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BAND CONTROL POLICY OF PLAYOUT SCHEDULING FOR VOICE OVER IP

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ABSTRACT

We study adaptive-playout scheduling for VoIP using the framework of stochastic impulse control theory. A Wiener process is introduced to model the fluctuation of the buffer length in the absence of control. In this context, the control signal consists of length units that correspond to inserting or dropping a pitch cycle. We define an optimality criterion that has an adjustable trade-off between average buffing delay and average control length (the length of the pitch cycles added plus the length of the pitch cycles dropped). The clock-drift effect is treated in a unified manner within this framework. A band control policy is shown to be optimal. The algorithm does not require knowledge of the clock drift. It maintains the buffer length within a band region by imposing impulse control (inserted or dropped pitch cycles) whenever the bounds of the band are reached. Our experiments show that the proposed method outperforms a popular reference method.

1. INTRODUCTION

Voice over internet protocol (VoIP), also known as internet telephony, has experienced rapid growth in recent years. It unifies communication between computer clients and mobile and regular phones. Encoded speech packets are delivered over the internet via UDP or RTP protocols aimed at real-time voice communications. The transmission mechanism introduces undesirable varying network delays, known as *jitter*, and packet loss. To combat unreliable packet delivery, packet loss concealment (PLC) techniques are commonly employed at the receiver to improve the perceived speech quality (e.g., [2]). To address the variation in network delay, a dejitter buffer is generally used. The received packets are temporally queued in the buffer to even out the network delay jitter. With increasing buffer length, the packet loss rate due to the late arrival decreases. However, a long buffer length negatively affects real-time communication. In designing a dejitter buffer an inevitable trade-off must be made between correcting the effect of delay jitter and maintaining the desirable low delay.

Packet networks typically exhibit time-varying characteristics stemming from dynamic packet-switched routing and from varying network load. In a situation with good network conditions, where the packets arrive in proper order and with little delay, a relatively short dejitter buffer length can be used, thus reducing end-to-end delay. Conversely, under poor network conditions, a long buffer length is desirable to avoid large underflows and, thus, improve the listening quality of the speech signal. This suggests the optimization of voice communication quality by adjusting the buffer length adaptively in response to the network conditions.

Adaptive playout scheduling for VoIP is the subject of a significant ongoing research effort. The objective of adaptive-playout scheduling is to pursue an optimal trade-off between packet-loss rate and the average buffering delay (e.g., [3], [4], [8]). The basic principle behind all reported methods is to predict future network delay based on the past packet-delay trajectory. This requires a time synchronization operation in the communication system to give accurate network delays and to allow removal of the *clock-drift* effect. The synchronization requirement is generally not trivial and reduces the transmission efficiency. Two schemes of playout adjustment have been proposed, each using a particular characteristic of the speech signal. The first scheme is based on the fact that the speech can be grouped into talkspurts separated by periods of silence. The scheme artificially elongates or shortens the silence duration on a per talkspurt basis (e.g., [4] and [8]) and is referred to as *silence-oriented* adjustment. This first scheme has the advantage that it does not affect the listening quality of speech. The second scheme exploits the pitch correlations of active speech. It stretches or compresses the active speech segment on a per packet basis (e.g., [3]). The difference between the processed speech segment duration and the original segment duration is constrained to be an integer multiple of the pitch period. This second scheme is referred to as *voice-oriented* adjustment. It generally introduces degradation of the listening quality. On the other hand, the scheme provides more flexibility in playout adjustment than the silence-oriented scheme.

This paper solves the problem of adaptive playout scheduling for VoIP rigorously using the framework of the well-established impulse control theory (e.g., [6]). The playout scheduling facilitates dynamic modification of the buffer length based on its historical trajectory up to the decision-making moment, referred to as a *non-anticipative* control scheme. A Wiener process (see [5]) is employed to model the variation of the buffer length in the absence of control. Thus, the Wiener process describes the network conditions. Motivated by [3], we use a voice-oriented playout adjustment to impose control on the dejitter buffer to obtain increased controllability of the buffer length. Our approach has two major advantages over earlier methods. First, the proposed system can operate without a time synchronization module and it deals with clock-drift in an integrated manner within the control framework. Second, the proposed system establishes an adjustable trade-off between the average rate of artificially generating or dropping speech (in samples per output sample) and the average buffering delay. Thus, our approach unifies the penalties on the packet loss rate and the playout adjustment operations, which are the two main causes of degradation of the quality of speech communication for a packet network.

