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BLIND MARGINALIZED PARTICLE FILTERING DETECTOR FOR THE SYSTEMS WITH IQ IMBALANCE AND CARRIER FREQUENCY OFFSET

Yuki Yoshida [1], *Kazunori Hayashi* [2], *and Hideaki Sakai* [3]

Graduate School of Informatics, Kyoto University
Yoshida Honmachi Sakyo–ku, Kyoto, 606–8501, JAPAN
phone: +81-75-753-9102, fax: +81-75-753-4755,
email: [1]yyoshida@sys.i.kyoto-u.ac.jp, [2]kazunori@i.kyoto-u.ac.jp, [3]hsakai@i.kyoto-u.ac.jp

ABSTRACT

Recently, marginalized particle filter (MPF) has been applied to blind symbol detection problems over selective fading channels. By marginalizing out the state appearing linearity and Gaussianity in the dynamics, the MPF can reduce the computational complexity, which is one of the main drawbacks of the standard particle filters. In this paper, we consider application of the MPF to the problem of blind detection in the presence of the In-phase/Quadrature-phase (IQ) imbalance and carrier frequency offset (CFO) which are inevitable performance degradation factors caused by the imperfection of analog front-end in wireless transceivers. Due to the existence of such impairments, the resulting state-space model of the problem is non-linear and non-Gaussian and the computationally efficient MPF is not applicable. To cope with this, we employ the auxiliary variable resampling technique to estimate IQ imbalance and CFO parameters. Simulations are provided that demonstrate the effectiveness of the proposed MPF detector.

1. INTRODUCTION

Particle filter (PF) [1, 2, 3], that has recently emerged in the fields of statistics and engineering, has shown great promise in solving a wide class of nonlinear and/or non-Gaussian problems. While the PF is fairly easy to implement and tune, the main drawback is its high computational complexity. One remedy to this problem is to analytically marginalize out the state appearing linearity and Gaussianity in the dynamics. The resulting PF is called as the marginalized particle filter (MPF), also known as a Kalman PF or Rao-Blackwellised PF [4]. The MPF is a potential combination of the standard PF and the Kalman filter [5] [6], and it is well known that the MPF can not only reduce the computational effort but also obtain better estimates compared with the standard PF in some cases [7]. Recently, the MPF has been applied to a blind detection of a symbol sequence transmitted over frequency selective fading channels [8] [9]. By assuming a linear and Gaussian state-space model to represent the channel distortion, the marginalized particle filtering detector (MPFD) can obtain the maximum a posterior (MAP) estimate directly without explicit channel estimation.

Meanwhile, one of the performance degradation factors in the implementation of wireless systems is the impairment caused by analog processing due to component imperfections. In most cases, such impairments cannot be efficiently or entirely eliminated in the analog domain due to power, area, and cost trade-offs. Therefore, efficient compensation techniques in the digital baseband domain are needed for the transceivers. Significant sources of such impairment are a carrier frequency offset (the CFO) and an In-phase/Quadrature-phase (IQ) imbalance [10]. Both of them are introduced at the up and down frequency conversion at the transceivers. The IQ imbalance is misalignment between the I and Q paths: the real and imaginary parts of the complex signal, and the CFO is mismatch of frequencis between the local oscillators (LOs) at the transceivers. Degrees of such imperfections depend on each transceiver and thus unknown to the receiver. When we consider the blind particle filtering detector for systems with such analog imperfections, since the resulting space-state model is non-linear and non-Gaussian due to the unknown CFO and IQ imbalance parameters, the computationally efficient MPFD can not be applied. Obviously, the standard PF is still applicable, however it is unattractive to lose the advantage of the MPFD due to such inherent analog imperfections.

In this paper, we propose the MPFD with auxiliary variable resampling for systems with the IQ imbalance and the CFO. The auxiliary variable resampling technique [11] is basically designed to cope with the essential weakness of the PF, i.e., performance degradation due to the existence of an outlier. Here, we use the technique for different purpose, i.e., to estimate the unknown parameters in the state-space model by exploiting approximate samples of the desired distribution at previous time instants. In the proposed method, by using the estimates obtained from the resampling procedure, the MPF can efficiently marginalize out the unknown channel and estimate the transmitted sequence. We demonstrate the significant performance improvement by using the proposed MPFD later in our computer simulations. In the followings, vectors are indicated in bold and scalar parameters in normal font. Superscripts *, ^{T}, and ^{H} represent conjugate, transpose, and Hermitian transpose, respectively.

2. PROBLEM FORMULATION

Let us consider the blind detection problem over frequency selective channels, where the original transmitted symbols $s_t \in \mathcal{A}$ (here, $\mathcal{A} = \{a_n\}$, $n = 1, \cdots, N$ denotes a complex signal constellation, and the time index $t = 0, 1, 2, \cdots,$) are complex modulated with differential coding in order to resolve the phase ambiguity inherent to any blind receiver. Before transmission, they suffer from transmitter (Tx) IQ imbalance in the analog domain and then IQ distorted signals $\hat{s}_t$ are transmitted in a frame of length $T+1$ symbols through the frequency selective fading channel. Furthermore, at the receiver front-end, the received signals are distorted by not only the receiver (Rx) IQ imbalance but also the CFO. We assume that the channel coefficients are time-invariant for the duration of the frame. In addition, the symbols preceding the current time frame: s_t $(t = \cdots, -1)$ are also assumed to be known.

