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# PERFORMANCE ANALYSIS OF PORT-STARBOARD DISCRIMINATION FOR TOWED TWIN-LINE ARRAY

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## ABSTRACT

*The performance of port-starboard discrimination (PSD) for a target detected by a towed twin-line sonar array is presented. The PSD performance is evaluated by a measure referred to as the port-starboard rejection ratio (PSRR), which is defined as the ratio of the output power in the desired signal direction to that in the mirror direction. Analytical expression for the PSRR is derived, in terms of the array separation, signal-to-noise-ratio (SNR), interference-to-noise-ratio (INR), signal-to-interference-ratio (SIR), angular separation and correlation between the desired signal and the interference, array size and finite sample size.*

## 1. INTRODUCTION

Towed horizontal arrays have received much attention in the past nearly ninety years. Single-line array receivers are lack of transverse aperture, which will result a cylindrically symmetric beampattern and, therefore, can't discriminate port from starboard. To address the problem of port-starboard ambiguity for single-line arrays, twin-line array containing two parallel towed arrays was developed [1]–[3]. Other method is to use directional sensors in a single-line array [4], [5]. Perhaps twin-line array is a most efficient way to resolve the port-starboard ambiguity. The only limitation of twin-line array is the difficulty to estimate the array shape, including the individual line shape and the inter-array separation.

In [5], three adaptive port-starboard beamforming algorithms of triplet arrays were analysed: optimum triplet beamformer, Cardioid Beamformer and adaptive triplet beamformer. Analysis results show that, the optimum beamformer gives good detection performance in all kinds of noise-limited environments, cardioid beamformer may be applied to areas where coastal reverberation masks detection in offshore bearings, and adaptive triplet beamforming is optimized for the actual estimated background, *i.e.*, estimations of correlation coefficients by beamformed data of three individual arrays. However, in practice, it is difficult to obtain the desired signal-free noise covariance matrix. And the interferers are not accounted for the performance analysis. In [6], some preliminary analyses of left-right ambiguity resolution performance for twin-line array was provided, the analysis is based on the twin-line directivity function, and the left-right resolution performance is compared for different array separations in a single narrowband signal only case, and consequently, the results in [6] is only effective in high enough SNR's.

The parameter of interest in the performance of port-starboard discrimination is the out power in the desired signal direction to that in the mirror direction, referred to as port-starboard rejection ratio (PSRR). In fact, this measure of PSRR is affected by many parameters. Apart from the array separation, other parameters including the SNR, the interference-to-noise-ratio (INR), the signal-to-interference-ratio (SIR), the angular separation and correlation between the desired signal and the interference, the array size and finite sample size.

In this paper, we will present a complete analysis of the PSRR as a function of all the parameters affecting performance.

## 2. TWIN-LINE MODEL

Twin-line array, comprised of two parallel line arrays in the horizontal plane, is depicted in Fig. 1. It can achieve port-starboard discrimination by phasing the signal fields received on the two lines so as to enhance signal power arriving from one side while suppressing the undesired another side power.

Consider a twin-line array towed in the horizontal $x$-$y$ plane in the direction of decreasing $x$, and let $\theta$ be the signal direction-of-arrival (DOA) measured clockwise from forward endfire, and $D$ denotes the array separation. For convenience, we let $\theta$ be positive in the starboard direction, whereas negative in the port direction.

Assume each line array contains $M$ sensors, thus the total number of sensors of a twin-line array is $N$=2$M$. The array received $N\times 1$ signal vector $\boldsymbol{x}(t)$ can be written as

$$\boldsymbol{x}(t) = \boldsymbol{a}(\theta_1)s_1(t) + \boldsymbol{v}(t) \quad (1)$$

where $\boldsymbol{a}(\theta)$ is the array steering vector toward direction $\theta$, $s_1(t)$ is the desired signal, and $\boldsymbol{v}(t)$ denotes the interference-plus-noise vector, and it can be written as [8]

$$\boldsymbol{v}(t) = \sum_{k=2}^{q} \boldsymbol{a}(\theta_k) s_k(t) + \boldsymbol{n}(t) \quad (2)$$