The remainder of the paper is organized as follows. Section 2 motivates the stochastic modeling of the dejitter buffer using a Wiener process, and the employment of impulse control. In Section 3, we define the stochastic impulse control problem and describe the optimal control policy. A rigorous argument of the optimality of our solution is provided in Section 4 and 5. Experimental results are presented in Section 6, followed by a short conclusion.

2. DEJITTER BUFFER MODELING

The fluctuation of the buffer length X_t in the absence of control can be modelled approximately as a Wiener process [5] with drift term μ_t and variance σ_t^2, for which the stochastic differential equation is expressed as

$$dX_t = \mu_t dt + \sigma_t dW_t, \tag{1}$$

where W_t', the first derivative of W_t, is zero-mean, white Gaussian noise. We interpret (1) informally by studying the properties of the

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increment ΔX_t in a time interval $[t, t+\Delta t]$ at the receiver. To start let us assume that there is no packet loss across the network. In a situation with zero clock drift, the speech consuming rate and the average speech arriving rate is the same, resulting in a zero mean for ΔX_t , i.e., $\mu_t = 0$. If affected only by network delay jitter, ΔX_t varies around zero. Our model (1) describes this variation with a Gaussian random variable with variance $\sigma_t^2 \Delta t$. The parameter σ_t describes the network condition: an increase in σ_t corresponds to an increased delay jitter. To facilitate the Wiener process formulation, the increments ΔX_t and ΔX_s over disjoint time intervals, i.e., $[t, t+\Delta t] \cap [s, s+\Delta s] = 0$, are assumed to be independent. Although this assumption may not hold for nearby time intervals, it simplifies the problem formulation and enables rigorous treatment. In the situation of nonzero clock-drift, the drift term $\mu_t dt$ in (1) is nonzero. A negative value of $\mu_t dt$ represents a faster clock rate in the receiver than in the transmitter, which corresponds to a larger consuming rate than producing rate.

An implicit condition of exploiting a Wiener process is the assumption of ordered arrival of the packets. This guarantees that the content in the buffer is always the expected speech to be played out. This assumption is satisfied in most cases in practice, as the packets for a particular network link are usually routed across the same path.

We are to enforce impulse control on the buffering system and denote the resulting buffer-length process by Z_t. The impulse control scheme assumes that the buffering system usually works fine without control. When the buffer is in abnormal status (e.g., at a high risk of underflow), a control signal u_t is added to the buffer length instantaneously, i.e., $Z_t = Z_{\{t-\}} + u_t$. At any time the buffer length can be adjusted upward or downward by impulse control signals that each consist of an integer multiple of the pitch period L_p, i.e., $Z_t = Z_{\{t-\}} + kL_p, k \in (\mathbb{Z} - \{0\})$. For simplicity, we assume the pitch period to be constant, which is accurate over short time intervals of voiced speech. The system operation is the same for voiced speech, unvoiced speech, and silence. This means that the impulse control signals are restricted to a set of discrete values also for unvoiced speech and silence.

We pursue an optimal control strategy to minimize the average buffering delay while keeping the average control length within a prescribed range, under the constraint that $Z_t \geq L_{\min}$. The lower bound $L_{\min}$ ($L_{\min} \geq 0$) on the length of the speech queued in the buffer is set as a safe-guard against underflows. A penalty on the control signal is used to measure the speech quality degradation caused by the imposed control. A so-called band control policy will be shown to be optimal, which indicates that the buffer length Z_t is always maintained within a band region.

In the situation that a packet is lost, i.e. the next packet has arrived before the current packet, the impulse control scheme can still work without interruption. In the case that a lost packet contains a non-integer number of pitch periods, exploiting the impulse control scheme alone can cause a signal discontinuity (mismatch) at the boundary of signal segments correspond the missing one and next received packets. To avoid the mismatch problem, a short segment is required to be extrapolated backward right before the received speech segment to bridge the two speech segments. We generalize the average control length to include the generated segment length.

The impulse control strategy is commonly applied in finance ([1]) and industrial inventory management ([6]). The Wiener process is used to model the fluctuation of an account or a storage system. An optimal regulating strategy is sought to balance the average cost of holding the content and the adjustment cost incurred by enforcing impulse control. The main difference between our problem and the problems in finance and inventory management is the constraint that the control signals are restricted to the countable set kL_p, where k is an integer.