Let ϵ_{tx} denotes the amplitude imbalance and ϕ_{tx} is the phase imbalance between the I and Q branches introduced at the transmitter, complex baseband expression for the IQ imbalance effect on the ideal symbol s_t is given by [10] as

$$\begin{aligned}\hat{s}_t &= (1+\epsilon_{tx})\cos(\phi_{tx})\Re\{s_t\} - (1+\epsilon_{tx})\sin(\phi_{tx})\Im\{s_t\} \\ &-i\cdot(1-\epsilon_{tx})\sin(\phi_{tx})\Re\{s_t\} + i\cdot(1-\epsilon_{tx})\cos(\phi_{tx})\Im\{s_t\} \\ &= \alpha s_t + \beta s_t^*, \end{aligned} \quad (1)$$

where $\Re(\cdot)$ and $\Im(\cdot)$ denotes the real and imaginary parts, respectably, and

$$\alpha := \cos(\phi_{tx}) + i\cdot\epsilon_{tx}\sin(\phi_{tx}), \quad (2)$$
$$\beta := \epsilon_{tx}\cos(\phi_{tx}) - i\cdot\sin(\phi_{tx}). \quad (3)$$

In general, as given by (1), the desired signal s_t is interfered by its own complex–conjugate or image s_t^*. The formal image rejection ratio (IRR) is given by,

$$\mathrm{IRR} = \mathrm{E}[|\alpha s_t|^2]/\mathrm{E}[|\beta s_t^*|^2] = |\alpha|^2/|\beta|^2. \quad (4)$$

With practical imbalance values, the IRR is in the order of 20–30 dB [12] and these IRR levels are insufficient in any practical transceiver with spectrally efficient modulation techniques, and thus, digital compensation is really needed. Using the channel impulse response $h(t)$ $(t = 0, \cdots L-1)$ of the length L and a circular complex Gaussian noise component $v_t \sim \mathcal{N}_c(0,\sigma^2)$, the received signal $\hat{r}_t$ before IQ distortion and the CFO at the receiver is given by

$$\hat{r}_t = \sum_{l=0}^{L-1} h(l)\hat{s}_{t-l} + n_t = \alpha\mathbf{h}^{\mathrm{T}}\mathbf{s}_t + \beta\mathbf{h}^{\mathrm{T}}\mathbf{s}_t^* + v_t, \quad (5)$$

where we define $\mathbf{s}_t = [s_t \cdots s_{t-L+1}]^{\mathrm{T}}$, $\mathbf{h} = [h(0)\cdots h(L-1)]^{\mathrm{T}}$. At the receiver front–end, the received signals suffer from both the IQ imbalance and the CFO. The time domain effect of the CFO ω on an incoming signal $\hat{r}_t$ is a phase rotation $\theta := \exp\{j2\pi\omega\}$ proportional with time. When the CFO is present together with Rx IQ imbalance, the resulting baseband signal $\hat{r}_t$ can be written as [13]

$$r_t = \gamma\theta^t\hat{r}_t + \delta\theta^{-t}\hat{r}_t^*, \quad (6)$$

where γ and δ are the Rx IQ imbalance parameters defined by using the amplitude imbalance ϵ_{rx} and the phase imbalances ϕ_{rx}, i.e.,

$$\gamma := \cos(\phi_{rx}) + i\cdot\epsilon_{rx}\sin(\phi_{rx}), \quad (7)$$
$$\delta := \epsilon_{rx}\cos(\phi_{rx}) - i\cdot\sin(\phi_{rx}). \quad (8)$$

Finally, we can formulate the dynamic state–space of the form

$$\begin{aligned}&\text{State equation:} \quad \mathbf{s}_t = \mathbf{T}\mathbf{s}_{t-1} + \mathbf{u}_t, \\ &\text{Observation equation:} \\ &r_t = \alpha\gamma\theta^t\mathbf{h}^{\mathrm{T}}\mathbf{s}_t + \beta^*\delta\theta^{-t}\mathbf{h}^{\mathrm{H}}\mathbf{s}_t \\ &\quad + \beta\gamma\theta^t\mathbf{h}^{\mathrm{T}}\mathbf{s}_t^* + \alpha^*\delta\theta^{-t}\mathbf{h}^{\mathrm{H}}\mathbf{s}_t^* + \gamma n_t + \delta n_t^*, \end{aligned} \quad (9)$$

where

$$\mathbf{T} := \begin{bmatrix} \mathbf{0}_{1\times L} \\ \mathbf{I}_{L-1}\ \mathbf{0}_{L-1\times 1}, \end{bmatrix} \quad (10)$$

denotes an $L\times L$ state–transition matrix, (here, $\mathbf{I}_L$ represents an $L\times L$ identity matrix and $\mathbf{0}_{I\times J}$ is an $I\times J$ all zero matrix) and $\mathbf{u}_t = [s_t\ \ 0\ \ \cdots\ \ 0]^{\mathrm{T}}$ is the state–perturbation, where the new symbol s_t is uniform random variables, i.e., $s_t \sim \mathcal{U}(\mathcal{A})$ and it is independent of previous and future symbols. For convenience, we define $n_t = \theta^t v_t$ and it is clear that the rotated version of v_t is still circular Gaussian, i.e., $n_t \sim \mathcal{N}_c(0,\sigma^2)$.

