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BLIND SOURCE SEPARATION OF CONVOLUTIVE MIXTURES OF NON CIRCULAR LINEARLY MODULATED SIGNALS WITH UNKNOWN BAUD RATES.

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ABSTRACT

This paper is devoted to the blind separation of convolutive mixtures of possibly non circular linearly modulated signals with unknown (and possibly different) baud rates and carrier frequencies. In this context, the received signal is sampled at any rate satisfying the Shannon sampling theorem. The corresponding discrete-time signal is cyclostationary with unknown cyclic frequencies. It is shown that if the various source signals do not share any cyclic frequency, the local minima of the constant modulus cost function are separating filters. In contrast with the circular sources case, the minimization of the Godard cost function in general fails if non circular sources have the same rates and the same carrier frequencies. It is shown that this is due to the existence of non separating local minima of the Godard cost function. If the frequency offsets of the sources are available at the receiver side, a simple modification of the Godard criterion is proposed. It achieves the separation of any non circular linearly modulated signals sources. The results of this paper show that the separation of unknown non circular linearly modulated signals is possible only if their frequency offsets can be blindly estimated prior to the separation scheme.

1. INTRODUCTION

The blind source separation of convolutive mixtures of linearly modulated signals has mainly be studied in the case where the signals share the same known baud rate, and when the sampling frequency of the multivariate received signal coincides with this baud-rate. In this context, to be refered to in the sequel as the *stationary case*, the discrete-time received signal coincides with the output of an unknown MIMO filter driven by the sequences of symbols sent by the various transmitters. In most cases, these sequences are independent and identically distributed, and several methods have been proposed in order to extract each of them from the observation. The source separation problems that are met in the context of passive listening are however more complicated because the transmitters are in general completely unknown from the receiver, and have no reason to transmit linearly modulated signals sharing the same baud-rates. It is therefore quite relevant to address the problem of blind separation of linearly modulated signals with unknown, and possibly different, baud rates. In this context, the received signal is sampled at any frequency satisfying the Shannon sampling theorem, so that the corresponding discrete-time signal is cyclostationary with unknown cyclic frequencies. If the cyclic frequencies were known at the receiver side, it would be easy to generalize the usual blind source separation approaches based on the optimization of contrast functions depending on higher order cumulants. However, when the cyclic frequencies are unknown, it is impossible to estimate consistently the cumulants, a conceptual problem first remarked by Ferreol and Chevalier ([3]) in the context of blind separation of instantaneous mixtures. An obvious approach would consist in estimating the unknown cyclic frequencies. However, this is a difficult task if the excess bandwidths of the transmitted signals are low and if the duration of observation is not large enough.

The minimization of the Godard constant modulus cost function allows to achieve source separation in the stationary context ([6]). In contrast with the cumulants, the constant modulus cost function can be consistently estimated in the cyclostationary context. In [4], we have shown that if the source signals transmit second-order circular symbol sequences, then, the minimization of the Godard cost function allows to extract the sources using a deflation approach if their baud-rates are different one from each others. If certain baud rates coincide, sufficient conditions for the separation have been established in [5]. Although we have not been able to prove that separation is achieved in the most general case, all the simulation we have performed strongly suggest that the minimization of the Godard cost function is successfull in the circular case. The purpose of this paper is to address this issue in the case of possibly non circular source signals. In order to simplify the presentation of our results, we just consider the case where the non circular signals are BPSK signals. In the non circular case, the so-called non conjugate cyclic frequencies, whose values depend on the carrier frequency offsets and the baud rates of the source signals, play an important role. We prove that the Godard cost function is still successful if the sources do not share the same cyclic and non conjugate cyclic frequencies. In contrast with the circular case, we however prove that the minimization of the Godard cost function fails to separate 2 BPSK signals sharing the same baud rate and the same carrier frequency. This is because the Godard cost function shows non separating local minima, and the minimization algorithms seem to converge quite often towards these points. However, we show that if the carrier frequency offsets of the transmitted signals are known or well estimated, then it is possible to modifify the Godard cost function in order to achieve source separation of any usual non circular linearly modulated signals. Our results thus show that in order to separate non circular linearly modulated signals, it is necessary to estimate their frequency offsets prior to the source separation. Fortunately, this is a much easier task than the estimation of baud rates, because the non conjugate cyclic correlation coefficents of the received signal at twice the frequency offsets are not affected by possible low excess bandwidths of the source signals (see [1]).