3. IMPULSE CONTROL OF THE DEJITTER BUFFER

In this section we formulate the problem rigorously. The time averaged cost function associated with average buffer length and average control length is formally defined. The optimal control policy is then provided.

3.1 Problem Formulation

In this subsection we introduce a stochastic differential equation for Z_t. The non-anticipative control strategy is formally defined. Finally, the two trade-off terms, average buffering delay and average control length are introduced.

Suppose the buffer length X_t without control fluctuates with drift μ, variance σ^2 and starting state x. Upon enforcing an impulse control, the dynamic system is given by

$$dZ_t = \mu dt + \sigma dW_t + \xi_{T_i} L_p 1_{\{T_i = t\}}. \tag{2}$$

The process Z_t is called a *semi-martingale* stochastic process as a result of the imposed impulse control. The term $1_{\{T_i=t\}}$ is a random indicator function, with one indicating that a control signal $\xi_{T_i} L_p$ is enforced on Z_t at time instant t.

Let $\mathscr{F} = \{g(X_s, 0 \leq s) | g \text{ is arbitrary}\}$. Thus, $\mathscr{F}$ is a set which contains all the random variables defined over the buffer length track X_s, $s \geq 0$. A particular $g \in \mathscr{F}$ might be a control signal $\xi_{T_i} L_p$. Similarly, we define $\mathscr{F}_t = \{g(X_s, 0 \leq s \leq t)\}$. Thus, for any $s < t$, $\mathscr{F}_s \subset \mathscr{F}_t$. To simplify notation, we denote $\mathbb{E}[g|X_0 = x]$ by $\mathbb{E}_x[g]$, for all $g \in \mathscr{F}$. A valid policy $\varphi = \{(T_n, \xi_{T_n} L_p) | n \geq 0, \xi_{T_n} \in (\mathbb{Z} - \{0\})\}$ consists of a random time sequence $\{T_0(=0), T_1, \ldots\}$ and a sequence of random variables $\{\xi_{T_0}, \xi_{T_1}, \ldots\}$ representing the sign and number of L_p for each adjustment such that

$$\begin{aligned} &Z_t \geq L_{\min} \text{ for all } t \geq 0. \\ &\xi_{T_i} \in \mathscr{F}_{T_i} \quad \text{for all} \quad i = 0, 1, \ldots. \end{aligned} \tag{3}$$

$T_0 = 0$ indicates the a control signal $\xi_{T_0} L_p$ is enforced on X_t at $t = 0$. Denote the policy space as $\mathscr{P}$, then $\varphi \in \mathscr{P}$. The constraint (3) ensures that the control signal ξ_{T_i} is a function $\xi_{T_i}(X_s, s \leq T_i)$, which indicates that $\mathscr{P}$ is a non-anticipative policy space.

Denote the last control index up to time t as N_t, i.e., $N_t = \max\{i \geq 0 | T_i \leq t\}$ for $t \in [0, \infty)$. Then the average buffering delay under a control policy φ is defined as

$$D_{ave}(\varphi) = \limsup_{T \to \infty} \frac{1}{T} \mathbb{E}_x \left[\int_0^T Z_t dt \right]. \tag{4}$$

The operator [sup] is used for technical reasons [6]. Similarly, define the average control length under the control policy φ as

$$L_{ave}(\varphi) = \limsup_{T \to \infty} \frac{1}{T} \mathbb{E}_x \left[\sum_{i=0}^{N_T} |\xi_{T_i}| L_p \right]. \tag{5}$$

We define the time averaged cost function as a Lagrangian:

$$\begin{aligned} J(x, \varphi) &= D_{ave}(\varphi) + \beta L_{ave}(\varphi) \\ &= \limsup_{T \to \infty} \frac{1}{T} \mathbb{E}_x \left[\left(\int_0^T Z_t dt + \beta L_p \sum_{i=0}^{N_T} |\xi_{T_i}| \right) \right]. \end{aligned} \tag{6}$$

The parameter β is the Lagrange multiplier that determines the trade-off between buffering delay and average control length. The optimal control policy is the one minimizing $J(x, \varphi)$ for a particular β, expressed as

$$\varphi^* = \arg \min_{\varphi \in \mathscr{P}} J(x, \varphi). \tag{7}$$

We denote the corresponding minimum average cost as $J_{\min}$.

