16th European Signal Processing Conference (EUSIPCO 2008), Lausanne, Switzerland, August 25-29, 2008, copyright by EURASIP

ON FAST IMPLEMENTATION OF HARMONIC MUSIC FOR KNOWN AND UNKNOWN MODEL ORDERS

Jesper Rindom Jensen, Jesper Kjær Nielsen, Mads Græsbøll Christensen, Søren Holdt Jensen and Torben Larsen

Department of Electronic Systems, Aalborg University
Fredrik Bajers Vej 7, DK-9220 Aalborg , Denmark
E-mail: {jesperrj,jkjaer,mgc,shj,tl}@es.aau.dk

ABSTRACT

This paper presents a fast implementation of the MUltiple SIgnal Classification (MUSIC) estimation criterion for fundamental frequency estimation of harmonic signals with known or unknown model orders. First, we reformulate the MUSIC estimator such that the MUSIC estimate can be computed directly from the signal subspace or just an arbitrary basis thereof. We also discuss the selection of a subspace tracker based on the known or unknown rank of the signal and noise subspaces. Second, we introduce an implementation of the MUSIC estimator that only involves one FFT for known model orders, and we extend it to the case of unknown model orders. The performance gain in terms of computation times obtained by the efficient implementation is significant which is demonstrated through simulations.

1. INTRODUCTION

The problem of estimating the fundamental frequency of a periodic signal in white Gaussian noise is a classical signal processing problem and is encountered in many speech and audio applications. This comprises among others speech and audio coding, automatic music transcription and musical genre classification. The signal model in these problems is

$$x[n] = \sum_{k=1}^{L} A_k e^{jk\omega_0 n} + e[n] \tag{1}$$

where $A_k = |A_k| e^{j\phi_k}$ is the complex amplitude of the kth harmonic, L is the model order, ω_0 is the fundamental angular frequency, and $e[n]$ is complex white Gaussian noise [1]. The MUSIC estimator is a subspace-based method that can be used for estimating the frequency of individual sinusoids corrupted with white Gaussian noise [2]. In [3] and [4] the MUSIC estimation criterion has been used for joint estimation of the fundamental frequency and the model order of a periodic signal as in (1). Further, this has been extended to estimation of a set of fundamental frequencies in multi-pitch signals [5]. The MUSIC algorithm for both known and unknown model orders suffers from high computational complexity. This is mainly due to two integral steps of the MUSIC algorithm: i) The forming of an estimate of the $M \times M$ auto-correlation matrix and the partitioning of its eigenspace into a signal and a noise subspace spanned by the most and the least dominant eigenvectors, respectively. ii) The minimization of a non-convex cost function which depends on the noise subspace eigenvectors. The first step was originally resolved by using an eigenvalue decomposition (EVD) to form the signal and noise subspaces, and it has a dominating cost of $O(M^3)$. Throughout the years extensive research has resulted in several subspace tracking algorithms with a lower computational complexity. An overview over some of these algorithms is given in [6] and [7]. The algorithms can be grouped into high, medium and low complexity subspace trackers with a computational complexity of $O(M^2L')$, $O(ML'^2)$ and $O(ML')$, respectively, where M is the length of a sample vector from the signal in (1) and L' is the rank of the subspace with $L' = L$ for the signal subspace and $L' = M - L$ for the noise subspace. The low complexity subspace trackers introduce approximations in order to achieve the $O(ML)$ cost, and this influences the performance of the subspace trackers with respect to orthonormality and convergence rate. Additionally, most of the low complexity subspace trackers only track an arbitrary orthonormal basis of the subspace, and not the subspace eigenvectors. Some examples are PAST [8], NIC [9], FAPI [10] and FDPM [7]. Only a few low complexity subspace trackers track the eigenvectors with some examples being PASTd[8], SWASVD3 [11], LORAF3 [12] and FST [13]. Efficient implementation of the minimization of the non-convex cost function in MUSIC has not been treated in many publications. In [14], root MUSIC is proposed where the MUSIC estimate is found by examining the roots of a polynomial, and in [4] an efficient implementation is proposed using an FFT for each noise subspace eigenvector.