where $s_k(t)$ denotes the signal of the $k$th source, and $\boldsymbol{n}(t)$ is the additive Gaussian noise vector, and

$$\boldsymbol{a}(\theta) = \boldsymbol{a}_T(\theta) \otimes \boldsymbol{a}_L(\theta) \quad (3)$$

$$\boldsymbol{a}_L(\theta) = \left[1, e^{-jkd\cos\theta}, \cdots, e^{-jk(M-1)d\cos\theta}\right]^T \quad (4)$$

$$\boldsymbol{a}_T(\theta) = [1, e^{jkD\sin\theta}]^T \quad (5)$$

where $d$ is the inter-sensor spacing, $k=2\pi f/c$ is the wavenumber, $f$ is frequency and $c$ is the sound speed, $q$ is the number of sources. $\boldsymbol{a}_L(\theta)$ is the steering vector of the left line array, $\boldsymbol{a}_T(\theta)$ is the steering vector of a transversal pair of $n$th port and starboard sensors, located at the positions with coordinates $((n-1)d, -D/2)$ and $((n-1)d, D/2)$. In this paper, $(\cdot)^T$ and $(\cdot)^H$ stand for the transpose and conjugate transpose, respectively. $\otimes$ denotes the Kronecker product.

### 2.1 Twin-line beampattern

Assume only one plane wave signal coming from $\theta_1$ with unit power exists, then using (3)–(5), the normalized output amplitude response (beampattern) of a twin-line array to the desired signal when steered to $\theta$ can be written as

$$\begin{aligned}
|B(f,\theta;\theta_1)| &= \frac{1}{N}\left|\boldsymbol{a}^H(\theta)\boldsymbol{a}(\theta_1)\right| \\
&= \frac{1}{N}\left|[\boldsymbol{a}_T(\theta)\otimes\boldsymbol{a}_L(\theta)]^H[\boldsymbol{a}_T(\theta_1)\otimes\boldsymbol{a}_L(\theta_1)]\right| \\
&= \frac{1}{2M}\left|\boldsymbol{a}_T^H(\theta)\boldsymbol{a}_T(\theta_1)\boldsymbol{a}_L^H(\theta)\boldsymbol{a}_L(\theta_1)\right| \\
&= \left|\frac{\sin(\pi f M d(\cos\theta-\cos\theta_1)/c)}{M\sin(\pi f d(\cos\theta-\cos\theta_1)/c)}\right| \times \\
&\qquad \left|\cos[\pi f D(\sin\theta-\sin\theta_1)/c\right| \quad (6)
\end{aligned}$$

It can be seen that the first term in (6) is the beampattern of a single-line array [6], [7], and the second term is the beampattern of a transversal two-element sensor pair.

Fig. 2 depicts the beampatterns of a twin-line array when the arriving directions of the desired signal are 90° and 60°, respectively. The array separation is $D=\lambda/4$. For signal coming from 90°, the second term in (6) equals to zero in the back-beam direction $\theta=-\theta_1$, referred to as mirror direction, *i.e.*, $|B(f,-\theta_1;\theta_1)|=0$, so it has the best port-starboard discrimination performance to the broadside for $D=\lambda/4$.

Thus, for narrowband signal, to achieve the best port-starboard discrimination performance, the array separation $D$ must be adjusted to satisfy $|B(f,-\theta_1;\theta_1)|=0$, *i.e.*,

$$D = (n \pm 1/4)\lambda / \sin\theta_1, n \in \mathbb{Z} \quad (7)$$

### 2.2 Port-starboard rejection ratio (PSRR)

We define a parameter named as port-starboard rejection ratio (PSRR) to evaluate the port-starboard discrimination performance. The PSRR is defined as the ratio of the array power response in the signal DOA to that in the mirror direction. It's obvious that, the larger the PSRR, the better the performance of the port-starboard discrimination. Using (6), the PSRR can be expressed as