At any time instant t, the unknowns of the problem are $\mathbf{s}_t, \mathbf{h}$ and analog imperfection parameters $\mathcal{B} := \{\epsilon_{tx}, \phi_{tx}, \epsilon_{rx}, \phi_{rx}, \omega\}$, and our main objective is to detect the transmitted sequence $s_{0:t}$ sequentially based on the MAP criterion from given observation $r_{0:t}$ (here $x_{0:t} := \{x_0,\ x_1,\ \cdots\ x_t\}$), i.e.,

$$s_{0:t}^{\mathrm{MAP}} = \arg\max_{s_{0:t}}\{p[s_{0:t}|r_{0:t}]\}. \quad (11)$$

One hasty solution is to approximate the joint posterior $p[s_{0:t}, \mathcal{B}, \mathbf{h}|r_{0:t}]$ via the PF. However, to jointly propagate particles for $s_{0:t}$, $\mathbf{h}$ and $\mathcal{B}$ based on the given state–space model is computationally intensive. In the following sections, we show an idea how to apply the computationally effective MPF to the problem by using the auxiliary variable resampling.

3. BLIND MPFD FOR KNOWN ANALOG IMPERFECTIONS

Firstly, we consider the case where $\mathcal{B}$ is a priori known to the receiver and approximate the posterior $p[s_{0:t}, \mathbf{h}|r_{0:t}, \mathcal{B}]$ via the MPF. From the Bayesian formulation, we have the following recursive computation of the posterior,

$$p[s_{0:t}|r_{0:t}, \mathcal{B}] \propto p[r_t|s_{0:t}, r_{0:t-1}, \mathcal{B}]\cdot p[s_{0:t-1}|r_{0:t-1}, \mathcal{B}]. \quad (12)$$

This provide the sequential computation of $p[s_{0:t}|r_{0:t}, \mathcal{B}]$, if we can calculate the likelihood $p[r_t|s_{0:t}, r_{0:t-1}, \mathcal{B}]$. Recalling that the channel $\mathbf{h}$ is unknown to the receiver, thus the likelihood can be written as

$$\begin{aligned}&p[r_t|s_{0:t}, r_{0:t-1}, \mathcal{B}] \\ &= \int_{\mathcal{C}^L} p[r_t|\mathbf{h}, \mathbf{s}_t, \mathcal{B}]\cdot p[\mathbf{h}|s_{0:t-1}, r_{0:t-1}, \mathcal{B}]d\mathbf{h}. \end{aligned} \quad (13)$$

It is well known that the above integration have a closed-form expression when the state-space model is linear and Gaussian. For given $\mathbf{s}_t$ and $\mathcal{B}$, the observation equation (9) is not a linear but a widely linear system [14] of $\mathbf{h}$. Therefore, we can modify (9) to a normal linear system by stacking signals and their complex conjugate, i.e.,

$$\begin{aligned}r_t &= \left(\left(\begin{bmatrix}\alpha\gamma\theta^t & \beta\gamma\theta^t \\ \alpha^*\delta\theta^{-t} & \beta^*\delta\theta^{-t}\end{bmatrix}\otimes\mathbf{I}_L\right)\begin{bmatrix}\mathbf{s}_t \\ \mathbf{s}_t^*\end{bmatrix}\right)^{\mathrm{T}}\begin{bmatrix}\mathbf{h} \\ \mathbf{h}^*\end{bmatrix} + [\gamma\ \ \delta]\begin{bmatrix}n_t \\ n_t^*\end{bmatrix} \\ &= ((\mathbf{\Lambda}_t\otimes\mathbf{I}_L)\,\bar{\mathbf{s}}_t)^{\mathrm{T}}\,\bar{\mathbf{h}} + z_t, \end{aligned} \quad (14)$$

where $\otimes$ denotes the Kronecker product, and

$$\begin{aligned}&\bar{\mathbf{s}}_t := \begin{bmatrix}\mathbf{s}_t \\ \mathbf{s}_t^*\end{bmatrix}, \bar{\mathbf{h}} := \begin{bmatrix}\mathbf{h} \\ \mathbf{h}^*\end{bmatrix}, z_t := [\gamma\ \ \delta]\begin{bmatrix}n_t \\ n_t^*\end{bmatrix}, \\ &\mathbf{\Lambda}_t := \begin{bmatrix}\alpha\gamma\theta^t & \beta\gamma\theta^t \\ \alpha^*\delta\theta^{-t} & \beta^*\delta\theta^{-t}\end{bmatrix}. \end{aligned} \quad (15)$$

Clearly, for given $\mathbf{s}_t$ and $\mathcal{B}$, (14) is a linear system of $\bar{\mathbf{h}}$. On the other hand, since $n_t \sim \mathcal{N}_c(0,\sigma^2)$, the noise z_t is not a proper but an improper complex Gaussian noise [15], therefore, by using $\bar{\mathbf{z}}_t = [z_t\ z_t^*]^{\mathrm{T}}$, we have

$$\begin{aligned}p[z_t] &= p[\bar{\mathbf{z}}_t] \\ &= \mathcal{N}_c\left(\begin{bmatrix}0 \\ 0\end{bmatrix}, \begin{bmatrix}(\gamma\gamma^* + \delta\delta^*)\sigma^2 & 2\gamma\delta\sigma^2 \\ 2\gamma^*\delta^*\sigma^2 & (\gamma\gamma^* + \delta\delta^*)\sigma^2\end{bmatrix}\right) \\ &:= \mathcal{N}_c\left([0\ 0]^{\mathrm{T}}, \mathbf{\Phi}\right). \end{aligned} \quad (16)$$