General notations: If $(u_n)_{n\in\mathbb{Z}}$ is a discrete-time sequence, we denote by $< u_n >$ the time average operator defined as

$$< u_n >= \lim_{N\to+\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} u_n$$

If $(x(n))_{n\in\mathbb{Z}}$ is a discrete-time cyclostationary sequence, we denote by $R_x^{(\alpha)}(\tau)$ the cyclic correlation coefficient at lag τ of signal x at cyclic frequency α, and by $S_x^{(\alpha)}(e^{2i\pi\nu})$ the corresponding cyclic spectrum. If x is non circular, the sequence $n \to \mathbb{E}(x(n+\tau)x(n))$ is non identically zero, and coincides with a sum of sinusoids. The frequencies of the corresponding Fourier-like expansion are called the non conjugate cyclic frequencies of x, and the related coefficients, denoted $R_{c,x}^{(\gamma)}(\tau)$ for each frequency γ, are called the non conjugate cyclic correlation coefficients.

2. PROBLEM STATEMENT.

We assume that K unknown transmitters send linearly modulated signals sharing the same frequency bandwidth. The receiver is equipped with a sensor of M–arrays, and the corresponding M–dimensional received signal is sampled at period T_e supposed to satisfy the Shannon sampling theorem. For any k, $k = 1,\dots,K$, the k-th transmitted signal is obtained by linearly modulating a unit variance zero mean i.i.d. sequence of symbols. In order to simplify the presentation of the results, each symbol sequence is assumed either second-order circular or binary (i.e. equal to ± 1 and therefore not circular). The corresponding symbol period is denoted by T_k, and it is assumed that the bandwidth of the complex envelope of the k-th transmitted signal is $[-\frac{1+\gamma_k}{2T_k}, \frac{1+\gamma_k}{2T_k}]$ where the so-called excess bandwidth factor γ_k belongs to $[0,1)$. The propagation channels between each transmitter and the receiver are assumed to be frequency selective. Moreover, the carrier frequencies of the various transmitted signals of course do not coincide with the center frequency of the receive filter of the receiver. Hence, the contribution of each transmitted signal at the receiver side is corrupted by a frequency offset. The frequency offset associated to source k is denoted Δf_k. The continuous-time received signal is sampled at a period T_e which is supposed to verify $T_e < \min_k \frac{T_k}{1+\gamma_k} + |\Delta f_k|^{-1}$. Under these assumptions, the M-dimensional discrete-time received signal $\mathbf{y}(n)$ can be written as

$$\mathbf{y}(n) = \sum_{k=1}^{K} e^{2i\pi n\delta f_k}\left(\sum_l \mathbf{h}_{k,l} s_k(n-l)\right) = \sum_{k=1}^{K} e^{2i\pi n\delta f_k}[\mathbf{h}_k(z)]s_k(n) \quad (1)$$

where for each k, $s_k(n)$ represents the sampled version of the $k-th$ transmitted signal, $\mathbf{h}_k(z) = \sum_{l\in\mathbb{Z}} \mathbf{h}_{k,l} z^{-l}$ is the transfer function of the 1-input / M outputs discrete time equivalent channel between transmitter k and the receiver, and δf_k is defined as $\delta f_k = \Delta f_k T_e$.

Each signal s_k is second order cyclostationary and its cyclic frequencies are $0, \alpha_k, -\alpha_k$ where $\alpha_k = \frac{T_e}{T_k}$. If s_k is a BPSK signal, the sequence $n \to \mathbb{E}(s_k(n+\tau)s_k(n))$ is non identically zero, and its non conjugate cyclic frequencies are also equal to $0, \alpha_k, -\alpha_k$. In the following, we assume without restriction that source signals $(s_k)_{k=1,\dots,K}$ are normalized in such a way that $R^{(0)}_{s_k}(0) =< \mathbb{E}|s_k(n)|^2 >= 1$ for each k.

In this paper, we study source separation methods based on the so-called deflation approach: one of the source signal is first extracted, its contribution is cancelled from the observation, and the procedure is iterated until extraction of all the sources. We therefore focus on the first step. In order to extract one of the source signal, $(\mathbf{y}(n))_{n\in\mathbb{Z}}$ is filtered by a M–inputs / 1–output filter $\mathbf{g}(z)$ to produce the scalar signal $r(n) = [\mathbf{g}(z)]\mathbf{y}(n)$. The filter $\mathbf{g}(z)$ is designed in such a way that $r(n)$ coincides with a filtered version of one of the source signals $(s_k)_{k=1,\dots,K}$. In the following, we denote by I and I_c the set of all cyclic and non conjugate cyclic frequencies of r. In other words, I and I_c are given by $I = \{0, (\pm\alpha_k)_{k=1,\dots,K}\}$ and $I_c = \{(2\delta f_k \pm \alpha_k)_{k=1,\dots,K}\}$. I^* is defined as $I - \{0\}$. If source k is circular, then, the non conjugate cyclic correlation coefficients of r at non conjugate cyclic frequency $2\delta f_k \pm \alpha_k$ are of course equal to 0.