In this paper, we propose efficient implementations of the MUSIC estimation criterion for the signal model in (1). First, the selection of the subspace tracker with respect to rank and whether or not the model order is known is treated. Second, we introduce a fast implementation of the cost function of the MUSIC estimator for known model orders. This involves just one FFT regardless of the model order without direct access to the signal or noise subspace eigenvectors, but only an arbitrary orthonormal basis of the subspace. This enables the use of a fast signal subspace tracker and MUSIC coherently. The algorithm is also extended for the case of unknown model orders. The paper is organized as follows. In Section 2 we restate the MUSIC estimator for known and unknown model orders. This serves as the starting point for the proposed performance improvements introduced in Section 3. These improvements are evaluated in Section 4, and Section 5 concludes the paper.

2. THE MUSIC ESTIMATION CRITERION

In this section, we present the basics of the MUSIC estimation criterion for known and unknown model orders. Consider the assumed model of the data in (1) and denote the M-dimensional signal vector $\boldsymbol{x}[n]$ as

$$\boldsymbol{x}[n] = \begin{bmatrix} x[n] & x[n+1] & \cdots & x[n+M-1] \end{bmatrix}^T \tag{2}$$

The work of M.G. Christensen was supported by the Parametric Audio Processing project, Danish Research Council for Technology and Production Sciences grant no. 274–06–0521

The work of S.H. Jensen was partly supported by the Danish Technical Research Council, through the framework project Intelligent Sound, www.intelligentsound.org (STVF No. 26-04-0092)

where $[\cdot]^T$ denotes the transpose. If the phases of the complex amplitudes are uniformly distributed on the interval $[-\pi, \pi]$, the auto-correlation matrix of the data vector $\boldsymbol{x}[n]$ is [1]

$$\boldsymbol{R}_{xx} = E\left\{\boldsymbol{x}[n]\boldsymbol{x}^H[n]\right\} = \boldsymbol{A}\boldsymbol{P}\boldsymbol{A}^H + \sigma_e^2\boldsymbol{I} \tag{3}$$

where $E\{\cdot\}$ denotes the statistical expectation, and $(\cdot)^H$ denotes the conjugate transpose or Hermitian. The scalar σ_e^2 is the variance of the complex white Gaussian noise assumed uncorrelated with the signal, $\boldsymbol{I}$ is the $M \times M$ identity matrix, and $\boldsymbol{P} = \text{diag}\{[P_1 \quad P_2 \quad \cdots \quad P_L]\}$ is a diagonal matrix with P_k being the power of the kth complex amplitude. Further, the matrix $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = [\boldsymbol{a}_1 \quad \boldsymbol{a}_2 \quad \cdots \quad \boldsymbol{a}_L] \tag{4}$$

where the vector $\boldsymbol{a}_i = \begin{bmatrix}1 & e^{j\omega_0 i} & \cdots & e^{j(M-1)\omega_0 i}\end{bmatrix}^T$. The EVD of (3) is [7]

$$\boldsymbol{R}_{xx} = [\boldsymbol{S} \quad \boldsymbol{G}] \begin{bmatrix}\boldsymbol{D}_S + \sigma_e^2\boldsymbol{I}_S & \boldsymbol{0} \\ \boldsymbol{0} & \sigma_e^2\boldsymbol{I}_G\end{bmatrix} \begin{bmatrix}\boldsymbol{S}^H \\ \boldsymbol{G}^H\end{bmatrix} \tag{5}$$

where $\boldsymbol{S} = [\boldsymbol{s}_1 \quad \boldsymbol{s}_2 \quad \cdots \quad \boldsymbol{s}_L]$ contains the L dominant orthonormal eigenvectors that span the signal subspace, and $\boldsymbol{G} = [\boldsymbol{g}_1 \quad \boldsymbol{g}_2 \quad \cdots \quad \boldsymbol{g}_{M-L}]$ contains the $M-L$ least dominant orthonormal eigenvectors that span the noise subspace. The columns of the unitary matrix $\boldsymbol{U} = [\boldsymbol{S} \quad \boldsymbol{G}]$ are the eigenvectors of $\boldsymbol{R}_{xx}$. The matrix $\boldsymbol{D}_S$ is diagonal and contains the L largest eigenvalues of $\boldsymbol{A}\boldsymbol{P}\boldsymbol{A}^H$ in decreasing order. The noise subspace is orthogonal to the signal subspace which is also spanned by the columns of $\boldsymbol{A}$. Therefore, the noise subspace is orthogonal to $\boldsymbol{A}$ which yields

