SUBSPACE INTERSECTION METHOD OF BEARING ESTIMATION IN SHALLOW OCEAN USING ACOUSTIC VECTOR SENSORS

K.G. Nagananda and G.V. Anand

Department of Electrical Communication Engineering
Indian Institute of Science, Bangalore 560012, INDIA
Phone: +91-80-22932277; Fax: +91-80-23600563
E-mail: kgnagananda@ece.iisc.ernet.in, anandgv@ece.iisc.ernet.in
Web: http://www.ece.iisc.ernet.in

1. INTRODUCTION

High-resolution bearing estimation or direction-of-arrival (DOA) estimation is one of the important steps in localization of acoustic sources in the ocean. Several methods have been used to solve this problem with MUSIC [1] and ESPRIT [2] algorithms being the prominent ones. These algorithms, however, assume plane-wave propagation leading to biased bearing estimates due to the multimode nature of acoustic propagation in the ocean. The bias increases with increasing number of propagating modes in the ocean. Matched field processing (MFP) techniques [3] overcome this problem at the expense of computational complexity, since MFP replaces the one-dimensional search of the above mentioned methods by a three-dimensional search in the bearing-range-depth space. The subspace intersection method (SIM) presented in [4] attempts to alleviate the problem of both bias and computational complexity. Bias is eliminated by replacing the invalid plane-wave propagation model by a more appropriate normal mode propagation model. SIM involves a one-dimensional search without any prior knowledge of the range and depth, thereby decreasing the computational complexity.

The above-mentioned methods make use of the conventional scalar sensors which measure only the acoustic pressure. Nehorai and Paldi [5] presented a method of source localization using an array of acoustic vector sensors (AVS), which can simultaneously measure acoustic pressure and the three Cartesian components of particle velocity at a point. The additional information provided by an AVS can be utilized for better localization. But most AVS array processing techniques including [5] consider a plane wave acoustic propagation model which is not valid for shallow ocean. A maximum likelihood estimation method, which was proposed recently [6], uses a valid model of shallow ocean acoustic propagation. But this method does not have the high resolution capability of SIM, and also it is highly computation intensive.

In this paper we attempt to unify the two frameworks - subspace intersection method and vector sensors, for high-resolution bearing estimation of multiple sources in shallow ocean. No assumption is made on the prior knowledge of source ranges and depths. We compare the performance of SIM with vector and scalar sensor arrays, and show that the use of vector sensors leads to a reduction in the number of sensors, lower rms estimation errors, elimination of spacing constraints, and improved localization performance in the end-fire direction.

The outline of the paper is as follows. In Section 2, we present the AVS array data model for a horizontally stratified ocean. The structure of the noise covariance matrix and a whitening transformation are discussed in Section 3. The subspace intersection method for a vector sensor array is presented in Section 4. Simulation results are presented in Section 5. Section 6 concludes the paper.

2. AVS ARRAY DATA MODEL

The shallow ocean is modeled as a horizontally stratified water layer of constant depth \( h \) overlying a horizontally stratified fluid bottom. The variation of its acoustic properties in the horizontal direction is assumed to be negligible in the range of interest. Consider \( J \) mutually uncorrelated narrowband point sources of center frequency \( \omega / 2\pi \) located at depths \( z_j \) and ranges \( r_j \) with respect to the reference sensor (first sensor) of a horizontal linear array of acoustic vector sensors, also known as vector hydrophones. Source azimuth angles \( \phi_j (j = 1,...,J) \) are measured with respect to the axis of the array. The array has \( M \) equipossed sensors at a spacing of \( d \) meters, all lying at a depth of \( z \) meters below the ocean surface. Assuming the sensor array to be in the far field region with respect to all sources, the expressions for pressure and particle velocity at the \( n^{th} \) source due to the \( j^{th} \) source are given by [6, 7]:

\[
p_{nj} = \sum_{n=1}^{N} b_{nj} e^{i(n-1)k_d\cos\phi_j - \omega tr},
\]

\[
v_{nxj} = \frac{\cos\phi_j}{\omega \rho} \sum_{n=1}^{N} k_d b_{nj} e^{i(n-1)k_d\cos\phi_j - \omega tr},
\]