3.2 Optimal Control Policy

We now show that a band control policy is optimal for the problem defined by (7). S is the upper bound of a band region which is determined by β. Specifically, the controlled buffer length Z_t is maintained within a band region $[L_{\min}, S]$, $L_{\min} < S - L_p$. Whenever the buffer length Z_t hits the lower bound $L_{\min}$, an additional speech segment of length L_p is generated and added to Z_t, incurring an increment of control length by L_p. On the other hand, whenever Z_t reaches the upper bound S, a segment of length L_p is removed from the buffer while the control length is increased by L_p. We denote the above policy as $\varphi(S)$ for simplicity.

Theorem 3.1 *The Lagrange cost function $J(x, \varphi)$ in (7) admits an optimal control policy $\varphi(S^*)$. S^* is determined by β via the unique negative value of a parameter Δ:*

$$\frac{\sigma^2}{2\mu^2} e^{-\frac{2\mu}{\sigma^2}\Delta} - \frac{\sigma^2}{2\mu^2} + \frac{\Delta}{\mu} = 2\beta, \tag{8}$$

$$S^* = L_{\min} + L_p + |\Delta|, \tag{9}$$

$$\beta \geq -\frac{\sigma^2}{2\mu^2} - \frac{L_p}{2\mu} + \frac{L_p}{\mu(1 - e^{-\frac{2\mu}{\sigma^2}L_p})}. \tag{10}$$

For the case that $\mu = 0$, the equations are reduced to

$$S^* = L_{\min} + L_p + \sqrt{2\beta\sigma^2}, \tag{11}$$

$$\beta \geq \frac{L_p^2}{6\sigma^2}. \tag{12}$$

Note that in *Theorem* 3.1 the band control policy is optimal regardless of the drift parameter μ. In fact, μ only affects the cost function $J(x, \varphi(S))$. This indicates that the band control policy can be performed without knowing the clock drift term.

The theorem is proved in a constructive manner. Section 4 constructs a lower bound on the cost function $J(x, \varphi)$ over all feasible $\varphi \in \mathscr{P}$. In Section 5, the performance of a band control policy is analyzed. The optimality conditions of a band control policy are then constructed by showing that they lead to achievement of the derived lower bound.

Next we introduce a fundamental tool: an expectation formula for a stochastic process $f(Z_t)$ that is defined on the semi-martingale process Z_t. The construction of the lower bound and the analysis of a particular band control policy are based on this formula. The proof of the formula is provided in [6].

Proposition 3.2 *Suppose that $f : [L_{\min}, \infty) \to \mathbb{R}$ is continuously differentiable, has a bounded derivative, and has a continuous second derivative at all but a finite number of points. Then for each time $T > 0$, initial state $x \in \mathbb{R}_+$ and a policy $\varphi = \{(T_n, \xi_{T_n}), n \geq 0\} \in \mathscr{P}$, the stochastic process $f(Z_t)$ satisfies*

$$\mathbb{E}_x[f(Z_T)] = \mathbb{E}_x[f(Z_0)] + \mathbb{E}_x\left[\int_0^T \mathscr{A} f(Z_t)dt + \sum_{n=1}^{N_T} \theta_n\right], \tag{13}$$

where

$$\theta_n = f(Z_{T_n^-} + \xi_{T_n} L_p) - f(Z_{T_n^-}), \text{ for } n = 1, 2, \ldots$$
$$\text{and} \quad \mathscr{A} f = \mu f' + \tfrac{1}{2}\sigma^2 f''.$$

The defined process $f(Z_t)$ in *Proposition* 3.2 is general in the sense that (13) holds for any f satisfying the stated requirements and for any policy $\varphi = \{(T_n, \xi_{T_n} L_p) | n \geq 0\} \in \mathscr{P}$. Furthermore, a process $f(Z_t)$ can be related with a particular control policy φ by specifying θ_n in (13).

4. LOWER BOUND OVER ALL POLICIES

In this section, we construct the lower bound on $J(x, \varphi)$ over all policies. The basic principle is to put more constraints on the function f in *Proposition* 3.2 in order to derive the lower bound from (13). The constraints are imposed carefully to allow the existence of solutions.

Proposition 4.1 *Suppose that $f : [L_{\min}, \infty) \to \mathbb{R}$ satisfies all the hypotheses of* Proposition *3.2 and additionally*

$$\begin{aligned} \mathscr{A} f(x) - x + h &\leq 0, & \quad (14)\\ f(x + kL_p) - f(x) &\leq k\beta L_p \quad \text{for all } k \in \mathbb{Z}_+, & (15)\\ f(x - kL_p) - f(x) &\leq k\beta L_p \quad \text{for all } k \in \mathbb{Z}_+, & (16)\\ |f(x)| &\leq M, & (17) \end{aligned}$$

where h and M are constants. Then $J_{\min} \geq h$.