The Godard cost function $J(r)$ is defined as $J(r) =< \mathbb{E}\left(|r(n)|^2 - 1\right)^2 >$, and can of course be written as $J(r) =< \mathbb{E}|r(n)|^4 > -2 < \mathbb{E}|r(n)|^2 > +1$. Using the relation $\mathbb{E}|r(n)|^4 = c_4(r(n)) + 2\left(\mathbb{E}|r(n)|^2\right)^2 + \left|\mathbb{E}(r^2(n))\right|^2$ and the Parseval identities $< \left(\mathbb{E}|r(n)|^2\right)^2 >= \sum_{\alpha\in I} |R^{(\alpha)}_r(0)|^2$ and $< \left|\mathbb{E}(r(n))^2\right|^2 >= \sum_{\alpha\in I_c} |R^{(\alpha)}_{c,r}(0)|^2$, we get immediately that

$$J(r) =< c_4(r(n)) > +2\sum_{\alpha\in I} |R^{(\alpha)}_r(0)|^2 + \sum_{\alpha\in I_c} |R^{(\alpha_c)}_{c,r}(0)|^2 - 2R^{(0)}_r(0) + 1 \quad (2)$$

In order to express $J(r)$ in a more convenient way, we remark that $r(n)$ can be written as

$$r(n) = \sum_{k=1}^{K} e^{2i\pi n\delta f_k}[f_k(z)]s_k(n) \quad (3)$$

where $f_k(z)$ is the transfer function $f_k(z) = \mathbf{g}(ze^{-2i\pi\delta f_k})\mathbf{h}_k(z)$. We denote by $\|f_k\|$ the norm of filter $f_k(z)$ defined by $||f_k||^2 = \int_{-1/2}^{1/2} |f_k(e^{2i\pi\nu})|^2 S^{(0)}_{s_k}(e^{2i\pi\nu})\, d\nu$ where $S^{(0)}_{s_k}(e^{2i\pi\nu})$ represents the spectral density of signal $(s_k(n))_{n\in\mathbb{Z}}$. We finally define filter $\tilde{f}_k(z)$ and signal $\tilde{s}_k(n)$ by

$$\tilde{f}_k(z) = \frac{f_k(z)}{\|f_k\|},\ \tilde{s}_k(n) = [\tilde{f}_k(z)]s_k(n)$$

If $\|f_k\| = 0$, we put $\tilde{f}_k(z) = 0$ and $\tilde{s}_k(n) = 0$.It is clear that $\|\tilde{f}_k\| = 1$, and that $< \mathbb{E}|\tilde{s}_k(n)|^2 >= 1$. $r(n)$ can be written as

$$r(n) = \sum_{k=1}^{K} \|f_k\| e^{2i\pi n\delta f_k}\tilde{s}_k(n) \quad (4)$$

and coincides with a filtered version of one of the source signal if and only if the coefficients $(\|f_k\|)_{k=1,\dots,K}$ satisfy $\|f_k\| = \delta(k-k_0)\|f_{k_0}\|$. After some algebra, we get that

$$J(r) = \sum_{k=1}^{K} \beta(\tilde{s}_k)\|f_k\|^4 + 2\sum_{k_1<k_2} l(\tilde{s}_{k_1},\tilde{s}_{k_2})\|f_{k_1}\|^2\|f_{k_2}\|^2 - 2\sum_{k=1}^{K}\|f_k\|^2 + 1 \quad (5)$$

where $l(\tilde{s}_{k_1},\tilde{s}_{k_2})$ and $\beta(\tilde{s}_k)$ are defined respectively by

$$2 + \mathrm{Re}\left[2\sum_{\alpha\in I_*} R^{(\alpha)}_{\tilde{s}_{k_1}}(0)\left(R^{(\alpha)}_{\tilde{s}_{k_2}}(0)\right)^* + \sum_{\alpha_c\in I_c} R^{(\alpha_c-2\delta f_{k_1})}_{c,\tilde{s}_{k_1}}(0)\left(R^{(\alpha_c-2\delta f_{k_2})}_{c,\tilde{s}_{k_2}}(0)\right)^*\right] \quad (6)$$

$$< \mathbb{E}|\tilde{s}_k(n)|^4 >=< c_4(\tilde{s}_k) > +2 + 2\sum_{l=-1,1}\left|R^{l\alpha_k}_{\tilde{s}_k}(0)\right|^2 + \sum_{l=-1,0,1}\left|R^{l\alpha_k}_{c,\tilde{s}_k}(0)\right|^2 \quad (7)$$