\[
v_{nyj} = \frac{\sin\phi_j}{\omega \rho} \sum_{n=1}^{N} k_d b_{nj} e^{i(n-1)k_d\cos\phi_j - \omega tr},
\]

\[
v_{nzj} = \frac{1}{\omega \rho} \sum_{n=1}^{N} \left[ \frac{-i\psi_n(z)}{\psi_n(z)} \right] b_{nj} e^{i(n-1)k_d\cos\phi_j - \omega tr},
\]

where

\[
b_{nj} = \frac{2\sqrt{2\pi e^{-2\pi^2/4}}}{h} \psi_n(z_j) \left[ e^{-\delta_n r_j} e^{i\delta_n r_j} \right] s_j(t),
\]

\( N \) is the number of normal modes in the oceanic waveguide, \( \delta_n; n = 1,...,N \) are the attenuation co-efficients,
\[ \psi_k(z); n = 1, \ldots, N \] are the eigenfunctions and \([k_n; n = 1, \ldots, N]\) are the eigenvalues obtained by solving the Sturm-Liouville type characteristic differential equation (with appropriate boundary conditions at \( z = 0 \) and \( z = h \)):

\[ \frac{d^2 \psi_k(z)}{dz^2} + \left[k^2(z) - k_n^2 \right] \psi_k(z) = 0, \]

\( k(z) = \omega/c(z), \) \( h \) is the depth of the water layer, \( c(z) \) is the sound speed in water at depth \( z, \) \( \rho(z) \) is the sound speed in bottom at depth \( z, \) \( \rho_0(z) \) are the densities of water and ocean bottom, respectively, at depth \( z, \) \( p_{mj} \) is the acoustic pressure at the \( m^th \) sensor due to the \( j^th \) source, \( \rho_{mj}(z) \) are the eigenvalues obtained by solving the Sturm-Liouville type

\[ \text{sign} \text{transform} \text{matrix} \text{of} \text{rank} \text{4} \text{noise} \text{covariance} \text{matrix} \text{at} \text{each} \text{vector} \text{sensor}. \text{We assume that} \text{noise is predominantly wind-generated noise, for which the covariance matrix can be shown to be [6]}

\[ \tilde{\psi}_k(z) = \sum_{n=1}^{N} \psi_k(z), \]

\( \xi_n(z) = \frac{1}{N} \sum_{n=1}^{N} \psi_n^2(z), \)

\[ \xi_n(z) = \frac{1}{N} \sum_{n=1}^{N} \psi_n^2(z), \]

\text{where} \( \eta = 4 \times 4 \text{block diagonal array noise covariance matrix} \text{where} \ Y = 4 \times 4 \text{noise covariance matrix at each vector sensor.} \text{We assume that noise is predominantly wind-generated noise, for which the covariance matrix can be shown to be [6]}

\[ \tilde{\psi}_k(z) = \sum_{n=1}^{N} \psi_k(z), \]

\( \xi_n(z) = \frac{1}{N} \sum_{n=1}^{N} \psi_n^2(z), \)

\[ \xi_n(z) = \frac{1}{N} \sum_{n=1}^{N} \psi_n^2(z), \]

\text{The array noise vector} \( \eta(t) \) \text{can therefore be whitened through the transformation} \( \eta(t) = W \eta(t) \text{where} \)

\[ W = \text{diag} \left( Y^{-1/2}, \ldots, Y^{-1/2} \right), \]

\text{is a depth-dependent} \text{4M} \times \text{4M} \text{whitening matrix.} \text{The whitened array data vector is given by}

\[ y_n(t) = W y_n(t) = W y_n(t), \]

\text{and the whitened array data covariance matrix is}

\[ C_{\text{wh}} = W y_n(t) y_n(t)^H = W y_n(t) y_n(t)^H + \sigma^2 I. \]

