signal applied in this experiment. The small degradation in convergence speed verified in the simulation results when introducing one more decomposition level ($M=4$ subbands) was caused by the increase in the total number of adaptive coefficients in relation to $M=2$ and $M=3$ subbands. Such an increase in the number of coefficients is negligible in the case of modeling very large order systems.

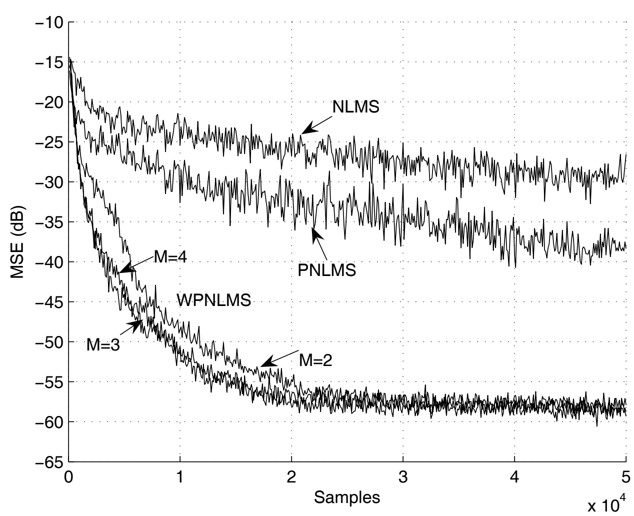

Figure 2: MSE evolution for the WPNLMS algorithm for Biorthogonal 4.4 wavelet with 1, 2 and 3 levels of decompositions, and for the NLMS and PNLMS algorithms.

The performance of the proposed WPNLMS algorithm is now examined for two-level decomposition ($M=3$ subbands) using the following wavelet functions: Daubechies 1 (Db1), Daubechies 4 (Db4), Biorthogonal 2.4 (Bior2.4), Biorthogonal 4.4 (Bior4.4) and Coiflet 4 (Coif4). With such wavelets, the increase in the complexity (compared to the PNLMS algorithm) and the delay introduced by the decomposition are not very large. Figure 3 shows the experimental

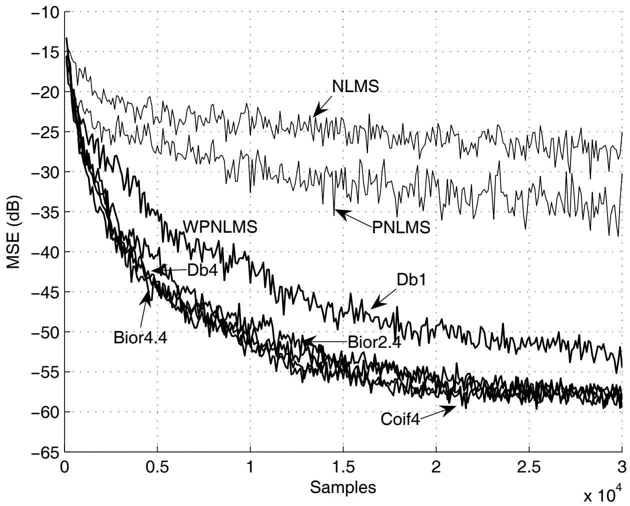

Figure 3: MSE evolution for the WPNLMS algorithm with different wavelet functions ($M = 3$), and for the NLMS and PNLMS algorithms.

MSE evolution of the WPNLMS algorithm for the different wavelets as well as of the NLMS and PNLMS algorithms. The plots show that Bior4.4, Coif4 and Db4 presented very similar performances, while Db1 produced the slowest convergence rate. Such results were already expected, since the Db1 wavelet has less selective frequency characteristics than the other wavelets.

Figure 4 displays the MSE evolution for the NLMS, PNLMS and WPNLMS (with two-level decomposition and Bior4.4) when the impulse response is abruptly changed from *gm*1 to *gm*7 (given in [7]). It can be observed that the proposed algorithm is able to track the impulse response change much faster than the NLMS and PNLMS algorithms.

We now compare the performance of the proposed algorithm with that of [4]. Figure 5 shows the MSE evolution of the wavelet-based PNLMS and SPNLMS algorithms with sparse filters (SF) proposed in this paper and the transform-domain (TD) approach presented in [4] for the channel model *gm*3 from [7]. In these simulations we have employed the 2-level Haar wavelet transform in the proposed method (in order to keep the computational and memory requirements low) and the 9-level Haar wavelet transform in the transform-domain method (as used in the simulations presented in [4]). From this figure we observe that the proposed method with the very simple Haar (or Db1) wavelet transform results in a significant improvement in the convergence rate of the PNLMS and SPNLMS algorithms. It can also be observed that the step-size normalization strategy adopted in the proposed method is advantageous when compared to that of [4], resulting in faster convergence.