$$\boldsymbol{A}^H\boldsymbol{G} = \boldsymbol{0}\ . \tag{6}$$

Define the cost function of the MUSIC estimator as

$$J = \|\boldsymbol{A}^H\boldsymbol{G}\|_F^2 = \text{Tr}\left\{\boldsymbol{A}^H\boldsymbol{G}\boldsymbol{G}^H\boldsymbol{A}\right\} \tag{7}$$

where $\text{Tr}\{\cdot\}$ denotes the trace, and $\|\cdot\|_F$ denotes the Frobenius norm. Moreover, let Ω be a set of candidate fundamental frequencies with cardinality $|\Omega|$. Then, the MUSIC estimator for known model orders is given by

$$\widehat{\omega}_0 = \arg\min_{\omega_0 \in \Omega} J\ . \tag{8}$$

Since the value of the estimator in (8) varies with the model order L, the MUSIC estimator must be normalized to enable joint estimation of fundamental frequency and model order. This yields the MUSIC estimator for unknown model orders [4]

$$(\widehat{\omega}_0, \widehat{L}) = \arg\min_{\omega_0 \in \Omega,\ L \in \mathcal{L}} \min \frac{J}{ML(M-L)} \tag{9}$$

where $\mathcal{L}$ is a set of candidate model orders with cardinality $|\mathcal{L}|$. The matrix $\boldsymbol{A}$ depends on both ω_0 and L while the noise subspace matrix $\boldsymbol{G}$ only depends on L. Additionally, the set of model orders $\mathcal{L}$ depends on ω_0 since the harmonics are bounded by the Nyquist frequency.

3. EFFICIENT IMPLEMENTATION

Recall from Section 2 that the MUSIC estimation process consists of two steps: i) Compute the noise subspace matrix $\boldsymbol{G}$ from the observed data. ii) Find the minimum of a non-convex cost function. In this section we discuss how to implement both steps in an efficient way with respect to the rank of the signal and noise subspace and taking into account whether the model order is known or unknown. In Subsection 3.1 we reformulate the first step to enable the use of a fast subspace tracker in situations with known model orders, and we discuss the limitation of fast subspace trackers in situations with unknown model orders. In Subsection 3.2 and 3.3 we propose an efficient implementation of the second step for known and unknown model orders, respectively.

3.1 Subspace Tracking

The estimators in (8) and (9) make use of the cost function in (7). This cost function is formulated to be dependent on the noise subspace matrix $\boldsymbol{G}$. If the rank of the matrix $\boldsymbol{G}$ is greater than the rank of the signal subspace matrix $\boldsymbol{S}$, the computational complexity can be lowered by rewriting (7) using $\boldsymbol{I} = \boldsymbol{S}\boldsymbol{S}^H + \boldsymbol{G}\boldsymbol{G}^H$ into

$$J = ML - \|\boldsymbol{A}^H\boldsymbol{S}\|_F^2 = ML - \text{Tr}\left\{\boldsymbol{A}^H\boldsymbol{S}\boldsymbol{S}^H\boldsymbol{A}\right\} \tag{10}$$

so that the cost function now depends on $\boldsymbol{S}$. Thus for the fastest evaluation of J, the formulation involving the subspace of the lowest rank, the minor subspace, should be used.

If only an arbitrary orthonormal basis $\boldsymbol{W}$ of the subspace spanned by $\boldsymbol{S}$ is available and the model order is known, (10) can be reformulated into

$$J = ML - \|\boldsymbol{A}^H\boldsymbol{W}\|_F^2 \tag{11}$$

since

$$\boldsymbol{S}\boldsymbol{S}^H = \boldsymbol{W}\boldsymbol{Q}(\boldsymbol{W}\boldsymbol{Q})^H = \boldsymbol{W}\boldsymbol{W}^H \tag{12}$$