\text{4. SUBSPACE INTERSECTION METHOD}

\text{In (30),} \( C \) \text{is a diagonal matrix of rank} \text{J and} W \text{is an invertible square matrix of rank} 4M. \text{It can be shown following an approach similar to that in reference [4] that the} J \text{column of the} 4M \times J \text{matrix} P \text{are linearly independent for every set of} J \text{distinct source positions if} 4M \geq NJ. \text{In the following analysis, we assume that the condition} 4M \geq NJ \text{is satisfied. It follows that the matrix} WPCP^H W^H \text{is of rank} J. \text{The signal subspace} S \text{is defined as}

\[ S = \text{span} \{ u_1, \ldots, u_J \} = \text{span} \{ Wp(\Phi_1, r_1, z_1), \ldots, Wp(\Phi_J, r_J, z_J) \}, \]
where \(u_1, \ldots, u_J\) are the ‘signal’ eigenvectors of \(C_m\) corresponding to the \(J\) largest eigenvalues. We define the modal subspace \(M(\phi)\) for azimuth \(\phi\) as
\[
M(\phi) = \operatorname{span}\{W_\phi(k_1), \ldots, W_\phi(k_N)\},
\]
where \(a(\phi, k_n)\) is the modal steering vector defined in (15) and (16). It is evident that \(M(\phi) \neq M(\phi)\) for \(\phi \neq \phi\). It follows from (12) and (32) that \(W_p(\phi, r_j, z_j) \in M(\phi)\) for \(j = 1, \ldots, J\). We also have \(W_p(\phi, r_j, z_j) \in S\) for \(j = 1, \ldots, J\). It follows that \(S\) and \(M(\phi)\) intersect if \(\phi \in \{\phi_1, \phi_2, \ldots, \phi_J\}\).

Finally, we note that \(S\) and \(M(\phi)\) intersect only if a nontrivial linear combination of the linearly independent basis vectors of \(S\) and \(M(\phi)\) is a null vector. Consider a linear combination with \(N + J - 1\) complex numbers \(c_1, \ldots, c_{N+J-1}\) and a real number \(\phi\) such that
\[
\sum_{j=1}^J c_j p(\phi, r_j, z_j) + \sum_{n=1}^{N-1} c_{j+n} a(\phi, k_n) + a(\phi, k_N) = 0. \tag{33}
\]
This set comprises \(4M\) complex equations with \(N + J - 1\) complex unknowns \(c_1, \ldots, c_{N+J-1}\) and one real number \(\phi\). For \(4M \geq N + J\), the number of equations exceeds the number of unknowns and a solution for (33) exists only if \(\phi \in \{\phi_1, \ldots, \phi_J\}\). It follows that \(S\) and \(M(\phi)\) intersect if and only if \(\phi \in \{\phi_1, \ldots, \phi_J\}\).

We now proceed to perform the bearing estimation of sources in shallow ocean using the acoustic vector array sensor. The procedure is similar to the subspace intersection method for a scalar sensor array. Let us construct a \(4M \times (N + J)\) matrix \(D(\phi)\) defined as follows:
\[
D(\phi) = [a_1 W_\phi(k_1) \ldots a_N W_\phi(k_N) u_1 \ldots u_J]. \tag{34}
\]
where \(a_1, \ldots, a_N\) are normalizing constants. Further, the first \(M\) columns of \(D(\phi)\) are linearly independent basis vectors of the modal subspace \(M(\phi)\) and the remaining \(J\) columns are the orthonormal basis vectors of the signal subspace \(S\). We then perform a QR decomposition to factorize the matrix \(D(\phi)\) as
\[
Q(\phi) = D(\phi) R(\phi). \tag{35}
\]
Here,
\[
Q(\phi) = [q_1(\phi), \ldots, q_{N+J}(\phi)] \tag{36}
\]
is an \(4M \times (N + J)\) matrix whose columns \(q_1(\phi), \ldots, q_{N+J}(\phi)\) are orthonormal vectors and \(R(\phi)\) is a \((N + J) \times (N + J)\) upper triangular matrix whose diagonal elements are the eigenvalues of the matrix \(D(\phi)\). The columns of \(D(\phi)\) are related to the columns of \(Q(\phi)\) through the equations
\[
d_i(\phi) = \sum_{k=1}^I r_{ki}(\phi) q_i(\phi), \quad i = 1, \ldots, N + J \tag{37}
\]
where \(d_i(\phi)\) is the \(i^{th}\) column of \(D(\phi)\) and \(r_{ki}(\phi)\) is the \((k, i)^{th}\) element of \(R(\phi)\). The elements \(r_{ki}(\phi)\) and the vectors \(q_i(\phi)\) can be determined recursively using the relations
\[
r_{11}(\phi) = ||d_1(\phi)||_2, \tag{38}
\]
\[
r_{1}(\phi) = \frac{1}{r_{11}(\phi)} d_1(\phi), \tag{39}
\]
\[
r_{ki}(\phi) = q_i^H(\phi) d_i(\phi), \quad 1 \leq k \leq i - 1; \quad i = 2, \ldots, (N + J) \tag{40}
\]
\[
r_{ii}(\phi) = \left\| d_i(\phi) - \sum_{k=1}^{i-1} r_{ki}(\phi) q_k(\phi) \right\|_2; \quad i = 2, \ldots, (N + J) \tag{41}
\]
\[
q_i(\phi) = \frac{d_i(\phi) - \sum_{k=1}^{i-1} r_{ki}(\phi) q_k(\phi)}{r_{ii}(\phi)}; \quad i = 2, \ldots, (N + J) \tag{42}
\]
where \(\| . \|_2\) denotes the Euclidean norm. A diagonal element \(r_{ij}(\phi)\) of the matrix \(R(\phi)\) is zero if and only if \(d_j(\phi)\) is spanned by \(\{d_1(\phi), \ldots, d_{i-1}(\phi)\}\). This property will be exploited during the formulation of the bearing estimation algorithm.