3. THE SOURCE SIGNALS DO NOT SHARE THE SAME CYCLIC AND NON CONJUGATE CYCLIC FREQUENCIES.

We first study the behaviour of $J(r)$ when the source signals do not share the same cyclic and non conjugate cyclic frequencies. This in particular implies that $\alpha_k \neq \alpha_l$ (i.e. $T_k \neq T_l$) and $\delta f_k \neq \delta f_l$ ($\Delta f_k \neq \Delta f_l$). In this context, the term $l(\tilde{s}_{k_1},\tilde{s}_{k_2})$ reduces to the constant term 2, and $J(r)$ is given by

$$J(r) = \sum_{k=1}^{K} \beta(\tilde{s}_k)\|f_k\|^4 + 2\sum_{k_1\neq k_2}\|f_{k_1}\|^2\|f_{k_2}\|^2 - 2\sum_{k=1}^{K}\|f_k\|^2 + 1 \quad (8)$$

We now study the conditions under which the minimum of $J(r)$ is reached for a filter such that $\|f_k\| = \delta(k-k_0)\|f_{k_0}\|$. For this, we first fix the unit norm filters $(\tilde{f}_k)_{k=1,\dots,K}$ or equivalently the coefficients $(\beta(\tilde{s}_k))_{k=1,\dots,K}$. Then, we minimize J w.r.t. the $(\|f_k\|)_{k=1,\dots,K}$. This is an easy task because, as a function of the $(\|f_k\|)_{k=1,\dots,K}$, $J(r)$ has a simple expression. It can be shown that if $\min_{k=1,\dots,K}\beta(\tilde{s}_k) < 2$, then the minimum of $J(r)$ w.r.t. the $(\|f_k\|)_{k=1,\dots,K}$ is reached for sequences such that $\|f_k\| = \delta(k-k_0)\|f_{k_0}\|$ where k_0 is one of the index for which $\beta(\tilde{s}_{k_0}) = \min_{k=1,\dots,K}\beta(\tilde{s}_k)$. We now define $\beta_{min,k}$ as $\beta_{min,k} = \min_{\tilde{f}_k,\|\tilde{f}_k\|=1}\beta(\tilde{s}_k)$. Using the above result, it can be shown that if $\min_{k=1,\dots,K}\beta_{min,k} < 2$, then the argument of the minimum of $J(r)$ is reached for a filter $(f^*_1(z),\dots,f^*_K(z))$ such that $f^*_k(z) = \delta(k-k_0)f^*_{k_0}(z)$ where k_0 is one of the index for which $\beta_{min,k_0} = \min_{k=1,\dots,K}\beta_{min,k}$. Moreover, $\|f^*_{k_0}\|^2 = \frac{1}{\beta_{min,k_0}}$ and the unit norm filter $\tilde{f}^*_{k_0}$ verifies $\beta_{min,k_0} = \beta([\tilde{f}^*_{k_0}(z)]s_{k_0}(n))$. Finally, the minimum of J is equal to $1 - \frac{1}{\beta_{min,k_0}}$, and the corresponding extracted signal $r^*(n)$ coincides with $r^*(n) = e^{2i\pi n\delta f_{k_0}}[f^*_{k_0}(z)]s_{k_0}(n)$. Using iteratively this result, we finally obtain the following proposition.

Theorem 1 *If the sources do not share the same cyclic and non conjugate cyclic frequencies, the global minimization of the Godard cost function allows to extract all the source signals using a deflation approach if*

$$\beta_{min,k} = \min_{\tilde{f}_k, \|\tilde{f}_k\|=1} \beta(\tilde{s}_k) < 2, \text{ for each } k = 1, \ldots, K \quad (9)$$

It remains to check if condition (9) holds. For circular linearly modulated signals, (9) has been proved analytically in [4]. In the case of BPSK signals, the following result can be proved using a similar approach.