Figure 6 presents the learning curves for the SF and TD WSPNLMS algorithms with Haar (Db1), Db2 and Db4 wavelet transforms, obtained with the channel model *gm*4. It can be seen that for the colored input signal employed

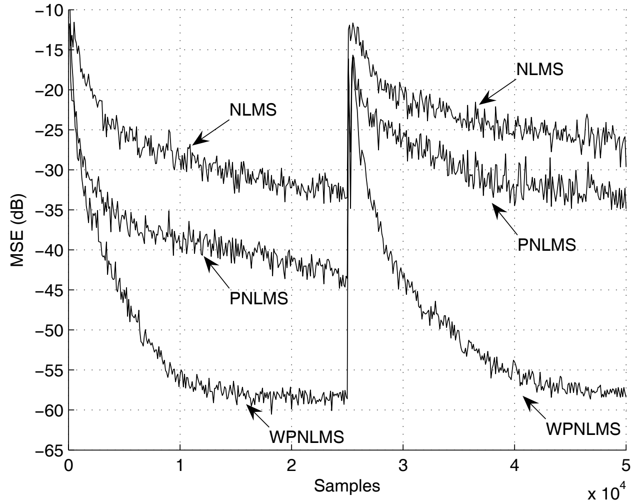

Figure 4: MSE evolution for the NLMS, PNLMS and WPNLMS algorithms for an abrupt change in the channel coefficients.

in the simulations, significantly faster convergence was obtained with the more selective Db2 wavelet (when compared to the use of Haar wavelet), while similar convergence rates were obtained with Db2 and Db4 wavelets.

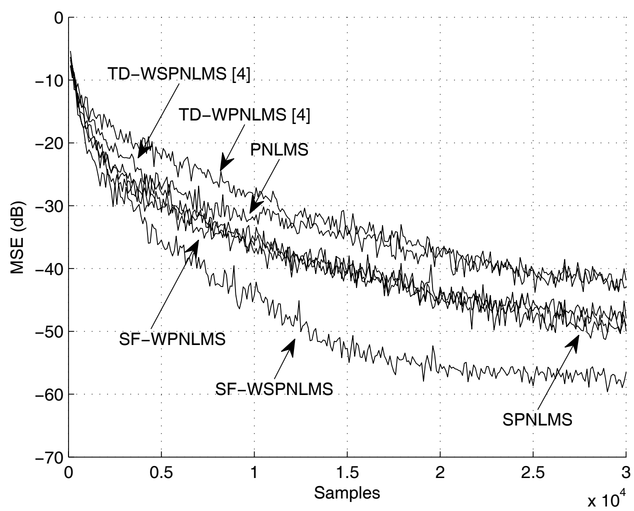

Figure 5: MSE evolution of the sparse filters (SF) and transform-domain (TD) WPNLMS and WSPNLMS algorithms with Haar wavelet.

5. CONCLUSIONS

In this paper we presented a novel proportionate adaptive algorithm that employs the wavelet transform and sparse subfilters. The step-size normalization takes into account the value of each subfilter coefficient as well as the input signal power in the corresponding frequency band. Simulations showed that the proposed method presents significantly faster convergence rate than do the NLMS and several recently pro-

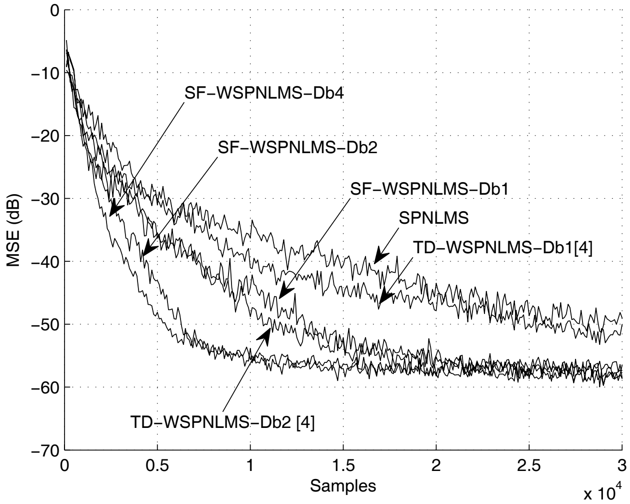

Figure 6: MSE evolution of the SF and TD WSPNLMS algorithms with different wavelet functions.

posed PNLMS-type algorithms, for applications in which the system has sparse impulse responses and is excited with colored input signal, such as in network echo cancellation structures.

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