Assuming that \(4M \geq NJ\) and \(4M \geq (N + J)\), we know that the subspaces \(M(\phi) = \operatorname{span}\{d_1(\phi), \ldots, d_{N+J}(\phi)\}\) and \(S = \operatorname{span}\{d_{N+1}(\phi), \ldots, d_{N+J}(\phi)\}\) intersect if and only if \(\phi \in \{\phi_1, \phi_2, \ldots, \phi_J\}\). Therefore, it follows that \(\phi \in \{\phi_1, \phi_2, \ldots, \phi_J\}\) if and only if one of the following conditions is satisfied:
\[
d_i(\phi) \in \operatorname{span}\{d_1(\phi), \ldots, d_{i-1}(\phi)\}; \quad i = 1, \ldots, (N + J), \quad \text{if} \quad r_{ii}(\phi) = 0; \tag{43}
\]
or
\[
d_{i+J}(\phi) \in \operatorname{span}\{d_1(\phi), \ldots, d_{i-1}(\phi)\}; \quad i = 1, \ldots, (N + J) \quad \text{if} \quad r_{ii}(\phi) \neq 0. \tag{44}
\]
Thus, we have the following result: \(d_i(\phi) \in \operatorname{span}\{d_1(\phi), \ldots, d_{i-1}(\phi)\}\) for some \(i \in \{N + 1, \ldots, N + J\}\) if and only if \(\phi \in \{\phi_1, \ldots, \phi_J\}\). Hence, the SIM response function
\[
B_{SIM}(\phi) = \left[ \min_{N+1 \leq i \leq N+J} \left| r_{ii}(\phi) \right| \right]^{-1} \tag{45}
\]
has sharp peaks at \(\phi = \phi_1, \phi_2, \ldots, \phi_J\).

5. RESULTS AND DISCUSSION

We consider a Pekeris model of the ocean [7], so that \(c(z), c_b(z), \rho(z)\) and \(\rho_0(z)\) are independent of \(z\). All simulations have been performed with the following values of channel parameters: \(c = 1500 \text{ m/s}, c_b = 1700 \text{ m/s}, \rho_0/\rho = 1.5\), and attenuation in the ocean bottom is \(\delta = 0.5 \text{ dB/λ}\), where \(\lambda\) is the free space wavelength. An array of vector sensors is assumed to be at a depth \(z = 25 \text{ m}\) below the surface of the ocean. The ocean depth \(h\) is 100 m and the source signal frequency is 50Hz. For this choice of parameters, the number of modes is \(N = 6\). Independent samples of uniformly distributed random phases were added to the signal from each source to render the sources noncoherent. Independent samples of a zero-mean complex Gaussian random vector with covariance matrix \(\sigma^2 \text{diag}(\Gamma_{\lambda\Gamma})\) were used to generate the noise vector \(n(t)\). The parameter \(\sigma^2\) was adjusted to yield the desired signal-to-noise ratio (SNR), defined as
\[
\text{SNR} = 10 \log_{10} \frac{\sum_{i=1}^{M} \sum_{j=1}^{N} \sigma_j^2 |p_{ij}|^2}{M \sigma^2 \xi}, \tag{46}
\]
where \(\sigma_j^2\) is the source power given by (6), \(\xi\) is defined in (24), and \(\sigma^2 \xi\) is the variance of pressure component of noise.