Proposition 1 *Consider a BPSK signal with symbol period T and excess bandwidth $0 < \gamma < 1$, and assume that the sampling period T_e does not belong to $\{T, T/2, 2T/3, 2T/3\}$. Denote by κ the kurtosis of the corresponding binary symbol sequence, $\kappa = -2$. Then, $\beta_{min} = \min_{\tilde{f}, \|\tilde{f}\|=1} \beta([\tilde{f}(z)]s(n))$ is given by*

$$\beta_{min} = \inf_{f_a \in \mathscr{F}([-\frac{1+\gamma}{2T}, \frac{1+\gamma}{2T}])} \Phi(f_a) \quad (10)$$

where $\Phi(f_a)$ is defined by

$$\Phi(f_a) = \kappa T \frac{\int_{\mathbb{R}} |f_a(t)|^4 dt}{(\int_{\mathbb{R}} |f_a(t)|^2 dt)^2} + 2 + 4\left(\frac{\int_{\mathbb{R}} |f_a(t)|^2 e^{-2i\pi\frac{t}{T}} dt}{\int_{\mathbb{R}} |f_a(t)|^2 dt}\right)^2 + \frac{\left|\int_{\mathbb{R}} f_a(t)^2 dt\right|^2}{(\int_{\mathbb{R}} |f_a(t)|^2 dt)^2} + \frac{\left|\int_{\mathbb{R}} f_a(t)^2 e^{-2i\pi\frac{t}{T}} dt\right|^2}{(\int_{\mathbb{R}} |f_a(t)|^2 dt)^2} + \frac{\left|\int_{\mathbb{R}} f_a(t)^2 e^{2i\pi\frac{t}{T}} dt\right|^2}{(\int_{\mathbb{R}} |f_a(t)|^2 dt)^2}$$

and where $\mathscr{F}([-\frac{1+\gamma}{2T}, \frac{1+\gamma}{2T}])$ represents the set of all square integrable functions $f_a(t)$ whose Fourier transforms $\hat{f}_a(\nu)$ are 0 outside the interval $[-\frac{1+\gamma}{2T}, \frac{1+\gamma}{2T}]$. Moreover, if we define η_{min} by $\eta_{min} = \min_{\|\tilde{f}\|=1} < c_4(\tilde{s}) >$, then

$$\eta_{min} = \inf_{f_a \in \mathscr{F}([-\frac{1+\gamma}{2T}, \frac{1+\gamma}{2T}])} \kappa T \frac{\int_{\mathbb{R}} |f_a(t)|^4 dt}{(\int_{\mathbb{R}} |f_a(t)|^2 dt)^2} \quad (11)$$

As $\mathscr{F}([-\frac{1+\gamma_1}{2T}, \frac{1+\gamma_1}{2T}]) \subset \mathscr{F}([-\frac{1+\gamma_2}{2T}, \frac{1+\gamma_2}{2T}])$ if $\gamma_1 < \gamma_2$, (10) implies that considered as a function of γ, $\beta_{min}(\gamma)$ is decreasing. Therefore, in order to check that $\beta_{min}(\gamma) < 2$ for $\gamma \in [0,1]$, it is sufficient to verify that $\beta_{min}(0) < 2$. If $f_a \in \mathscr{F}([-\frac{1}{2T}, \frac{1}{2T}])$ ($\gamma = 0$), the integrals

$$\int_{\mathbb{R}} |f_a(t)|^2 e^{-2i\pi\frac{t}{T}} dt, \ \int_{\mathbb{R}} f_a(t)^2 e^{-2i\pi\frac{t}{T}} dt, \ \int_{\mathbb{R}} f_a(t)^2 e^{2i\pi\frac{t}{T}} dt$$

vanish. Moreover, $\left|\int_{\mathbb{R}} f_a(t)^2 dt\right|^2 \le \left(\int_{\mathbb{R}} |f_a(t)|^2 dt\right)^2$. This implies that $\beta_{min}(0) \le \eta_{min}(0) + 3$. $\eta_{min}(0)$ can be evaluated numerically (see [4] for more details), and it turns out that $\eta_{min}(0) \simeq -1.36$. Therefore, $\beta_{min}(0) \le 1.64$, thus showing that $\beta_{min}(0) < 2$, and that $\beta_{min}(\gamma) < 2$ for $\gamma \in [0,1]$. $\beta_{min}(\gamma)$ can also be evaluated numerically for each γ. It is represented in Figure 1

The above results show that the global minimum of the Godard cost function corresponds to separating filters. However, in pratice, minimization algorithms may converge towards local minima. Under a technical assumption, a positive result can be established.