where $\boldsymbol{Q}$ is an arbitrary unitary matrix. The last reformulation is very important since it allows the use of fast subspace trackers that only track an arbitrary orthonormal basis of $\boldsymbol{S}$ or $\boldsymbol{G}$. The MUSIC estimator for unknown model orders in (9) requires the cost function in (7) to be evaluated for each candidate model order. If for example $\boldsymbol{S}$ is known for the largest candidate model order, this reevaluation of J can be performed by successively removing the last column of $\boldsymbol{S}$ until J is evaluated for all candidate model orders. If, however, only an arbitrary orthonormal basis $\boldsymbol{W}$ of the subspace spanned by $\boldsymbol{S}$ is available for the largest candidate model order, this approach fails. This is because projection matrices of subsets of $\boldsymbol{S}$ cannot uniquely be recovered from subsets of $\boldsymbol{W}$. To demonstrate this, partition $\boldsymbol{S} = [\boldsymbol{S}_1 \quad \boldsymbol{S}_2]$ and $\boldsymbol{W} = [\boldsymbol{W}_1 \quad \boldsymbol{W}_2]$ into two subsets whose dimensions are pairwise equal. Inserting this into (12) readily yields

$$\boldsymbol{S}_1\boldsymbol{S}_1^H + \boldsymbol{S}_2\boldsymbol{S}_2^H = \boldsymbol{W}_1\boldsymbol{W}_1^H + \boldsymbol{W}_2\boldsymbol{W}_2^H \tag{13}$$

from which $\boldsymbol{S}_1\boldsymbol{S}_1^H$ cannot be recovered since only $\boldsymbol{W}_1$ and $\boldsymbol{W}_2$ are known. That is, the subspace trackers that track the eigenvectors like PASTd[8], LORAF3 [12] and FST [13], and not just any arbitrary orthonormal basis, must be used in the case of unknown model orders. Alternatively, the model order can be estimated before the frequency estimation, but this approach suffers from a less accurate estimate as compared to the joint estimation in (9).

3.2 Known Model Orders

In this section we propose an efficient implementation of the cost function in (7) for known model orders. The algorithm is extended in Subsection 3.3 to the case of unknown model orders. We consider the case where an arbitrary basis $\boldsymbol{W}$ of the signal subspace matrix $\boldsymbol{S}$ is known. Define the matrix $\boldsymbol{M}$ as $\boldsymbol{M} = \boldsymbol{I} - \boldsymbol{W}\boldsymbol{W}^H$ and note that this matrix is Hermitian. Evaluating the trace of the cost function in (7) yields

$$J = \sum_{i=1}^{L} \boldsymbol{a}_i^H\boldsymbol{M}\boldsymbol{a}_i = \sum_{i=1}^{L} J_i \tag{14}$$

where J_i denotes the ith partial cost function. The partial cost function J_i can be rewritten into

$$J_i = \begin{bmatrix} 1 \\ e^{-j\omega i} \\ \vdots \\ e^{-j(M-1)\omega i} \end{bmatrix}^T \begin{bmatrix} m_{11} & \cdots & m_{1M} \\ \vdots & \ddots & \vdots \\ m_{M1} & \cdots & m_{MM} \end{bmatrix} \begin{bmatrix} 1 \\ e^{j\omega i} \\ \cdots \\ e^{j(M-1)\omega i} \end{bmatrix}$$

$$= \begin{bmatrix} m_{1M} \\ \vdots \\ m_{12} + m_{23} + \cdots + m_{(M-1)M} \\ m_{11} + m_{22} + m_{33} + \cdots + m_{MM} \\ m_{21} + m_{32} + \cdots + m_{M(M-1)} \\ \vdots \\ m_{M1} \end{bmatrix}^T \begin{bmatrix} e^{j(M-1)\omega i} \\ \vdots \\ e^{j\omega i} \\ 1 \\ e^{-j\omega i} \\ \vdots \\ e^{-j(M-1)\omega i} \end{bmatrix}$$

$$= \sum_{r=-(M-1)}^{M-1} c[r] e^{jr\omega i} \quad (15)$$

where $c[r]$ is the sum of the elements on the rth diagonal of $\boldsymbol{M}$. Exploiting the Hermitian property of $\boldsymbol{M}$, (15) can be rewritten as

$$J_i = c[0] + \sum_{r=1}^{M-1} c[r] e^{jr\omega i} + \sum_{r=-(M-1)}^{-1} c[r] e^{jr\omega i}$$

$$= c[0] + \left[\sum_{r=1}^{M-1} c^*[r] e^{-jr\omega i} \right]^* + \sum_{r=1}^{M-1} c^*[r] e^{-jr\omega i}$$