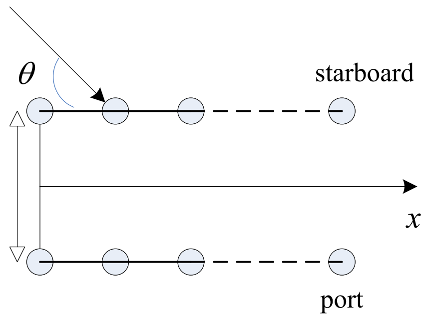



Figure 1 – Geometry of twin-line array

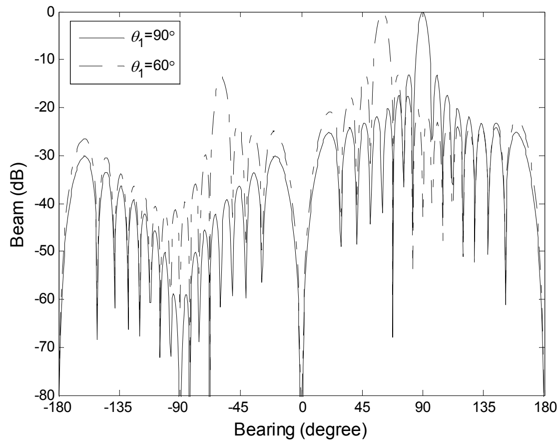



Figure 2 – Beampatterns of a twin-line array for signal arriving from 90° and 60°, respectively. ($D=\lambda/4$)

$$\text{PSRR}=\frac{|B(f,\theta_1;\theta_1)|^2}{|B(f,-\theta_1;\theta_1)|^2} = \frac{1}{\cos^2(2\pi f D\sin\theta_1/c)} \quad (8)$$

The value of PSRR characterizes the ability to reject or to cancel a plane wave coming from the back-beam direction. According to (8), we can see that, the PSRR is determined by the signal frequency, the signal arriving direction and the twin-line array separation. If the array separation $D$ satisfies (7), then the value of PSRR will approach infinite, as in Fig. 2, there is a null in the mirror direction of $\theta_1$=90°, whereas the backlobe is higher for the signal coming from 60° because the denominator in (8) is not equal to zero under this array separation.

The above conclusion about the performance of port-starboard discrimination is theoretically optimal because the influence of interference and noise are not considered. In the next section, we'll analyze the more general expression for the PSRR with more parameters considered, such as SNR, INR, SIR, the angular separation and correlation between the desired signal and the interference, etc.

## 3. GENERAL PSRR ANALYSIS

The output of the conventional plane-wave beamforming is given by

$$y(t) = \boldsymbol{w}^H \boldsymbol{x}(t) \quad (9)$$

where $\boldsymbol{w}$ is the weight vector, for conventional delay-and-sum beamforming, $\boldsymbol{w}$ is the same as the steering vector toward the desired signal.

From (1), we can express the sample covariance matrix (SCM) as [8]

$$\hat{\boldsymbol{R}} = \frac{1}{K}\sum_{t=1}^{K} \boldsymbol{x}(t)\boldsymbol{x}^H(t) = \hat{\sigma}_{s_1}^2 \boldsymbol{a}_1\boldsymbol{a}_1^H + \boldsymbol{a}_1\hat{\boldsymbol{r}}^H + \hat{\boldsymbol{r}}\boldsymbol{a}_1^H + \hat{\boldsymbol{Q}} \quad (10)$$

where $K$ is the number of samples, $\boldsymbol{a}_1$ is the abbreviation for $\boldsymbol{a}(\theta_1)$, and $\hat{\sigma}_{s_1}^2$ denotes the sample-mean of the desired signal power

$$\hat{\sigma}_{s_1}^2 = \frac{1}{K}\sum_{t=1}^{K} |s_1(t)|^2$$

$\hat{\boldsymbol{r}}$ denotes the sample-mean of the correlation between the desired signal and the interference-plus-noise:

$$\hat{\boldsymbol{r}} = \frac{1}{K}\sum_{t=1}^{K} s_1^*(t)\boldsymbol{v}(t) \quad (11)$$

and $\hat{\boldsymbol{Q}}$ is the sample covariance matrix of the interference-plus-noise

$$\hat{\boldsymbol{Q}} = \frac{1}{K}\sum_{t=1}^{K} \boldsymbol{v}(t)\boldsymbol{v}^H(t) \quad (12)$$