Proposition 2 *Assume that at least one of the function $\tilde{f}_k \to \beta([\tilde{f}_k(z)]s_k(n))$ defined on the set of all unit norm filters has no local minimum $\tilde{f}_k^*$ such that $\beta([\tilde{f}_k^*(z)]s_k(n)) \ge 2$. Then, the argument of each local minimum of the Godard cost function is a separating filter.*

Proof. We just sketch the proof of this result. For this, we write $J(r)$ given by (8) as $J(r) = u^4 T(\mathbf{v}, \beta) - 2u^2 + 1$ where $u = (\sum_{k=1}^K \|f_k\|^2)^{1/2}$, $v_k = \frac{\|f_k\|}{u}$, $\mathbf{v} = (v_1, \ldots, v_K)^T$, $\beta = (\beta(\tilde{s}_1, \ldots, \beta(\tilde{s}_K))^T$ and $T(\mathbf{v}, \beta) = \sum_{k=1}^K (\beta(\tilde{s}_k) - 2)v_k^2 + 2$. We consider a local minimum $(f_1^*(z), \ldots, f_K^*(z))^T$ of $J(r)$, and denote by u_*, $\mathbf{v}_*$, $\tilde{f}_k^*$, $\tilde{s}_k^*$, β_* the corresponding values of $u, \mathbf{v}, \tilde{f}_k, \tilde{s}_k, \beta$. It is easy to check that the point $\mathbf{v}_*$ is a local minimum of the function $\mathbf{v} \to T(\mathbf{v}, \beta_*)$. As at least one the coefficients $(\beta(\tilde{s}_k^*) - 2)$ is strictly negative, $v_k^* = \delta(k - k_0)v_{k_0}^*$ where k_0 is one of the index for which $\beta_{k_0,*} - 2 < 0$ (see e.g. [2]). This implies that $\|f_{k,*}\| = \delta(k - k_0)\|f_{k_0}^*\|$, and that the local minimum $f_{1,*}(z), \ldots, f_{K,*}(z)$ is a separating filter.

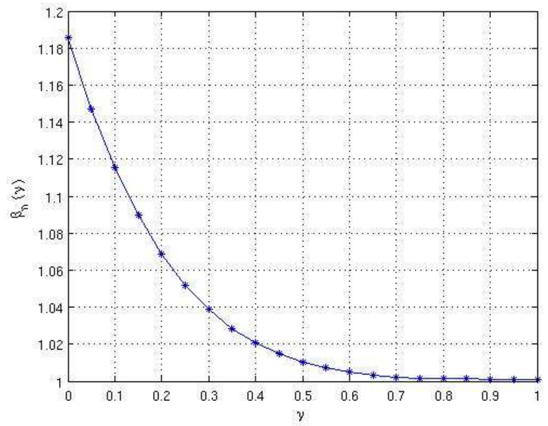

Figure 1: β_{min} w.r.t. γ, BPSK case.

It is difficult to check analytically whether it exists k for which $\tilde{f}_k \to \beta([\tilde{f}_k(z)]s_k(n))$ has no local minimum $\tilde{f}_k^*$ such that $\beta([\tilde{f}_k^*(z)]s_k(n)) \ge 2$. However, this condition probably holds because the gradient based minimization algorithms of the functions $\tilde{f}_k \to \beta([\tilde{f}_k(z)]s_k(n))$ we have run always converge towards a point for which $\beta([\tilde{f}_k(z)]s_k(n)) < 2$.

In sum, the above results indicate that if the source signals do not share the same cyclic and non conjugate cyclic frequencies, then, the minimization of the Godard cost function allows to extract circular and BPSK source signals. In this context, it is therefore possible to separate the cyclostationary signals without any knowledge on their cyclic and non conjugate cyclic frequencies.

4. 2 BPSK SOURCES SHARING THE SAME BAUD-RATE AND THE SAME CARRIER FREQUENCY.