Consequently, $p[r_t|\mathbf{h},\mathbf{s}_t,\mathcal{B}]$ also becomes an improper complex Gaussian and, by using $\bar{\mathbf{r}}_t = [r_t\ r_t^*]^{\mathrm{T}}$, it can be written as

$$p[r_t|\mathbf{h},\mathbf{s}_t,\mathcal{B}] = p[\bar{\mathbf{r}}_t|\mathbf{h},\mathbf{s}_t,\mathcal{B}] = \mathcal{N}_c\left(\begin{bmatrix}((\mathbf{\Lambda}_t\otimes\mathbf{I}_L)\,\bar{\mathbf{s}}_t)^{\mathrm{T}}\\((\mathbf{\Lambda}_t\otimes\mathbf{I}_L)\,\bar{\mathbf{s}}_t)^{\mathrm{H}}\mathbf{D}\end{bmatrix}\bar{\mathbf{h}},\ \mathbf{\Phi}\right), \quad (17)$$

where

$$\mathbf{D} := \begin{bmatrix}\mathbf{0}_{L\times L} & \mathbf{I}_L\\ \mathbf{I}_L & \mathbf{0}_{L\times L}\end{bmatrix}. \quad (18)$$

Meanwhile, the posterior of the channel $p[\mathbf{h}|s_{0:t},r_{0:t},\mathcal{B}]$ is also proportional to the integrand in (13)

$$p[\mathbf{h}|s_{0:t},r_{0:t},\mathcal{B}] \propto p[r_t|\mathbf{h},\mathbf{s}_t,\mathcal{B}]\cdot p[\mathbf{h}|s_{0:t-1},r_{0:t-1},\mathcal{B}]. \quad (19)$$

Thus, if we assume that the extended channel vector $\bar{\mathbf{h}}$ is a priori distributed according to a circular complex Gaussian $\bar{\mathbf{h}} \sim \mathcal{N}_c(\bar{\mathbf{h}}_{-1},\bar{\mathbf{R}}_{-1})$, then $p[\mathbf{h}|s_{0:t},r_{0:t},\mathcal{B}]$ is proportional to a product of Gaussian densities and it is also Gaussian. Let $\bar{\mathbf{h}}_t$ and $\bar{\mathbf{R}}_t$ denote the posterior mean and covariance of $\bar{\mathbf{h}}$ given $s_{0:t}$, $r_{0:t}$, and $\mathcal{B}$. The integrand of (13) can be written as

$$\begin{aligned}&p[r_t|\mathbf{h},\mathbf{s}_t,\mathcal{B}]\cdot p[\mathbf{h}|s_{0:t-1},r_{0:t-1},\mathcal{B}]\\&= \pi^{-1-L}|\mathbf{\Phi}|^{-\frac{1}{2}}|\bar{\mathbf{R}}_{t-1}|^{-\frac{1}{2}}\exp\{-\frac{1}{2}[(\bar{\mathbf{h}}-\bar{\mathbf{h}}_t)^{\mathrm{H}}\bar{\mathbf{R}}_t^{-1}(\bar{\mathbf{h}}-\bar{\mathbf{h}}_t)\\&+\bar{\mathbf{r}}_t^{\mathrm{H}}\mathbf{\Phi}^{-1}\bar{\mathbf{r}}_t+\bar{\mathbf{h}}^{\mathrm{H}}\bar{\mathbf{R}}_{t-1}^{-1}\bar{\mathbf{h}}\\&-(\mathbf{\Gamma}_t^{\mathrm{H}}\mathbf{\Phi}^{-1}\bar{\mathbf{r}}_t+\mathbf{R}_{t-1}^{-1}\bar{\mathbf{h}}_{t-1})^{\mathrm{H}}\bar{\mathbf{R}}_t(\mathbf{\Gamma}_t^{\mathrm{H}}\mathbf{\Phi}^{-1}\bar{\mathbf{r}}_t+\mathbf{R}_{t-1}^{-1}\bar{\mathbf{h}}_{t-1})]\}\end{aligned} \quad (20)$$

where

$$\mathbf{\Gamma}_t = \begin{bmatrix}((\mathbf{\Lambda}_t\otimes\mathbf{I}_L)\,\bar{\mathbf{s}}_t)^{\mathrm{T}}\\((\mathbf{\Lambda}_t\otimes\mathbf{I}_L)\,\bar{\mathbf{s}}_t)^{\mathrm{H}}\mathbf{D}\end{bmatrix}, \quad (21)$$