Now we redefine the PSRR as the ratio between the practical output power toward the desired signal arriving direction and that in the back-beam direction, as follows

$$\widehat{\text{PSRR}} = \frac{\boldsymbol{w}_+^H \hat{\boldsymbol{R}} \boldsymbol{w}_+}{\boldsymbol{w}_-^H \hat{\boldsymbol{R}} \boldsymbol{w}_-} \quad (13)$$

where $\boldsymbol{w}_+$ and $\boldsymbol{w}_-$ denote the weight vector steered to the desired signal direction and to the mirror signal direction, respectively. For conventional beamforming, $\boldsymbol{w}_+ = \boldsymbol{a}_1$, and $\boldsymbol{w}_- = \boldsymbol{a}_{1-}$, here $\boldsymbol{a}_{1-}$ stands for $\boldsymbol{a}(-\theta_1)$, then (13) can be rewritten as

$$\widehat{\text{PSRR}} = \frac{\boldsymbol{a}_1^H \hat{\boldsymbol{R}} \boldsymbol{a}_1}{\boldsymbol{a}_{1-}^H \hat{\boldsymbol{R}} \boldsymbol{a}_{1-}} = \frac{\boldsymbol{a}_1^H[\hat{\sigma}_{s_1}^2 \boldsymbol{a}_1\boldsymbol{a}_1^H + \boldsymbol{a}_1\hat{\boldsymbol{r}}^H + \hat{\boldsymbol{r}}\boldsymbol{a}_1^H + \hat{\boldsymbol{Q}}]\boldsymbol{a}_1}{\boldsymbol{a}_{1-}^H[\hat{\sigma}_{s_1}^2 \boldsymbol{a}_1\boldsymbol{a}_1^H + \boldsymbol{a}_1\hat{\boldsymbol{r}}^H + \hat{\boldsymbol{r}}\boldsymbol{a}_1^H + \hat{\boldsymbol{Q}}]\boldsymbol{a}_{1-}} = \frac{\hat{\sigma}_{s_1}^2 N^2 + 2N\,\mathrm{Re}(\boldsymbol{a}_1^H\hat{\boldsymbol{r}}) + \boldsymbol{a}_1^H\hat{\boldsymbol{Q}}\boldsymbol{a}_1}{\hat{\sigma}_{s_1}^2 |\boldsymbol{a}_{1-}^H\boldsymbol{a}_1|^2 + 2\,\mathrm{Re}(\boldsymbol{a}_{1-}^H\hat{\boldsymbol{r}}\boldsymbol{a}_1^H\boldsymbol{a}_{1-}) + \boldsymbol{a}_{1-}^H\hat{\boldsymbol{Q}}\boldsymbol{a}_{1-}} \quad (14)$$

where Re(·) means the real part, and

$$|\boldsymbol{a}_{1-}^H\boldsymbol{a}_1| = N\cos(2\pi fD\sin\theta_1/c)$$

Substituting (2) into (11), we have

$$\hat{\boldsymbol{r}} = \sum_{k=2}^{q} \boldsymbol{a}(\theta_k)\left(\frac{1}{K}\sum_{t=1}^{K} s_1^*(t)s_k(t)\right) + \frac{1}{K}\sum_{t=1}^{K} s_1^*(t)\boldsymbol{n}(t)$$

$$= \sum_{k=2}^{q} \boldsymbol{a}(\theta_k)\hat{\sigma}_{s_1}\hat{\sigma}_{s_k}\hat{\rho}_{1k} + \hat{\sigma}_{s_1}\hat{\boldsymbol{\sigma}}_{\boldsymbol{n}} \odot \hat{\boldsymbol{\rho}}_{1\boldsymbol{n}} \quad (15)$$

where $\hat{\rho}_{1k}$ is the sample-mean of the correlation coefficient between $s_1(t)$ and $s_k(t)$, $\hat{\boldsymbol{\sigma}}_{\boldsymbol{n}}$ is the noise standard deviation vector, and $\hat{\boldsymbol{\rho}}_{1\boldsymbol{n}}$ is the sample-mean of the correlation coefficient vector between $s_1(t)$ and $\boldsymbol{n}(t)$, $\odot$ denotes the element-wise vector or matrix product. When the desired signal and the noise is uncorrelated, then the second term in (15) can be omitted.