In this section, we consider the opposite situation, namely that all the source signals are BPSK signals with the same baud rate T, the same carrier frequency Δf, and the same excess bandwidth γ. We also denote by α and δf the terms $\alpha = T_e/T$ and $\delta f = \Delta f T_e$. We recall that T_e represents the sampling period, and that we assume that T_e does not belong to $\{T, T/2, T/3, 2T/3\}$. We also assume that the number of sources is $K = 2$. Our purpose is to show that the Godard cost function has non separating local minima, and that the corresponding minimization algorithms often converge towards these spurious points. In order to demonstrate this, we assume that the common excess bandwidth γ of the 2 source signals is equal to 0. In this context, the cyclic and non conjugate cyclic correlations coefficients at frequencies $\pm\alpha$ are zero. The expression (5) of $J(r)$ thus reduces to:

$$J(r) = \beta(\tilde{s}_1)\|f_1\|^4 + \beta(\tilde{s}_2)\|f_2\|^4 + 2\|f_1\|^2\|f_2\|^2 \left(2 + \mathrm{Re}(R_{c,\tilde{s}_1}^{(0)}(0) R_{c,\tilde{s}_2}^{(0)}(0)^*)\right) - 2\left(\|f_1\|^2 + \|f_2\|^2\right) + 1 \quad (12)$$

where $\beta(\tilde{s}_i)$ is given by

$$\beta(\tilde{s}_i) = < c_4(\tilde{s}_i) > + 2 + |R_{c,\tilde{s}_i}^{(0)}(0)|^2$$

for $i=1,2$. This expression is formally similar to the expression of J in the case where the 2 sources are circular with a non zero excess bandwidth (see [4]), except that the cyclic correlation coefficients $R^{(0)}_{c,\tilde{s}_i}(0)$ have to be replaced by $2R^{(\alpha)}_{\tilde{s}_i}(0)$. $|2R^{(\alpha)}_{\tilde{s}_i}(0)|$ turns out to be less than 1 in the context of [4] and it is easy to show that $|R^{(0)}_{c,\tilde{s}_i}(0)|<1$. Therefore, it is possible to use the results of [4] established in the circular case to prove if β_{min} and η_{min} defined in Proposition 1 verify

$$\begin{aligned} -3\beta_{\min}+5+\eta_{\min} &> 0 \\ 2(\beta_{\min}-1)\beta_{\min}-4\left(1-\frac{1}{2}\sqrt{2(\beta_{\min}-1)-1-\eta_{\min}}\right)^2 &< 0, \end{aligned}$$

then, the argument of the global minimum of $J(r)$ is a separating filter, and the minimum value of $J(r)$ coincides with $1-1/\beta_{min}$. For $\gamma=0$, $\beta_{min}\simeq 1.19$, $\eta_{min}\simeq -1.36$, and it is easily checked that the above 2 conditions are satisfied. Therefore, the global minimization of $J(r)$ allows to separate the 2 BPSK signals. Moreover, $1-1/\beta_{min}\simeq 0.16$. However, $J(r)$ has non separating local minima, and as shown below, a gradient minimization algorithm of $J(r)$ often converges towards these local minima. In order to define these local minima, we denote by $\tilde{f}_1^*(z)$ one of the argument of the global minimum of $\beta([\tilde{f}_1(z)]s_1(n))$ over the set of unit norm filters **with real coefficients**. We denote by $\beta_{1,min}$ the corresponding minimum. It is easy to show that $\beta_{1,min}$ can be evaluated using Proposition 1 for $\gamma=0$, but in which the minimum of function Φ_a is evaluated over **the real elements** of $\mathscr{F}([-1/2T,1/2T])$. Using this, it can be shown that $\beta_{1,min}$ coincides with $\eta_{min}+3$, i.e. that $\beta_{1,min}\simeq 1.64$. We consider the unit norm filter with imaginary coefficients $\tilde{f}_2^*(z)=i\tilde{f}_1^*(z)$. It is clear that $\beta([\tilde{f}_2^*(z)]s_2(n))$ coincides with $\beta_{1,min}$. We finally define filters $f_i^*(z)$ for $i=1,2$ by

$$f_i^*(z)=\frac{1}{(1+\beta_{1,min})^{1/2}}\tilde{f}_i^*(z)$$

If $r_*(n)=[f_1^*(z)]s_1(n)+[f_2^*(z)]s_2(n)$, one can check that $J(r_*)=1-2/(1+\beta_{1,min})\simeq 0.25$. This non separating point can be shown to be a local minimum of J. Moreover, the gradient minimization algorithm of $J(r)$ seems to converge very often to one this point rather than towards the argument of the separating global minimum of J. To verify this, we present in Figure 5 an histogram of the values of $J(r)$ at convergence of the gradient minimization algorithm. 1000 experiments have been used. It is seen that the value of $J(r)$ at convergence is around 0.25 in most cases, showing that the gradient algorithm converges very often towards one of the above local minimum.