$|\cdot|$ denotes the determinant of a matrix, and

$$\bar{\mathbf{R}}_t^{-1} = \mathbf{\Gamma}_t^{\mathrm{H}}\mathbf{\Phi}^{-1}\mathbf{\Gamma}_t + \bar{\mathbf{R}}_{t-1}^{-1}, \quad (22)$$

$$\bar{\mathbf{h}}_t = \bar{\mathbf{R}}_t(\mathbf{\Gamma}_t^{\mathrm{H}}\mathbf{\Phi}^{-1}\bar{\mathbf{r}}_t + \mathbf{R}_{t-1}^{-1}\bar{\mathbf{h}}_{t-1}). \quad (23)$$

Consequently, the integrate (13) can be analytically found as

$$\begin{aligned}&p[r_t|s_{0:t},r_{0:t-1},\mathcal{B}]\\&= \pi^{-1}|\mathbf{\Phi}|^{-\frac{1}{2}}||\bar{\mathbf{R}}_t|^{\frac{1}{2}}|\bar{\mathbf{R}}_{t-1}|^{-\frac{1}{2}}\exp\{-\frac{1}{2}[\bar{\mathbf{r}}_t^{\mathrm{H}}\mathbf{\Phi}^{-1}\bar{\mathbf{r}}_t+\bar{\mathbf{h}}^{\mathrm{H}}\bar{\mathbf{R}}_{t-1}^{-1}\bar{\mathbf{h}}\\&-(\mathbf{\Gamma}_t^{\mathrm{H}}\mathbf{\Phi}^{-1}\bar{\mathbf{r}}_t+\mathbf{R}_{t-1}^{-1}\bar{\mathbf{h}}_{t-1})^{\mathrm{H}}\bar{\mathbf{R}}_t(\mathbf{\Gamma}_t^{\mathrm{H}}\mathbf{\Phi}^{-1}\bar{\mathbf{r}}_t+\mathbf{R}_{t-1}^{-1}\bar{\mathbf{h}}_{t-1})]\}.\end{aligned} \quad (24)$$

This makes it possible to compute the posterior $p[s_{0:t}|r_{0:t},\mathcal{B}]$ sequentially according to (12) without concerning the channel $\mathbf{h}$. Such mariginalization is also known in the context of the prediction and update steps of the Mixture Kalman filters [6]. Using the sequential importance sampling (SIS) [2] and based on the marginalization, we have the blind MPFD algorithm as shown in Table 1 where the superscript $*^{(i)}$ denotes the state trajectories of the i-th particle and M is the number of particles. On the other hand. the importance function and weights are given by $q[\cdot|\cdot]$ and $w_t^{(i)}$. The choice of the importance function is essential because it determines the efficiency as well as the complexity of the PF. Two major examples are prior and posterior importance functions. The posterior importance function is known as the optimal function which minimize the variance of the importance weights.

Table. 1 Blind MPFD

for $i = 1,\cdots,M$ **do**
  Initialize $\bar{\mathbf{R}}_{-1}^{(i)} = \bar{\mathbf{R}}_{-1}$ and $\bar{\mathbf{h}}_{-1}^{(i)} = \bar{\mathbf{h}}_{-1}$
end for
for $t = 0,\cdots,T$ **do**
  for $i = 1,\cdots,M$ **do**
    –Sample $s_t^{(i)}$ from the set $\mathcal{A}$ with $q[s_t|s_{0:t-1}^{(i)},r_{0:t}]$
    –Update $\bar{\mathbf{R}}_t^{(i)}$ and $\bar{\mathbf{h}}_t^{(i)}$ according to (22) and (23)
    –Calculate the weights by $\tilde{w}_t^{(i)} = w_{t-1}^{(i)}\frac{p[r_t|s_{0:t}^{(i)},r_{0:t-1}]}{q[s_t^{(i)}|s_{0:t-1}^{(i)},r_{0:t}]}$
  end for
end for
for $i = 1,\cdots,M$ **do**
  –Normalize the weights by $w_t^{(i)} = \frac{\tilde{w}_t^{(i)}}{\sum_{j=1}^{M}\tilde{w}_t^{(j)}}$
end for
return $\hat{s}_{0:T}^{(MAP)} = s_{0:T}^{(imax)}$ where $imax = \arg\max_i w_T^{(i)}$

However it is often analytically intractable. Thus one would usually resort to using the prior importance function without exploiting informations from the observations, and it is often ineffective and leads to poor performance.

In our case, it is possible to employ the optimal one, that is given by,

$$q[s_t|s_{0:t-1}^{(i)},r_{0:t},\mathcal{B}] = \frac{p[r_t|s_t,s_{0:t-1}^{(i)},r_{0:t},\mathcal{B}]}{\sum_{n=1}^{N}p[r_t|s_t=a_n,s_{0:t-1}^{(i)},r_{0:t},\mathcal{B}]}. \quad (25)$$

Clearly, we can derive (25) by using (24). The absence of sampling $\mathbf{h}$ and utilize the optimal importance function make the MPF quite effective.

4. BLIND MPFD WITH AUXILIARY VARIABLE RESAMPLING

Next, we consider the PF solution for the system with the unknown analog imperfections. In this case, due to the presence of unknown $\mathcal{B}$, the system (9) becomes non-linear and the posterior importance function is intractable. As mentioned above, the PF has capability to cope with such a non-linear problem, and we can somehow design the PF detector using the prior importance function. However the resulting PF detector sacrifices the efficiency of the MPFD and it is less attractive. Therefore, we propose the efficient MPF solution for the problem by using the auxiliary variable resampling technique.