From (14) and (15), we can see that the performance of port-starboard discrimination is determined by a few parameters, such as the power of the desired signal, the interference and noise, the correlation between the desired signal and the interference, the array separation, and so on.

## 4. THEORETICAL PSRR IN SOME SPECIAL CASES

In section 3, we obtain the sample-based expression for PSRR, see (14). Now, replacing the sample covariance matrix $\hat{\boldsymbol{R}}$ with the exact covariance matrix $\boldsymbol{R}$, we obtain the theoretical PSRR expression for large sample condition. The expectation of (14) can be written as

$$\text{PSRR} = \frac{\boldsymbol{w}_+^H \boldsymbol{R} \boldsymbol{w}_+}{\boldsymbol{w}_-^H \boldsymbol{R} \boldsymbol{w}_-} = \frac{\sigma_{s_1}^2 N^2 + 2N\,\mathrm{Re}(\boldsymbol{a}_1^H\boldsymbol{r}) + \boldsymbol{a}_1^H\boldsymbol{Q}\boldsymbol{a}_1}{\sigma_{s_1}^2 |\boldsymbol{a}_{1-}^H\boldsymbol{a}_1|^2 + 2\,\mathrm{Re}(\boldsymbol{a}_{1-}^H\boldsymbol{r}\boldsymbol{a}_1^H\boldsymbol{a}_{1-}) + \boldsymbol{a}_{1-}^H\boldsymbol{Q}\boldsymbol{a}_{1-}} \quad (16)$$

### 4.1 Desired signal only

In the case that only the desired signal is present, we have

$$\boldsymbol{v}(t) = \boldsymbol{n}(t)$$

Assume $\boldsymbol{n}(t)$ is independent of $s_1(t)$, and has zero mean and covariance matrix $\sigma_n^2\boldsymbol{I}$, it gives

$$\boldsymbol{r} = E\{s_1^*(t)\boldsymbol{v}(t)\} = E\{s_1^*(t)\boldsymbol{n}(t)\} = \boldsymbol{0} \quad (17)$$

where **0** is a all-zero vector, and

$$\boldsymbol{Q} = E\{\boldsymbol{n}(t)\boldsymbol{n}^H(t)\} = \sigma_n^2\boldsymbol{I} \quad (18)$$

Substituting (17)–(18) into (16), and after some algebraic manipulation, we readily get

$$\text{PSRR} = \frac{N\sigma_{s_1}^2 + \sigma_n^2}{N\sigma_{s_1}^2\cos^2(2\pi fD\sin\theta_1/c) + \sigma_n^2} \quad (19)$$

Denoting by SNR as the signal-to-noise ratio of the desired signal at a single omnidirectional sensor

$$\text{SNR} = \frac{\sigma_{s_1}^2}{\sigma_n^2}$$

we can rewrite (19) as

$$\text{PSRR} = \frac{N\text{SNR} + 1}{N\text{SNR}\cos^2(2\pi fD\sin\theta_1/c) + 1}. \quad (20)$$

Thus, for the SNR is fixed, we have

$$1 \le \text{PSRR} \le N\text{SNR} + 1 \quad (21)$$

and for fixed $D$, $f$ and $\theta_1$, when

$$\text{SNR} \gg 1/[N\cos^2(2\pi fD\sin\theta_1/c)],$$

we have

$$\text{PSRR} \simeq \frac{1}{\cos^2(2\pi fD\sin\theta_1/c)} \quad (22)$$

that is, (20) is reduced to (8), while for

$$\text{SNR} \ll 1/[N\cos^2(2\pi fD\sin\theta_1/c)],$$

we have

$$\text{PSRR} \simeq N\text{SNR} + 1 \quad (23)$$

From (22) and (23), we can conclude that, the PSRR is linearly increasing with SNR up to (22) where it levels up and fixed.