A simple modification of the Godard cost function allows to overcome the above problems. We assume that the carrier frequency offset δf is known or correctly estimated at the receiver side, and consider the cost function $J^{'}(r)$ defined by

$$J^{'}(r)=J(r)-|R^{(2\delta f)}_{c,r}(0)|^2 \tag{13}$$

In our particular context of 2 BPSK signals with $\gamma=0$, $J^{'}(r)$ is given by

$$J^{'}(r)=\sum_{k=1}^{K}\beta^{'}(\tilde{s}_k)\|f_k\|^4+2\sum_{k_1\neq k_2}\|f_{k_1}\|^2\|f_{k_2}\|^2-2\sum_{k=1}^{K}\|f_k\|^2+1$$

where $\beta^{'}(\tilde{s}_i)$ is defined by $\beta^{'}(\tilde{s}_i)=<c_4(\tilde{s}_i)>+2$. The expression of $J^{'}(r)$ is thus similar to (8), except that $\beta(\tilde{s}_i)$ is now replaced by $\beta^{'}(\tilde{s}_i)$. It is easy to check that $<c_4(\tilde{s}_i)><0$, so that $\beta^{'}(\tilde{s}_i)<2$ for each i. Theorem 1 and Proposition 2 thus imply that the global minimum and the local minima of $J^{'}$ are separating filter. This shows that the minimization of $J^{'}(r)$ allows to separate the 2 BPSK signals if $\gamma=0$. This result can be partially extended if $\gamma>0$.

5. FINAL REMARKS AND CONCLUSION.

In more general contexts, $J^{'}(r)$ is defined by substracting from $J(r)$ the sum of modulus squares of the non conjugate cyclic correlations of r at lag 0 at the "significant" non conjugate cyclic frequencies $2\delta f_k$ corresponding to twice the carrier frequency offsets of the BPSK signals . It can be shown that the minimization of $J^{'}$ still allows to separate source signals that do not share the same cyclic and non conjugate cyclic frequencies, and that positive results can also be obtained in more complicated scenarii (groups of BPSK signals sharing the same baud rate and carrier frequency). The minimization of $J^{'}$ thus appears as a reliable approach to separate convolutive mixtures of possibly non circular linearly modulated signals. In order to use $J^{'}$, it is of course necessary to estimate the significant non conjugate cyclic frequencies of the received signal prior the source separation. Fortunately, this is a much easier task than the estimation of the baud rates, because the non conjugate cyclic correlation coefficents of the received signal at twice the frequency offsets are not affected by possible low excess bandwidths of the source signals.

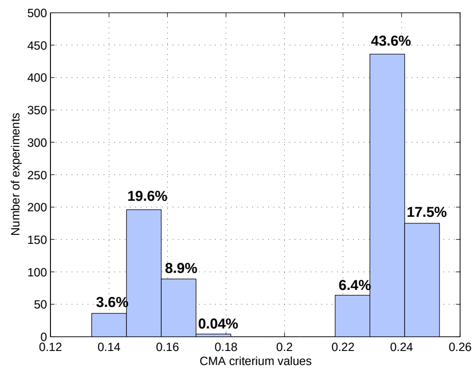

Figure 2: Values of $\hat{J}(r)$ at convergence of the gradient algorithm.

REFERENCES

[1] Ph. Ciblat, Ph. Loubaton, E. Serpedin, G.B. Giannakis, "Performance analysis of blind carrier offset estimation for non-circular transmissions through frequency-selective channels", IEEE Trans. on Signal Processing, vol. 50, no. 1, pp. 130-140, January 2002.

[2] N. Delfosse, Ph. Loubaton, "Adaptive blind separation of independent sources : a deflation approach", *Signal Processing*, 45, 1995, pp. 59-83.

[3] A. Ferréol, P. Chevalier, "On the behavior of current second and higher-order blind source separation methods for cyclostationary sources", IEEE Trans. on Signal Processing, vol. 48, no. 6, pp. 1712-1725, June 2000.

[4] Pierre Jallon, Antoine Chevreuil, Philippe Loubaton, Pascal Chevalier, "Separation of convolutive mixtures of linear modulated signals using constant modulus algorithmcyclostationary sources", *Proc. ICASSP-2005*, vol. III, pp. 453-456, Philadelphia, March 2005.

[5] P. Jallon, "Séparation de Mélanges Convolutifs de Sources Cyclostationnaires", PhD Thesis, Université de Marne la Vallée, Mai 2006.

[6] J.K. Tugnait, "Blind spatio-temporal equalization and impulse response estimation for MIMO channels using Godard cost functiion, IEEE Trans. on Signal Processing, vol. 45, no. 1, pp. 268-271, January 1997.