$$= 2\text{Re} \left[\sum_{r=0}^{N-1} g[r] e^{-jr\omega i} \right] = 2\text{Re} \left[G(e^{j\omega i}) \right] \quad (16)$$

where $[\cdot]^*$ denotes the complex conjugate and $g[r]$ is given by

$$g[r] = \begin{cases} c[0]/2 & \text{for } r = 0 \\ c^*[r] & \text{for } r = 1, 2, \cdots, M-1 \\ 0 & \text{for } r = M, M+1, \cdots, N-1 \end{cases} . \quad (17)$$

If the discrete-time Fourier transform in (16) is sampled with $\omega = 2\pi k/N$, the discrete Fourier transform $J_i[ki]$ is obtained. This is desirable from a computational complexity point of view since it can be computed by an FFT of $g[r]$ with ki as the frequency index. The dominant cost for evaluating (16) is thus $O(N \log_2 N)$ where N is the FFT-length.

The cost function in (14) can be computed from (16). First, we simply evaluate (16) for $i = 1$ and obtain $J_1[k]$ from which we create the partial cost function vector $\boldsymbol{J}_1$ containing the values corresponding to the desired subset Ω of candidate fundamental frequencies. Next, for $i = 2, 3, \cdots, L$ we extract every ith sample from $J_1[ki]$ for $k \mapsto \omega \in \Omega$ to obtain $\boldsymbol{J}_2, \boldsymbol{J}_3, \cdots, \boldsymbol{J}_L$, all of the same length as $\boldsymbol{J}_1$. Note, that since we extract the samples from $J_1[k]$, we need to restrict the maximum frequency in Ω to be less than π/L. The discrete version of the cost function in (14) defined on the subset Ω of candidate fundamental frequencies is thus the sum of the partial cost function vectors with the resolution determined by the FFT-length, i.e.

$$\boldsymbol{J} = \sum_{i=1}^{L} \boldsymbol{J}_i . \quad (18)$$

Using this approach, the dominant cost of computing the cost function in (14) from the sequence $g[r]$ is still $O(N \log_2 N)$ computations.

Input: - Orthonormal basis of the signal subspace $\boldsymbol{W} \in \mathbb{C}^{M \times L}$ - FFT-length N - Candidate frequencies $\Omega = \{\omega_{\min}, \cdots, \omega_{\max}\}$ with cardinality $\lvert\Omega\rvert$		
Step 1: Find $g[r]$		
$\boldsymbol{c}' = M\boldsymbol{e}_1 - \sum_{k=1}^{M} [\boldsymbol{W}_a]_{k:k+M-1,:} [\boldsymbol{W}]^H_{k,:}$	(19)	$O(M^2 L)$
$g[r] = \begin{cases} c[0]/2 & \text{for } r = 0 \\ c^*[r] & \text{for } 1 \leq r < M \\ 0 & \text{for } M \leq r < N \end{cases}$	(17)	$O(1)$
Step 2: Compute $J_1[k]$ from an FFT of $g[r]$		
$J_1[k] = 2\text{Re} \left[\sum_{r=0}^{N-1} g[r] e^{-j2\pi rk/N} \right]$	(16)	$O(N \log_2 N)$
Step 3: Downsample $J_1[k]$		
$J[k] = \sum_{i=1}^{L} J_1[ki] \quad \forall k \mapsto \omega \in \Omega$	(18)	$O(L\lvert\Omega\rvert)$

Table 1: Fast evaluation of the MUSIC estimator for known model orders.

In (16) the discrete Fourier transform is computed from the sequence $g[r]$ defined in (17). Recall, that $c^*[r]$, $r = 0, 1, \cdots, M-1$ is computed from the lower triangular part of $\boldsymbol{M}$. Therefore, we reformulate $\boldsymbol{M} = \boldsymbol{I} - \boldsymbol{W}\boldsymbol{W}^H$ to yield $c^*[r]$ directly by

$$\boldsymbol{c}' = \begin{bmatrix} c[0] \\ c^*[1] \\ \vdots \\ c^*[M-1] \end{bmatrix} = \begin{bmatrix} m_{11} + m_{22} + m_{33} + \cdots + m_{MM} \\ m_{21} + m_{32} + \cdots + m_{M(M-1)} \\ \vdots \\ m_{M1} \end{bmatrix}$$