### 4.2 Desired signal and single interference

In the case of a single interference, we have

$$\boldsymbol{v}(t) = \boldsymbol{a}_2 s_2(t) + \boldsymbol{n}(t) \quad (24)$$

and also, $\boldsymbol{n}(t)$ is independent of $s_2(t)$ and has zero mean and covariance matrix $\sigma_n^2\boldsymbol{I}$, then

$$\begin{aligned}\boldsymbol{r} &= E\{s_1^*(t)\boldsymbol{v}(t)\} \\ &= \boldsymbol{a}_2 E\{s_1^*(t)s_2(t)\} \\ &= \boldsymbol{a}_2\sigma_{s_1}\sigma_{s_2}\rho_{12}\end{aligned} \quad (25)$$

where $\rho_{12}$ is the correlation coefficient between $s_1(t)$ and $s_2(t)$

$$\rho_{12} = \frac{E\{s_1^*(t)s_2(t)\}}{\sigma_{s_1}\sigma_{s_2}} \quad (26)$$

And

$$\boldsymbol{Q} = \sigma_{s_2}^2\boldsymbol{a}_2\boldsymbol{a}_2^H + \sigma_n^2\boldsymbol{I}. \quad (27)$$

Using (3) and (25), we can get

$$\begin{aligned}\text{Re}(\boldsymbol{a}_1^H\boldsymbol{r}) &= \sigma_{s_1}\sigma_{s_2}\,\text{Re}(\boldsymbol{a}_1^H\boldsymbol{a}_2\rho_{12}) \\ &= N\sigma_{s_1}\sigma_{s_2}\,\text{Re}(\alpha\rho_{12})\end{aligned} \quad (28)$$

$$\boldsymbol{a}_1^H\boldsymbol{Q}\boldsymbol{a}_1 = \sigma_{s_2}^2 N^2|\alpha|^2 + N\sigma_n^2 \quad (29)$$

$$\boldsymbol{a}_{1-}^H\boldsymbol{Q}\boldsymbol{a}_{1-} = \sigma_{s_2}^2|\boldsymbol{a}_{1-}^H\boldsymbol{a}_2|^2 + N\sigma_n^2 \quad (30)$$

where

$$\alpha = \boldsymbol{a}_1^H\boldsymbol{a}_2/|\boldsymbol{a}_1||\boldsymbol{a}_2| = \boldsymbol{a}_1^H\boldsymbol{a}_2/N \quad (31)$$

is a complex scalar characterizes the *spatial correlation coefficient* between the steering vectors of the desired signal and the interference. And using (3)–(5), we readily have

$$\begin{aligned}\boldsymbol{a}_{1-}^H\boldsymbol{a}_2/N &= \boldsymbol{a}_T^H(-\theta_1)\boldsymbol{a}_T(\theta_2)\boldsymbol{a}_L^H(-\theta_1)\boldsymbol{a}_L(\theta_2)/N \\ &= \boldsymbol{a}_T^H(-\theta_1)\boldsymbol{a}_T(\theta_2)\boldsymbol{a}_L^H(\theta_1)\boldsymbol{a}_L(\theta_2)/N \\ &= e^{jk(M-1)d(\cos\theta_1-\cos\theta_2)/2}\frac{\sin(kMd(\cos\theta_1-\cos\theta_2)/2)}{M\sin(kd(\cos\theta_1-\cos\theta_2)/2)} \\ &\quad\times e^{jkD(\sin\theta_1+\sin\theta_2)/2}\cos[kD(\sin\theta_1+\sin\theta_2)/2] \\ &= B(f,-\theta_1;\theta_2)\end{aligned} \quad (32)$$

So, we get

$$|\boldsymbol{a}_{1-}^H\boldsymbol{a}_1|^2/N^2 = \cos^2(kD\sin\theta_1) \quad (33)$$

$$\frac{\text{Re}(\boldsymbol{a}_{1-}^H\boldsymbol{r}\boldsymbol{a}_1^H\boldsymbol{a}_{1-})}{N^2} = \sigma_{s_1}\sigma_{s_2}\beta \quad (34)$$

and $\beta$ is shown in the bottom of this page.