The covariance matrix \(C_{yMLE}\) was estimated from the whitened data vector \(y_w(t)\) by averaging \(y_w(t) y_w^H(t)\) over \(L\) snapshots
\[
\hat{C}_{yMLE} = \frac{1}{L} \sum_{l=1}^{L} y_w(t) y_w^H(t). \tag{47}
\]
Eigenvalues of \(\hat{C}_{yMLE}\) corresponding to the \(J\) largest eigenvalues provide estimates of the signal eigenvectors \(\phi_1, \ldots, \phi_J\). Simulations have been performed by considering data obtained using both conventional scalar sensors and vector sensors to localize the sources. The sources are located at equal ranges \(r_j = 5000\) m and equal depths \(z_j = 37.5\) m. The spacing between sensor elements is \(d = \lambda/2 = 15\) m, unless otherwise stated. The number of snapshots is \(L = 350\) unless otherwise stated. All results have been generated by averaging over 250 simulations.

A comparative performance analysis of SIM with AVS and scalar sensor arrays is presented in Figs.1-10. A typical SIM
For unambiguous bearing estimation, the steering vectors have to satisfy the condition
\[ a(\phi, k_n) = a(\phi', k_n') \text{ only if } \phi = \phi', \quad n = n'. \]  

For a scalar sensor array, the vector
\[ a(\phi, k_n) = [1 \ e^{id_n \cos \phi} \ \cdots \ e^{(M-1)d_n \cos \phi} ]^T \]
may not satisfy condition (48) if \( d > \pi / k_1 = \lambda_d / 2 \). For an AVS array, \( a(\phi, k_n) \in \mathbb{C}^{M \times 1} \) has a more complex structure as seen from (15) and (16), and hence condition (48) is always satisfied even if \( d > \lambda_d / 2 \). The absence of any constraint on the intersensor spacing \( d \) can be exploited to enhance the resolution of an AVS array by choosing a larger value of \( d \). Figures 9 and 10 show the SIM response function for AVS array and scalar sensor array respectively for \( d = 3 \lambda_d \approx 3 \lambda_1 \), when two sources are located at 80° and 82°. The AVS array resolves the two sources and estimates their bearing angles unambiguously. For a scalar sensor array, unambiguous estimation is not possible due to the presence of a large number of sidelobes.
6. CONCLUSIONS

In this paper, we have presented the subspace intersection method (SIM) of high-resolution bearing estimation in a range-independent shallow ocean using a horizontal linear array of acoustic vector sensors (AVS). The bearing angles are estimated using a one-dimensional search algorithm which does not assume prior knowledge of source ranges and depths. The minimum number of sensors required by the SIM algorithm reduces by a factor of 4 if scalar sensors are replaced by vector sensors; the requirement is \( M \geq NJ \) for scalar sensors and \( M \geq \frac{1}{4} NJ \) for vector sensors. For an AVS array there is no constraint on interelement spacing \( d \), while for a scalar sensor array \( d \) cannot exceed \( \lambda/2 \). Hence, with an AVS array, resolution can be enhanced by increasing the spacing \( d \). The performance of SIM with a scalar sensor array degrades drastically for sources near the broadside direction, but SIM with AVS array does not suffer from this drawback. Simulation results show that the bias and rms error for an AVS array are significantly lower than those for a scalar sensor array, especially at low SNR. The AVS array also provides greater resolution than the scalar sensor array.

ACKNOWLEDGMENT

This work was partially supported by the DRDO-IISc Programme of Advanced Research in Mathematical Engineering.

REFERENCES