$$= M\boldsymbol{e}_1 - \sum_{k=1}^{M} [\boldsymbol{W}_a]_{k:k+M-1,:} [\boldsymbol{W}]^H_{k,:} \quad (19)$$

where $\boldsymbol{e}_1 = [1 \quad 0 \quad \cdots \quad 0]^T$ is the unit vector and $\boldsymbol{W}_a = [\boldsymbol{W} \quad \boldsymbol{0}]^T$ is an augmented matrix obtained by combining $\boldsymbol{W}$ and the $M-1 \times L$ zero matrix. The notation $[\cdot]_{a:b,c:d}$ indicates that a submatrix is created from the rows running from a to b and the columns running from c to d of the matrix. If all the rows or columns are used from the matrix, we simply write ':'. From (19) we readily obtain $g[r]$ defined in (17).

This concludes our proposed fast implementation of the evaluation of the MUSIC estimator in (8). The algorithm is summarized in Table 1 where the different dominant costs are given. The dominant cost is $O(M^2 L) + O(N \log_2 N)$. This cost should be compared to the dominant cost $O((M-L) N \log_2 N) + O((M-L) L \lvert\Omega\rvert)$ of the fast implementation proposed in [4]. Another obvious advantage of our algorithm is that it computes the MUSIC estimator from an arbitrary orthonormal basis of the signal subspace and not the noise subspace eigenvectors.

3.3 Unknown Model Orders

We now extend our fast implementation of the MUSIC estimator for known model orders to the case with unknown model orders in (9). The extension is straightforward besides that the eigenvectors of the auto-correlation matrix in (5) spanning the signal subspace must be known instead of just an arbitrary basis of it.

The MUSIC estimator for unknown model orders is obtained by extending the MUSIC estimator for known model orders to be evaluated for each candidate model order in the

Input: - Signal subspace eigenvectors $\boldsymbol{S} \in \mathbb{C}^{M \times L_{\max}}$ - FFT-length N - Candidate frequencies $\Omega = \{\omega_{\min}, \cdots, \omega_{\max}\}$ with cardinality $\lvert\Omega\rvert$ - Candidate model orders $\mathcal{L} = \{L_{\min}, \cdots, L_{\max}\}$ with cardinality $\lvert\mathcal{L}\rvert$	
for $l = L_{\min}, \cdots, L_{\max}$	
Step 1: Find $g_l[r]$	
if $l = L_{\min}$	
$\boldsymbol{c}'_l = M\boldsymbol{e}_1 - \sum_{k=1}^{M} [\boldsymbol{S}_a]_{k:k+M-1,1:l} [\boldsymbol{S}]^H_{k,1:l}$ (20)	$O(M^2 l)$
else	
$\boldsymbol{c}'_l = \boldsymbol{c}'_{l-1} - \sum_{k=1}^{M} [\boldsymbol{S}_a]_{k:k+M-1,l} [\boldsymbol{S}]^H_{k,l}$ (21)	$O(M^2)$
end	
$g_l[r] = \begin{cases} c_l[0]/2 & \text{for } r = 0 \\ c_l^*[r] & \text{for } 1 \leq r < M \\ 0 & \text{for } M \leq r < N \end{cases}$ (17)	$O(1)$
Step 2: Compute $J_{1,l}[k]$ from an FFT of $g_l[r]$	
$J_{1,l}[k] = 2\text{Re}\left[\sum_{r=0}^{N-1} g_l[r] e^{-j2\pi rk/N}\right]$ (16)	$O(N \log_2 N)$
Step 3: Downsample $J_{1,l}[k]$	
$J_l[k] = \sum_{i=1}^{l} J_{1,l}[ki] \quad \forall k \mapsto \omega \in \Omega_l$ (18)	$O(l\lvert\Omega_l\rvert)$
end	

Table 2: Fast evaluation of the MUSIC estimator for unknown model orders.