Proof $f(Z_t)$ defines a new process on the buffer length. Equations (15) and (16) implicitly put constraints on θ_n of (13) over all $\varphi \in \mathscr{P}$. Combining (13) and (14)–(16) produces

$$\mathbb{E}_x[f(Z_T)] \leq \mathbb{E}_x[f(Z_0)] - hT + \mathbb{E}_x\left[\int_0^T Z_t dt + \beta L_p \sum_{n=1}^{N_T} |\xi_n|\right]. \tag{18}$$

Dividing both sides of (18) by T and letting $T \to \infty$ results in

$$\limsup_{T\to\infty} \frac{1}{T}\left[\mathbb{E}_x\left(f(Z_T) - f(Z_0) + \beta L_p|\xi_0|\right)\right] + h \leq J(x, \varphi). \tag{19}$$

As the process $f(Z_t)$ is bounded by M from (17), it follows that $h \leq J(x, \varphi)$, for each policy $\varphi \in \mathscr{P}$ and each initial state $x \in \mathbb{R}_+$. Thus, there is a $J_{\min} \geq h$, proving the lower bound inequality. The function f might have more than one solution. It is obvious that h can take many values for a particular f. However only one value of h is relatively tight w.r.t. (14), i.e. the equality holds for some region of $[L_{\min}, \infty)$. This value is determined by f (denoted as $h(f)$) and possibly does not attain $J_{\min}$.

Note that the function f in *Proposition* 4.1 is restricted to provide a lower bound on (6). The imposed constraints on f do not narrow $\mathscr{P}$ to a smaller space. The crucial step in finding an optimal control policy w.r.t. (7) is to construct a particular function f^* satisfying (14)-(17) and to show that some control policy $\varphi^* \in \mathscr{P}$ reaches $J_{\min}$ by proving $h(f^*) = J(x, \varphi^*)$.

5. BAND CONTROL POLICY AND ITS OPTIMALITY CONDITIONS

In this section, we first discuss properties of a band control policy. We then derive *Theorem* 3.1 by determining the optimal conditions of a band control policy.

5.1 Band Control Policy

Now we study a band control policy $\varphi(S)$, $L_{\min} < S - L_p$, by investigating the computation of the associated cost function $J(x, \varphi(S))$.

The initial state $x \in \mathbb{R}_+$, ξ_{T_0} is selected to satisfy $Z_0 = x + \xi_{T_0} L_p \in (L_{\min}, S)$. The control sequence $(T_i, \pm L_p)$, $i = 1, 2, \ldots$, is then constructed recursively as Z_t evolves. The sign of the control signal depends on the boundary it reaches. To compute the cost function $J(x, \varphi(S))$, a process $V(Z_t)$ is constructed in a manner that $J(x, \varphi(S))$ can be derived from (13) by essentially taking $f = V$. We define the function $V(x) \in C^2$ (i.e. the second derivative of V is continuous) as

$$\begin{aligned} \mathscr{A} V(x) &= \frac{1}{2}\sigma^2 V''(x) + \mu V'(x)\\ &= x - l,\ L_{\min} \leq x \leq S, \end{aligned} \tag{20}$$

subject to the boundary conditions

$$V(L_p + L_{\min}) - V(L_{\min}) = \beta L_p, \tag{21}$$

$$V(S - L_p) - V(S) = \beta L_p. \tag{22}$$

The parameter l is a constant in (20). The two conditions (21)–(22) essentially specify θ_n in (13) for the band control policy $\varphi(S)$. $V(x)$ is called the *relative value function* associated with $\varphi(S)$.

Proposition 5.1 *Let $V : [L_{\min}, S] \to \mathbb{R}$, a function on buffer length Z_t, be defined by (20)–(22). Then the cost function $J(x, \varphi(S))$ is independent of the starting state x and is given by*

$$J(x,\varphi) = -\frac{\sigma^2}{2\mu} - \mu\beta + \frac{L_p}{2} + L_{\min} + \frac{2\beta\mu - L_p - L_{\min} + S}{1 - e^{-\frac{2\mu}{\sigma^2}(S - L_p - L_{\min})}}. \quad (23)$$

Proof The general solution to the ordinary differential equation (ODE) (20) is

$$V(x) = Ax - Be^{-\frac{2\mu}{\sigma^2}x} + \frac{x^2}{2\mu} + E, \quad (24)$$

with $l = -(\frac{\sigma^2}{2\mu} + \mu A)$. The two constants A and B are determined from (21) and (22) uniquely. As a result, l is uniquely specified. The parameter E in (24) can take any value.