We begin with considering two step sampling of $\{s_t^{(i)},\mathcal{B}_t^{(i)}\}$ based on the following decomposition,

$$p[s_{0:t},\mathcal{B}|r_{0:t}] = p[\mathcal{B}|r_{0:t}]p[s_{0:t}|r_{0:t},\mathcal{B}]., \quad (26)$$

Clearly, the second component in right-hand side of (26) is the same as the distribution of our interest in the previous section. Hence, if the detector can firstly draw a set of samples $\mathcal{B}_t^{(i)} = \{\epsilon_{tx\ t}^{(i)},\phi_{tx\ t}^{(i)},\epsilon_{rx\ t}^{(i)},\phi_{rx\ t}^{(i)},\omega_t^{(i)}\}$ distributed as $p[\mathcal{B}|r_{0:t}]$ at each time instant, we can propagate samples $\{s_t^{(i)}\}$ distributed as $p[s_{0:t}|\mathcal{B},r_{0:t}]$ by using the MPF in Table 1.

Now, our interest is how to draw the samples $\{\mathcal{B}_t^{(i)}\}$ from $p[\mathcal{B}_t^{(i)}|r_{0:t}]$. For this purpose, we employ the auxiliary variable resampling technique which has been proposed by Pitt and Shephard [11]. Basically, it has been proposed to cope with the essential weakness of PF, i.e., performance degradation due to the existence of outliers. Here, we use the method in a slightly different purpose, i.e., to propagate the approximate samples $\{\mathcal{B}_t^{(i)}\}$ distributed as $p[\mathcal{B}_t^{(i)}|r_{0:t}]$ by using particles of previous time instant.

Here we try to perform the sampling in a higher dimension $p[\mathcal{B},k|r_{0:t}]$ instead of $p[\mathcal{B}|r_{0:t}]$, where integer $k = 1,\cdots,M$ is so-called auxiliary variable and refers to the index of the particles at time $t-1$. By applying Bayesian rule, a proportionality can be derived as

$$\begin{aligned} p[\mathcal{B},k|r_{0:t}] &\propto p[r_t|\mathcal{B}]p[\mathcal{B},k|r_{0:t-1}] \\ &= p[r_t|\mathcal{B}]p[\mathcal{B}|k,r_{0:t-1}]p[k|r_{0:t-1}] \\ &= p[r_t|\mathcal{B}]p[\mathcal{B}|\mathcal{B}_{t-1}^{(k)}]w_{t-1}^{(k)}. \end{aligned} \quad (27)$$

If we can draw samples from this joint density and then discard the index k, then we can produce samples from the desired density $p[\mathcal{B}|r_{0:t}]$. In practice, the optimal importance function to draw $\{\mathcal{B}_t^{(i)},k^{(i)}\}$ is intractable, so we use the importance sampling density defined to satisfy the proportionality,

$$q_{\text{aux}}[\mathcal{B},k|r_{0:t}] \propto p[r_t|\hat{\mathcal{B}}_t^{(k)}]p[\mathcal{B}|\mathcal{B}_{t-1}^{(k)}]w_{t-1}^{(k)}, \quad (28)$$

where $\hat{\mathcal{B}}_t^{(i)} = \{\hat{\epsilon}_{tx\ t}^{(i)},\hat{\phi}_{tx\ t}^{(i)},\hat{\epsilon}_{rx\ t}^{(i)},\hat{\phi}_{rx\ t}^{(i)},\hat{\omega}_t^{(i)}\}$ are some characterization of $\mathcal{B}$ for given $\mathcal{B}_{t-1}^{(k)}$, i.e., the averages, modes or medians of $p[\mathcal{B}|\mathcal{B}_{t-1}^{(k)}]$. To avoid confusion, we denote the importance function as auxiliary importance function $q_{\text{aux}}[\cdot|\cdot]$. By writing,

$$q_{\text{aux}}[\mathcal{B},k|r_{0:t}] = q_1[\mathcal{B}|k,r_{0:t}]q_2[k|r_{0:t}], \quad (29)$$

and defining

$$q_1[\mathcal{B}|k,r_{0:t}] := p[\mathcal{B}|\mathcal{B}_{t-1}^{(k)}], \quad (30)$$

it follows that

$$q_2[k|r_{0:t}] \propto p[r_t|\hat{\mathcal{B}}_t^{(k)}]w_{t-1}^{(k)}. \quad (31)$$

This means that we can sample from $q_{\text{aux}}[\mathcal{B},k|r_{0:t}]$ by firstly simulating the index k with probability $p[r_t|\hat{\mathcal{B}}_t^{(k)}]w_{t-1}^{(k)}$, and then sampling from the transition density $p[\mathcal{B}|\mathcal{B}_{t-1}^{(k)}]$ for given k.

Consequently, combining the MPFD in Table 1 and the auxiliary variable resampling, the MPFD successfully applied to the system with unknown analog imperfections. Interestingly, in [9](see also [16]), the authors have proposed a similar suboptimal importance function, called hybrid importance function, through the different approach and proposed the blind particle filtering detector over selective fading channels with unknown channel order.