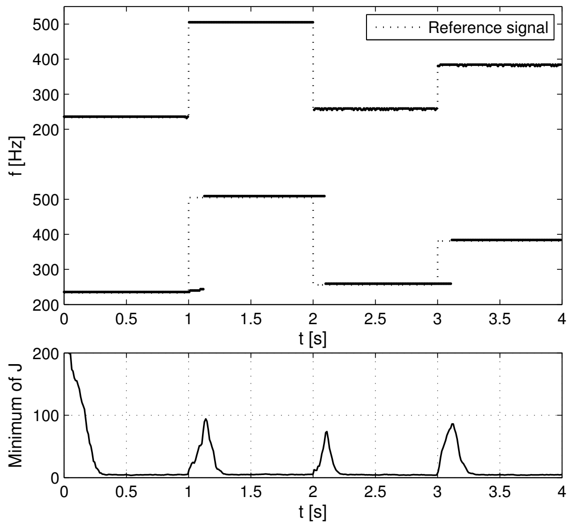

Figure 1: MUSIC frequency estimates of a synthetic signal with a known model order and proposed cost function implementation (top), using an EVD (top curve) and an FDPM subspace tracker (bottom curve), respectively. The bottom plot shows the value of the MUSIC cost function when using FDPM.

subset $\mathcal{L}$ and then seeking the joint minimum among the candidate fundamental frequencies and model orders. Thus for all $L \in \mathcal{L}$, the MUSIC estimator for known model orders is evaluated using the proposed implementation in Subsection 3.2 for a subset of candidate fundamental frequencies Ω_L dependent on the model order where the set Ω_L is a subset of Ω such that $\Omega_L = \{\omega \vert \omega \in \Omega, \omega < \pi/L\}$. This dependence is required since the largest harmonic in the observed signal in (1) is bounded by the Nyquist frequency.

The proposed implementation structure entails recalculation of the sequence $g[r]$ for all $L \in \mathcal{L}$. This can be done recursively if equation (19) is rewritten into

$$\boldsymbol{c}' = M\boldsymbol{e}_1 - \sum_{l=1}^{L}\sum_{k=1}^{M} [\boldsymbol{S}_a]_{k:k+M-1,l} [\boldsymbol{S}]^H_{k,l} \quad (20)$$

from which the recursive form for $l = 2, \cdots, L_{\max}$ is obtained as

$$\boldsymbol{c}'_l = \boldsymbol{c}'_{l-1} - \sum_{k=1}^{M} [\boldsymbol{S}_a]_{k:k+M-1,l} [\boldsymbol{S}]^H_{k,l} \quad (21)$$

where $\boldsymbol{c}'_1$ is found from (20) for $L = 1$.

Table 2 summarizes our fast implementation of the MUSIC estimator for unknown model orders. The recursive calculation of the sequence $g[r]$ ensures that the dominant cost of the algorithm in Table 2 only increases due to the need for computing an FFT $\lvert\mathcal{L}\rvert$ times instead of one time which was the case for the implementation of the MUSIC estimator for known model orders in Table 1. Thus, the dominant cost is $O(M^2 L_{\max}) + O(\lvert\mathcal{L}\rvert N \log_2 N)$ operations.

4. SIMULATION RESULTS

In this section, the MUSIC estimator for known and unknown model orders is evaluated using different implementations. First, we evaluate the performance improvement in the case of known model orders using a synthetic signal. Second, we evaluate the performance improvement in the case of unknown model orders using a violin signal.

Different implementations of the MUSIC estimator for known model orders were tested on a synthetic signal since the model order had to be known. The synthetic signal was generated as a harmonic signal with a model order of three, a fundamental frequency changing in steps, a sampling frequency $f_s = 8000$ kHz and an SNR of 10 dB. The set of candidate frequencies Ω was chosen to be in the interval from 60 Hz to 1000 Hz. An FFT-length of 2048 was chosen which resulted in a frequency resolution of 3.9 Hz, and the sample size was $M = 82$. In the top plot of Fig. 1 two curves are shown. The upper curve was generated using an EVD while the lower curve was generated using the FDPM subspace tracker [7]. The bottom plot shows the minimum value of the MUSIC cost function when the FDPM subspace tracker was used. It is seen that the use of the FDPM tracker resulted in a less accurate estimate due to the convergence time in the frequency transitions. The computation time, however, was significantly reduced as shown in Table 3. The table shows the computation time for the old implementation and the new implementation of the MUSIC cost function for both the EVD and the FDPM tracker. Notice that the main source of the computational complexity was the use of an EVD instead of the FDPM subspace tracker. The performance improvement obtained using the new implementation of evaluation of the MUSIC cost function had a significant impact in the case of the FDPM subspace tracker, but less impact in the case of an EVD.