To apply *Proposition* 3.2, $V(x)$ is extended to $[L_{\min}, \infty)$ to be

$$\hat{V}(x) = \begin{cases} V(x) & x \in [L_{\min}, S] \\ V(S) & x \in [S, \infty) \end{cases}. \quad (25)$$

Then, using *Proposition* 3.2 with $f = \hat{V}$, we obtain

$$\mathbb{E}_x[\hat{V}(Z_T) - \hat{V}(Z_0)] = \mathbb{E}_x\left[\int_0^T \mathscr{A}\hat{V}(Z_t)dt\right] + \mathbb{E}_x\left[\sum_{n=1}^{N_T} \theta_n\right], \quad (26)$$

where

$$\theta_n = \begin{cases} \hat{V}(L_{\min} + L_p) - \hat{V}(L_{\min}) \\ \hat{V}(S - L_p) - \hat{V}(S) \end{cases}.$$

The value of θ_n at $t = T_n^-$ depends on the boundary Z_t reaches. Since Z_t is always maintained within $[L_{\min}, S]$, this implies from (20) that $\mathscr{A}\hat{V}(Z_t) = Z_t - l$ in (26). Thus, it is further simplified as

$$\mathbb{E}_x\left[\hat{V}(Z_T) - \hat{V}(Z_0) + \beta|\xi_0|\right] + lT = \mathbb{E}_x\left[\int_0^T Z_t dt + \beta(L_p N_T + |\xi_0|)\right].$$

Dividing both sides by T and letting $T \to \infty$ results in $J(x,\varphi) = l$. The term $\limsup_{T\to\infty} \frac{1}{T}\mathbb{E}_x[\hat{V}(Z_T) - \hat{V}(Z_0)]$ vanishes in the above derivation as $\hat{V}(x)$ is bounded within $[L_{\min}, S]$. l takes the form as the expression of $J(x,\varphi)$ in (23).

Note that l is the constant, which makes that $\mathscr{A}\hat{V}(Z_t) = Z_t - l$ holds within the band region $[L_{\min}, S]$. It is proved in *Proposition* 5.1 that l is also the average cost $J(x,\varphi(S))$. This property facilitates the analysis of the optimal control policy w.r.t. (7).

5.2 Optimal Control Conditions and Parameters

In this subsection, we first provide optimal conditions for a band control policy. Then we show that the constructed band control policy is optimal w.r.t. $J(x,\varphi)$.

As is stated in *Proposition* 5.1, the average cost of a band control policy $\varphi(S)$ is given by (23). We are to find a particular band control policy $\varphi(S^*)$ with an average cost that forms a lower bound on all the feasible policies $\varphi \in \mathscr{P}$. Specifically, a solution of f satisfying all the inequalities in *Proposition* 4.1 is constructed explicitly. The resulting policy $\varphi(S^*)$ is shown to attain $J_{\min}$.

Following (24), $V'(x)$ takes the form of

$$V'(x) = A + B\frac{2\mu}{\sigma^2}e^{-\frac{2\mu}{\sigma^2}x} + \frac{x}{\mu}, \quad x \in [L_{\min}, S].$$

Now we impose optimal conditions on $V'(x)$ with

$$V'(S) = V'(S - L_p), \quad (27)$$
$$V'(S) \leq 0. \quad (28)$$

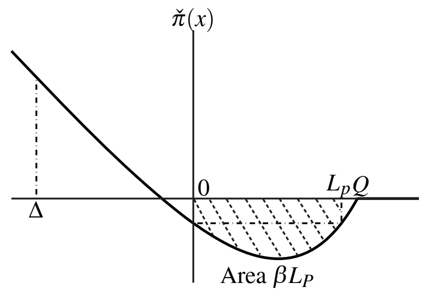

Figure 1: Demonstration of the function $\breve{\pi}(x)$.