From (25) and (28), a set of samples $\{\mathcal{B}_t^{(i)},s_t^{(i)}\}$ approximate the total importance distribution:

$$\begin{aligned} &q_{\text{tot}}[s_t,\mathcal{B}|s_{0:t-1}^{(k)},\mathcal{B}_{t-1}^{(k)},r_{0:t}] \\ &\quad = q_{\text{aux}}[\mathcal{B},k|r_{0:t}]q[s_t|s_{0:t-1}^{(k)},\mathcal{B},r_{0:t}] \\ &\quad \propto p[r_t|\hat{\mathcal{B}}_t^{(k)}]p[\mathcal{B}|\mathcal{B}_{t-1}^{(k)}]w_{t-1}^{(k)}p[r_t|s_t,s_{0:t-1}^{(k)},\mathcal{B},r_{0:t-1}]. \end{aligned} \quad (32)$$

Corresponding to the total importance function, the weights update procedure to approximate the posterior $p[s_t,\mathcal{B}|s_{0:t-1}^{(k)},\mathcal{B}_{t-1}^{(k)},r_{0:t}]$ is obtained as

$$\begin{aligned} w_t^{(i)} &\propto w_{t-1}^{(k^{(i)})}\frac{p[r_t|\mathbf{s}_t^{(i)},\mathcal{B}_t^{(i)},r_{0:t-1}]p[s_t^{(i)},\mathcal{B}_t^{(i)}|s_{0:t-1}^{(k^{(i)})},\mathcal{B}_{t-1}^{(k^{(i)})}]}{q_{\text{tot}}[s_t^{(i)},\mathcal{B}_t^{(i)}|s_{0:t-1}^{(k^{(i)})},\mathcal{B}_{t-1}^{(k^{(i)})},r_{0:t}]} \\ &= \frac{\sum_{n=1}^N p[r_t|s_t=a_n,s_{0:t-1}^{(k^{(i)})},\mathcal{B}_t^{(i)},r_{0:t-1}]}{\sum_{n=1}^N p[r_t|s_t=a_n,s_{0:t-1}^{(k^{(i)})},\hat{\mathcal{B}}_t^{(k^{(i)})},r_{0:t-1}]}, \end{aligned} \quad (33)$$

where $k^{(i)}$ denotes the auxiliary variable at the i–th particle. We summarize the algorithm of the proposed blind MPFD in Table 2. Now, the rest of the problem is the choice of $\hat{\mathcal{B}}_t^{(i)}$

Table. 2 Blind MPFD with auxiliary variable resampling

for $i=1,\cdots,M$ **do**
  Initialize $\bar{\mathbf{R}}_{-1}^{(i)} = \bar{\mathbf{R}}_{-1}$ and $\bar{\mathbf{h}}_{-1}^{(i)} = \bar{\mathbf{h}}_{-1}$
  Draw initial samples $\mathcal{B}_{-1}^{(i)} \sim p_0[\mathcal{B}]$
end for

(Step 1. Auxiliary variable resampling)
for $t=0,\cdots,T$ **do**
  for $i=1,\cdots,M$ **do**
    –Compute $\hat{\mathcal{B}}_t^{(i)}$
  end for
  for $i=1,\cdots,M$ **do**
    –Sample $k^{(i)}$ from the set $\{1,\cdots,M\}$ with probability proportional to $p[r_t|\hat{\mathcal{B}}_t^{(k)},s_{0:t-1}^{(k)},r_{0:t-1}]w_{t-1}^{(k)}$
    –Sample $\mathcal{B}_t^{(i)}$ from $p[\mathcal{B}|\mathcal{B}_{t-1}^{(k^{(i)})}]$
  end for

  (Step 2. Sequential importance sampling)
  for $i=1,\cdots,M$ **do**
    –Sample $s_t^{(i)}$ from the set $\mathcal{A}$ with $q[s_t|s_{0:t-1}^{(k^{(i)})},\mathcal{B}_t^{(i)},r_{0:t}]$
    –Update $\bar{\mathbf{R}}_t^{(i)}$ and $\bar{\mathbf{h}}_t^{(i)}$ according to (22) and (23)
    –Calculate the weight $\tilde{w}_t^{(i)}$ with (33)
  end for
  for $i=1,\cdots,M$ **do**
    –Normalize the weights by $w_t^{(i)} = \frac{\tilde{w}_t^{(i)}}{\sum_{j=1}^M \tilde{w}_t^{(j)}}$
  end for
  –let $s_{0:t}^{(i)} = \{s_t^{(i)},s_{t-1}^{(k^{(i)})},\cdots,s_0^{(k^{(i)})}\}$ for each i
end for
return $\hat{s}_{0:T}^{(MAP)} = s_{0:T}^{(imax)}$ where $imax = \arg\max_i w_T^{(i)}$

and $p[\mathcal{B}|\mathcal{B}_{t-1}^{(i)}]$. In fact, since parameters $\epsilon_{tx},\phi_{tx},\epsilon_{rx},\phi_{rx}$, and ω are all static, the simplest choice will be $\hat{\mathcal{B}}_t^{(i)} = \mathcal{B}_t^{(k^{(i)})}$ and just set $\mathcal{B}_t^{(i)} = \mathcal{B}_{t-1}^{(k^{(i)})}$ without sampling. However, the inability to rejuvenate $\mathcal{B}$ with an arrival of new observations makes the accuracy of the final estimate greatly sensitive to the initial samples. To overcome this drawback, in [16], authors applied smoothing kernel [3]. For example, for the CFO parameter ω,