Different implementations of the MUSIC estimator for unknown model orders were tested on a violin signal with an SNR of 10 dB. The spectrogram of the violin signal is shown in top plot of Fig. 2. The middle plot shows the estimates of the fundamental frequency and the bottom plot shows the normalized minimum of the MUSIC cost function using an EVD and the PASTd subspace tracker [8], respectively. In the simulations a sample size of $M = 110$ at a sampling fre-

Obtaining signal subspace	EVD		FST	
Imp. of cost function	Old	New	Old	New
Known model order	11.6 s	11.2 s	0.85 s	0.48 s
Unknown model order	38.9 s	34.7 s	11.9 s	7.1 s

Table 3: Simulation times on a 2.2 GHz Intel® Core^{TM}2 Duo laptop for the simulation with known and unknown model orders, respectively. The table shows the simulation times for the old and the new implementation of the cost function for the signal subspace where each of the implementations were tested with an eigenvalue decomposition (EVD) and a fast subspace tracker (FST), respectively.

quency of $f_s = 11025$ Hz was chosen. The set of candidate fundamental frequencies Ω was chosen to be in the interval from 60 Hz to 1000 Hz, and the set of candidate model orders $\mathcal{L}$ was in the interval from 4 to 8. The FFT-length was 4096 corresponding to a frequency resolution of 2.7 Hz. It is seen that the use of the PASTd tracker resulted in a less accurate estimate as compared to the use of an EVD. The computation time, however, is again significantly reduced as can be seen from Table 3. We notice again that the main source of computational complexity was the use of an EVD instead of the PASTd subspace tracker. The new implementation had only a significant impact on the simulation time when the PASTd subspace tracker.

5. CONCLUSION

In this paper, we presented fast implementations of the MUSIC estimator for known and unknown model orders. We showed how fast subspace tracker could be applied by reformulating the MUSIC estimator. For known model order only an arbitrary basis of the subspace must be known whereas the subspace eigenvectors must be known for unknown model orders. The fast subspace trackers should be chosen accordingly. We also proposed a new way to calculate the MUSIC cost function by use of only one FFT for known model orders and by use of L FFTs for unknown model orders. The simulations showed that these improvements could lower the computation time significantly, especially when fast subspace trackers were applied. The precision of the frequency estimate, however, was reduced due to approximations introduced by the subspace trackers. The new implementation of the cost function entailed no approximations.

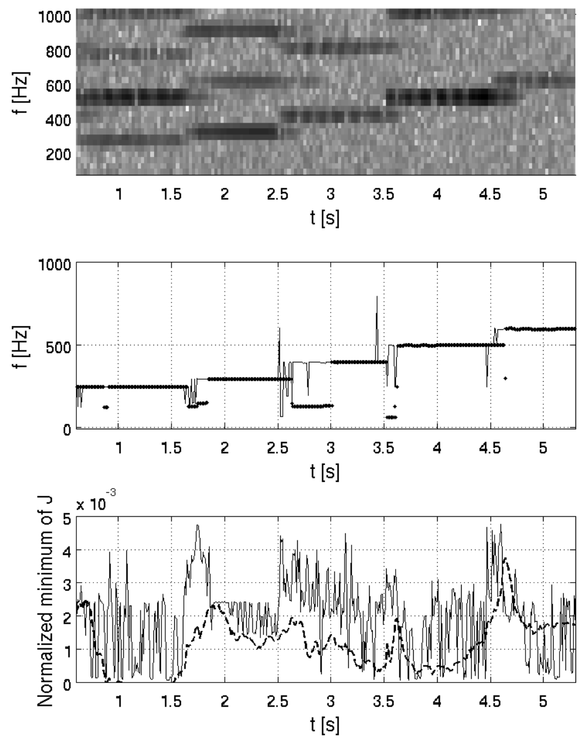

Figure 2: Spectrogram of a violin signal (top), MUSIC frequency estimates of it (middle) using an EVD (line) and a PASTd subspace tracker (dots), and normalized value of the MUSIC cost function (bottom) using an EVD (line) and PASTd (dotted line).

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