Equ. (27) imposes a constraint on S, denoted as S^*. To facilitate the analysis of this band control policy, we define

$$\begin{aligned} \pi(x) &= V'(x + (S^* - L_p)) \\ &= A' + B'\frac{2\mu}{\sigma^2}e^{-\frac{2\mu}{\sigma^2}x} + \frac{x}{\mu}. \end{aligned} \quad (29)$$

Correspondingly, the constraints on $\pi(x)$ are

$$\pi(0) - \pi(L_p) = 0, \quad (30)$$
$$\int_0^{L_p} \pi(x)dx = -\beta L_p, \quad (31)$$
$$\int_\Delta^{\Delta + L_p} \pi(x)dx = \beta L_p, \quad (32)$$
$$\pi(0) \leq 0, \quad (33)$$

The upper bound S^* takes the form of (9). Equ. (30)–(31) determine A' and B' uniquely. Thus, $\pi(x)$ is uniquely specified. By studying (29) and (30), it is concluded that $\pi(x)$ is a convex function. This ensures that (32) uniquely determines Δ. Solving (32) produces an equation on Δ as specified by (8). Next solving $\pi(0) \leq 0$ produces an inequality on β as (10). In this situation, the constant l_{opt} which makes that $\mathscr{A}V(Z_t) = Z_t - l_{opt}$ holds within $[L_{\min}, S^*]$ is

$$l_{opt} = \beta\mu + S^* - \frac{L_p}{2}. \quad (34)$$

As discussed in subsection 5.1, l_{opt} is also the associated average cost $J(x,\varphi(S^*))$.

By proper extension of $V(x)$ to $[L_{\min}, \infty)$, it can be shown that the extended function $\check{V}(x)$ is one solution of f in *Proposition* 4.1. The constant l_{opt} provides a tight lower bound of $J(x,\varphi)$, i.e., $l_{opt} = J_{\min}$. Equ. (27) essentially guarantees that $\check{V}(x)$ satisfies (15) and (16). The inequality (28) makes $\check{V}(x)$ bounded over $[L_{\min}, \infty)$.

Proof of Theorem 3.1: Now we extend $\pi(x)$ to $[\Delta, \infty)$ as

$$\breve{\pi}(x) = \begin{cases} \pi(x) & \Delta \leq x \leq Q, \\ 0 & x > Q, \end{cases} \quad (35)$$

where $\pi(Q) = 0$, $L_p \leq Q$, as illustrated in Figure 1. The extended function $\check{V}(x)$ over $[L_{\min}, \infty)$ can thus be constructed as

$$\check{V}(x) = \int_{L_{\min}}^x \breve{\pi}(w + \Delta - L_{\min})dw.$$

Now we take $f = \check{V}$ in *Proposition* 4.1. Observe that $\check{V}(x)$ is continuously differentiable and has bounded derivative on $[L_{\min}, \infty)$. Its second derivative is also continuous except at $x = S^* + (Q - L_p)$. Therefore, $\check{V}(x)$ satisfies all the hypotheses of *Proposition* 3.2. Note that the construction of $\check{V}(x)$ is different from $\hat{V}(x)$ in (25) to enable the optimality argument.

Next, we study $\check{V}(x)$ by investigating the inequalities in *Proposition* 4.1. It is immediate from (29) and (35) that $\check{V}(x)$ is bounded over $[L_{\min}, \infty)$. As $\check{\pi}(x)$ is constructed as (35), this guarantees that $|\check{V}(x+L_p) - \check{V}(x)| \leq \beta L_p$ for all $x \geq L_{\min}$. Thus, the inequalities (15)–(16) follow directly. For $x \in [L_{\min}, (S^* + Q - L_p)^-]$, there is $\mathscr{A}\check{V}(x) = x - l_{opt}$. As $x = (S^* + Q - L_p)^+$, $\check{V}$ drops from a positive value to 0, resulting in $\mathscr{A}\check{V}((S^* + Q - L_p)^+) < (S^* + Q - L_p) - l_{opt}$. Increasing x further, $\mathscr{A}\check{V}(x)$ is fixed and $x - l_{opt}$ increases linearly. Thus, $\mathscr{A}\check{V}(x) < x - l_{opt}$ still holds for $x > S^* + Q - L_p$. This indicates that $l_{opt} \leq J_{\min}$ from *Proposition* 4.1. As l_{opt} is also the average cost $J(x, \varphi(S^*))$, thus $l_{opt} = J_{\min}$. This indicates that the band control policy specified by (8)–(10) is optimal.

For the case that $\mu = 0$, the solution of S^* and the inequality on β are obtained by taking the limits of (8)–(10) as $\mu \to 0$, as specified by (11)–(12). Alternatively, one can analyze the degenerate system (i.e., $\mu = 0$) similarly as the case for the original one.