$$\hat{\omega}_t^{(i)} = \eta\omega_{t-1}^{(i)} + (1-\eta)\bar{\omega}_{t-1}, \quad (34)$$

$$\omega_t^{(i)} \sim \mathcal{N}(\hat{\omega}_t^{(k^{(i)})},\xi^2\rho_{t-1}), \quad (35)$$

where $\bar{\omega}_{t-1}$ is weighted average of $\{\omega_{t-1}^{(i)}\}$,i.e., $\bar{\omega}_{t-1} = \sum_{i=1}^M w_{t-1}^{(i)}\omega_{t-1}^{(i)}$, ρ_{t-1} is weighted sample covariance and $\mathcal{N}(\cdot)$ denotes real Gaussian pdf. The same procedure can be used for the other imperfection parameters. It is suggested in [3] that $\eta = \sqrt{1-\xi^2}, \xi^2 = 1-((3\nu-1)/2\nu)$, and ν is a discount factor typically from the set [0.95, 0.99]. What is more, in the system described in (9), since the IQ imbalance and the CFO is somehow limited in a certain region by the manufacturer, it would be more effective to use truncated real Gaussian $\mathcal{TN}(\hat{\omega}_t^{(k^{(i)})},\xi^2 v_{t-1},\omega_l,\omega_u)$ in which samples constrained in the region $[\omega_l,\omega_u]$, instead of $\mathcal{N}(\hat{\omega}_t^{(k^{(i)})},\xi^2\rho_{t-1})$.

5. SIMULATION RESULTS

Here, we evaluate the bit error rate (BER) performance of the proposed MPFD via computer simulations. In our experiment, we simulate a scenario of a time-invariant frequency selective Rayleigh fading channel of length $L = 3$. The circular Gaussian prior is assumed for the channel coefficients, i.e., $\mathbf{h} \sim \mathcal{N}_c(\mathbf{h}_{-1}, \mathbf{R}_{-1})$ where

$$\mathbf{h}_{-1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{R}_{-1} = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.05 \end{bmatrix}. \qquad (36)$$

We employ differentially coded QPSK for modulation and demodulation schemes and transmit $T = 200$ symbols over the channel. The received signal is distorted by Tx IQ imbalance and the CFO, where the priors for these analog imperfection parameters are

$$\epsilon_{tx}, \epsilon_{rx} \sim \mathcal{TN}(0, 0.01, 0, 0.1), \qquad (37)$$

$$\phi_{tx}, \phi_{rx} \sim \mathcal{TN}(0, 0.01, -0.05\pi, 0.05\pi), \qquad (38)$$

$$\omega \sim \mathcal{TN}(0, 0.01, -0.01\pi, 0.01\pi), \qquad (39)$$

and the resulting IRR is about 30dB. At the receiver, we test the conventional MPFD [8], which is basically designed for the systems without analog imparfections, and the proposed MPFD. For comparison, we also evaluate the performance of the conventional MPFD in the case where the analog front-end of the transceivers are ideal, i.e., $\epsilon_{tx} = \epsilon_{rx} = \phi_{tx} = \phi_{rx} = \omega = 0$. All the priors are assumed to be known to the receiver and both the detectors employ $M = 300$ particles. In the proposed MPFD, we employ the smoothing kernel in the auxiliary variable resampling step and, for fair comparison, the resampling procedure is conducted each time step in the conventional MPFD.

Fig. 1 shows the BER performances of the conventional and proposed MPFD versus the transmitted signal power to the noise power ratio (SNR) . All the results are average of 500 realizations. From the figure, the analog imperfections, which inherent in any analog front-end, seriously degrade the performance of the conventional MPF detector. On the other hand, the proposed MPFD can efficiently compensate the imperfections and improve the performance significantly. It should be mentioned that, in the low SNR region, the propose MPFD shows the superior performance compared with the conventional MPFD with the ideal analog front-end. This is because of the essential performance improvement by the use of the auxiliary variable resampling instead of sequential importance resampling [2] used in the conventional MPFD.

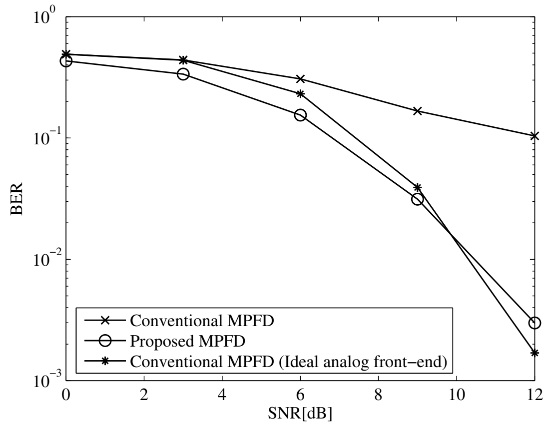

Figure 1: The BER performance versus SNR of the conventional and proposed MPFDs

6. CONCLUSIONS

The blind particle filtering detector in the presence of the IQ imbalance and the CFO has been proposed. By using the auxiliary variable resampling, the MPF can efficiently applied to such non-linear and non-Gaussian problem. Also, we show the effectiveness of the proposed MPFD via computer simulations.

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