Substituting the results (28)–(35) into (16), and let both the nominator and denominator be divided by $N^2$, we get

$$\text{PSRR} = \frac{P_+}{P_-} \quad (36)$$

where

$$P_+ = \sigma_{s_1}^2 + 2\sigma_{s_1}\sigma_{s_2}\,\text{Re}(\alpha\rho_{12}) + \sigma_{s_2}^2|\alpha|^2 + \sigma_n^2/N \quad (37)$$

$$\begin{aligned}P_- &= \sigma_{s_1}^2\cos^2(kD\sin\theta_1) + 2\sigma_{s_1}\sigma_{s_2}\beta \\ &\quad + \sigma_{s_2}^2|B(f,-\theta_1;\theta_2)|^2 + \sigma_n^2/N\end{aligned} \quad (38)$$

Let $\text{INR} = \sigma_{s_1}^2/\sigma_n^2$ denote the interference-to-noise ratio, and then dividing (37) and (38) by $\sigma_n^2$, we get

$$\begin{aligned}P_+ &= \text{SNR} + 2\sqrt{\text{SNR}\cdot\text{INR}}\,\text{Re}(\alpha\rho_{12}) \\ &\quad + \text{INR}|\alpha|^2 + 1/N\end{aligned} \quad (39)$$

$$\begin{aligned}P_- &= \text{SNR}\cos^2(kD\sin\theta_1) + 2\sqrt{\text{SNR}\cdot\text{INR}}\,\beta \\ &\quad + \text{INR}|B(f,-\theta_1;\theta_2)|^2 + 1/N\end{aligned} \quad (40)$$

That is, the PSRR is affected by a few parameters in practice, such as SNR, INR, or signal-to-interference ratio (SIR, SIR=SNR/INR), the correlation coefficient between the desired signal and the interference, the spatial correlation coefficient related to the angular separation between the desired signal and the interference, array separation and the array size, *etc*.

From (39)–(40), when the INR is much smaller than the SNR, or when the spatial correlation coefficient $\alpha$ approaches zero, then (36) will reduce to (20).

## 5. SIMULATION RESULTS

In our simulations, we consider an $N$=32 twin-line array with half-wavelength inter-element spacing, and the array separation is a quarter of wavelength. The experimental results were computed using (14) by 100 Monte-Carlo runs, and the analytical results were computed by (36), (39)–(40), the number of samples is 100.

In the first experiment, we consider there is an correlated interference near the endfire direction, and the desired signal is in the broadside with arriving angle 20°.

$$\begin{aligned}\beta &= \cos(kD\sin\theta_1)\cos[kD(\sin\theta_1+\sin\theta_2)/2]\frac{\sin(kMd(\cos\theta_1-\cos\theta_2)/2)}{M\sin(kd(\cos\theta_1-\cos\theta_2)/2)} \\ &\quad\times\text{Re}\left\{\rho_{12}e^{jk[(M-1)d(\cos\theta_1-\cos\theta_2)-D(\sin\theta_1-\sin\theta_2)]/2}\right\}\end{aligned} \quad (35)$$

Fig. 3 shows the analytical PSRR and simulated experimental PSRR versus the input SNR, for SIR=20dB and SIR=—20dB, respectively. For high SIR, the analytical curve coincides with the practical result precisely, and we can obtain the satisfactory port-starboard discrimination performance. However, when there are strong interferences, the PSRR can not increase with SNR, and it will level up when the SNR is large enough.

In the second experiment, we consider the effect of the correlation between the desired signal and the interference, to the value of the PSRR. Fig. 4 depicts the resulting PSRR versus the SNR. For low SIR, the PSRR will decrease when the interference is correlated with the desired signal, while the correlation has little effect to the PSRR for high SIR.

## 6. CONCLUSIONS

We have derived a general expression for the PSRR of a twin-line towed array based on the array output power in the signal direction and the mirror direction. Simulation results demonstrate that the PSRR performance was mainly affected by the array separation, the signal-to-interference ratio, the correlation between the desired signal and the interference, and the SNR.