6. EXPERIMENTAL RESULTS

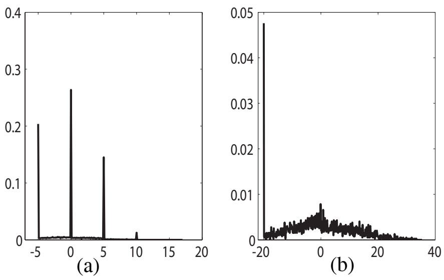

Figure 2: The estimated PDF for the increment ΔX_t. (a): $\Delta t = 5$ ms, $\hat{\sigma}^2 = 2.85$. (b): $\Delta t = 20$ ms, $\hat{\sigma}^2 = 6.4$.

We have tested our proposed band control policy for playout scheduling (referred to as *band-control* algorithm) using synthetic data. The packet transmission over internet was emulated according to the TIA-921 standard [9]. The network configuration was selected to be case 104D, representing a commonly encountered network link. In our system, a packet was generated every 20 ms. To model the buffer length X_t using a Wiener process, X_t must vary continuously. This is realized by assuming that the content of a packet does not arrive instantaneously. Instead, it takes 20 ms to fully receive its content.

The distribution of ΔX_t without control was examined. As the network condition remained constant, the probability density function (PDF) of ΔX_t was estimated by computing its normalized histogram. Figure 2 displays the estimated PDF of ΔX_t for some Δt. One observes that as the considered time interval Δt is increasing, its distribution is approximating the Gaussian PDF with increasing accuracy. This confirms that the usage of the Wiener process in modeling X_t is reasonable. The parameter $\hat{\sigma}^2$ of the Wiener process was estimated by normalizing the variance of the PDF by Δt. It is seen from Figure 2 that the PDFs are asymmetric. This is because for a fixed Δt, ΔX_t can never be less than $-\Delta t$, which forms a lower bound. This effect weakens as Δt increases, resulting in a relative spreading of the PDF. This explains that the estimated $\hat{\sigma}^2$ increases with increasing Δt.

We studied the comparative performance of our proposed band-control algorithm for a range of trade-offs between average buffer length and average control length (length of dropped and generated pitch periods per unit time). The pitch period was set to be L_p=6 ms. The lower bound was $L_{\min}$=0. The time resolution for monitoring Z_t for abnormal status was 1 ms. As a packet contains three pitch periods (corresponding to 18 ms) plus 2 ms, the boundary mismatch was accounted for for each lost packet. The duration of the generated segment right before the succeeding received one was set to 8 ms to preserve speech periodicity. The average buffering delay and the average control length were computed for each band region $[0, S]$ over a sequence of 1000 packets. Algorithm 1 of [8] was applied to the same network delay trace to provide a reference for performance. It uses silence-oriented adjustment. The end-to-end delay for packet i (the first packet in a silence period) was set to $\hat{d}_i + \beta \hat{v}_i$, where $\hat{d}_i$, $\hat{v}_i$ are estimated parameters in the algorithm. We adjusted β to control the delay/packet loss ratio instead of taking a fixed value as in [8]. The talkspurt and silence periods were simulated according to ITU-T recommendation P.59 [7], with a mean of 596 ms and 227 ms, respectively, without hangover time. Figure 3 displays the two obtained curves. For the reference algorithm, the average control length is the same as the packet loss ratio. It is observed that our algorithm outperforms the reference algorithm, especially for the interesting average control length range [0.05, 0.2].

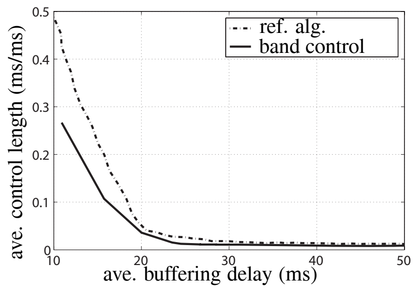

Figure 3: Performance comparison of the algorithms.

7. CONCLUSION

We conclude that stochastic modeling of the dejitter buffer length is reasonable. The necessary information for the derived optimal control scheme of the dejitter buffer to be performed is only the buffer length and the pitch period. Thus, no knowledge of the clock drift is needed. This renders our algorithm more flexible than reported methods, and enables straightforward integration of the proposed algorithm with many existing codecs. The experimental results indicate that our method can provide good performance compared to a popular reference system.

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