## REFERENCES

[1] S. G. LEMON, "Towed-array History, 1917–2003," *IEEE J. of Oceanic Eng.*, vol. 29, pp. 365–373, Apr. 2004.
[2] I. W. Schurman, "Reverberation rejection with a dual-line towed array," *IEEE J. Oceanic Eng.*, vol. 21, pp. 193–204, Apr. 1996.
[3] J. P. Feuillet, W. S. Allensworth and B. K. Newhall, "Nonambiguous beamforming for a high resolution twin-line array," *J. Acoust. Soc. Amer.*, vol. 97, p. 3292, May 1995.
[4] Y. Doisy, "Port-starboard discrimination performances on active towed array systems," in *Proc. Underwater Defense Technology (UDT)*, Cannes, France, 1995, pp. 125–129.
[5] J. Groen, S. P. Beerens and et. al, "Adaptive port-starboard beamforming of triplet sonar arrays," *IEEE J. of Oceanic Eng.*, vol. 30, pp. 348–359, Apr. 2005.
[6] Q.-H. Li, "Preliminary analysis of left-right ambiguity resolution performance for twin-line array," *ACTA ACUSTICA*, pp. 385–388, May 2006.
[7] R. O. Nielsen, *Sonar Signal Processing*. Norwood, MA: Artech House, 1991.
[8] M. Wax and Y. Anu, "Performance analysis of the minimum variance beamformer," *IEEE Trans. Signal Process.* Vol. 44, pp. 928–937, Apr. 1996.

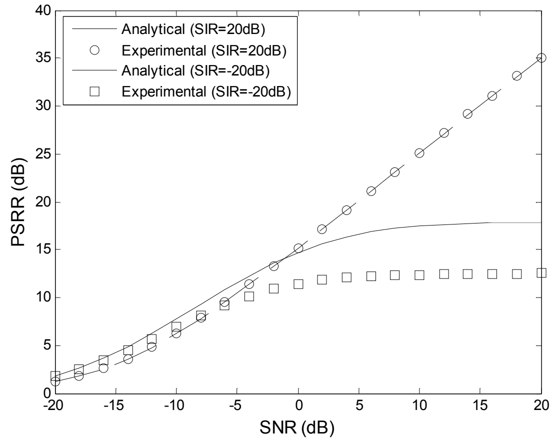
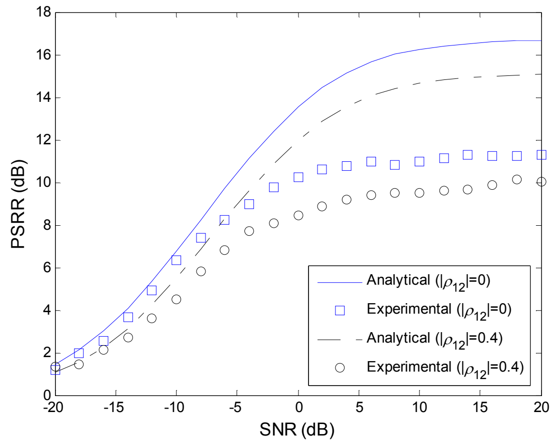
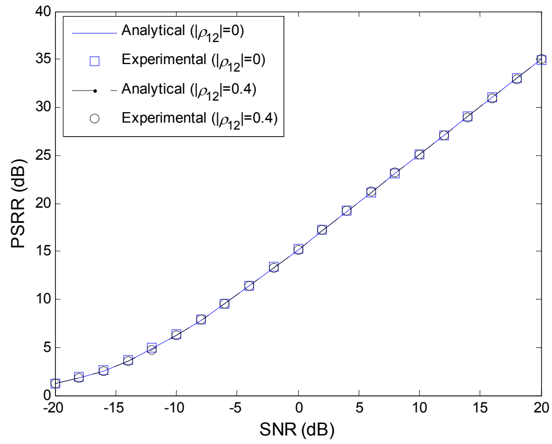



Figure 3 – The PSRR of the desired signal versus the SNR ($D=\lambda/4$, $\theta_1=90°$, $\theta_1=20°$, $|\rho_{12}|=0.4$)




(a) SIR=—20dB




(b) SIR=20dB

Figure 4 – The PSRR of the desired signal versus the SNR ($D=\lambda/4$, $\theta_1=90°$, $\theta_1=20°$)