16th European Signal Processing Conference (EUSIPCO 2008), Lausanne, Switzerland, August 25-29, 2008, copyright by EURASIP

ANALYTICAL RATE OPTIMIZATION FOR MULTICAST

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ABSTRACT

We consider the problem of multicast rate optimization for a given number of multicast groups. The efficient solution of this problem is particularly relevant for video multicast. Motivated by practical considerations, such as the need for adaptivity to changing network conditions, we develop an analytical solution that minimizes the expected distortion of a receiver. Unlike previous work, we recognize variability in existing networks and model the receiver reception capacities by a continuous stochastic variable. Our analytical solution is optimal under the assumption of a large number of multicast groups. We investigate the distortion overhead when this assumption is not satisfied and find that it is low. We compare our results to the state of the art, iterative optimization for a realization of receiver reception capacities, and find that our analytical solution performs better for any number of multicast groups.

1. INTRODUCTION

Multicast is a bandwidth efficient technique for transmission of a source to many receivers. Its efficiency is rooted in the fact that the source encoding is transmitted only once over any given link in the network. The potential increase in network throughput is at its largest when multicast is used for transmission of data requiring high bit-rates to a large number of receivers. The most obvious application is live video transmission of popular events, such as sports. However, the bandwidth efficiency comes at the cost that all receivers must settle for the same encoding of the source. Hence, all receivers perceive the same quality, which is decided by the bit-rate of the encoding, independently of their reception capacity.

Better adaptation to receiver reception capacities can be obtained if the number of source encodings is increased [1]. This technique is referred to as replication. The receivers are assigned to different multicast groups, each group receiving one source encoding at a particular bit-rate. A trade-off exists between the level of receiver reception capacity accommodation and bandwidth efficiency. This trade-off is controlled by the number of multicast groups, which in practice is constrained by issues such as storage capacity for streaming applications and encoding complexity for live communication applications. In this paper, we study the optimization of system performance for a trade-off specified by the number of multicast groups. Specifically, we optimize the bit-rates of the source encodings analytically, with respect to the expected distortion over all receivers, in the limit of a large number of multicast groups. We also study the overhead incurred by the assumption used.

A key aspect of the problem is the time variance of the transmission system. Variations of network load, imperfections in the receiver reception capacity estimation and the process of receivers joining and leaving the transmission session give constantly changing solution requirements. Therefore, finding an analytic expression for the optimal bit-rates of the source encodings is essential, as it allows for fast adaptation to the current state of the system.

Since time variance is important, we assume in our work that receiver reception capacities are realizations of an underlying stochastic variable. To the best of our knowledge, this approach to multicast rate optimization has not been investigated so far. Our assumption is supported by results from network capacity estimation literature, e.g., the work by Paxson [2] and Dovrolis et al. [3]. Existing multicast rate optimization solutions are based on the assumption that receiver reception capacities are deterministic, e.g., [4–6]. Yang et al. [4] first solved the problem of rate optimization, with respect to a general definition of receiver utility, for deterministic receiver reception capacities. Their solution is in the form of a dynamic programming algorithm. For reliable channels, the complexity of the algorithm grows linearly with the number of multicast groups and quadratically with the number of receivers. Recognizing the limitation of this solution in a practical system, Yousefi'zadeh et al. [5] decreased the complexity for a specific utility function. Both of these solutions are incapable of handling receiver capacity uncertainties. We quantify the overhead caused by erroneous capacity estimation for the algorithm by Yang et al. and show that it cannot be neglected. Further, since the solutions are in the form of algorithms, their adaptivity to the current system state is dependent of the computational complexity assigned to the system optimization.

The optimization objective is an important aspect of the problem formulation. Maximization of the sum of receiver utilities was used in [4, 5]. The receiver utility is defined as general class of functions that is dependent of the receiver reception capacity and the rate at which data is received. Liu et al. [6] maximize the sum of receiver specific functions that vary linearly with the receiving rate. In our work, we use the distortion as the measure of performance. By doing so, we assume that receivers are interested in the least absolute distortion, independently of their own reception capacity. We characterize the distortion with the distortion-rate function of the encoder.

A natural extension of the work proposed herein, is to include the scenario of an unreliable network. For completeness, we note that the problem of rate optimization for multicast over unreliable networks has not been solved. The authors of [4, 5] constrain their rate optimization problem by loss tolerance, defined as the largest loss rate a receiver can tolerate. However, the loss rate alone is not enough to characterize video quality [7, 8], why these solutions cannot be applied to the most relevant case of multicast. Another approach to combating the unreliable network is to utilize forward error correction. This approach is used in [9, 10], but the problem of rate optimization is not explicitly solved.

It has been noted in existing literature, e.g., [1], that the availability of an embedded encoder at the sender can increase the bandwidth efficiency for multicast with several groups. Transmission of the redundancy between the source encodings in replication can be avoided if each layer of the embedded encoder is assigned to a separate multicast group.

The receivers can then subscribe to the desired number of multicast groups to obtain the best quality for their reception capacity. This setup is called layered multicast. Kim and Ammar [11] studied this possibility, given the notion that embedded coding in general incurs an overhead. The solution that we propose is also applicable to layered multicast. We show this by formulating the problem in accordance with other authors [4–6].

The paper is organized as follows. In Section 2, we introduce the notation and present the problem formulation. The analytical solution to the problem, optimal asymptotically in the number of multicast groups, is presented in Section 3. Section 4 presents simulation results. Concluding remarks are found in Section 5.

2. PROBLEM FORMULATION

In this section we introduce the notation and present the rate optimization problem formulation. We consider multicast of a video source, although the problem is applicable to any source for which the distortion-rate performance of its encoder is known.

Consider the multicast of a video source, modeled as a stochastic process X. Let $d(r)$ represent the distortion obtained when the source is encoded with a given encoder at the bit-rate r. The analysis of this paper can be applied to any distortion-rate function. We choose the empirical distortion-rate model for video, first introduced in [12],

$$d(r) = \frac{\theta}{r - R_0} + D_0. \tag{1}$$

The distortion is measured in terms of the MSE and θ, R_0 and D_0 are model parameters. We assume that the model parameters can be chosen such that the model represents the performance of an embedded coder as well. This is true if the embedded coder refines the quality independently of the distribution of rate between layers. We point to [13] for important results on successive refinement in general and to [14, 15] for video in particular.

The multicast is set up using I streams with rates s_i, $i \in \{1, \dots, I\}$. The streams are coded either independently or hierarchically. In the case of independent coding, replication, each receiver subscribes to one multicast group, i, and, thus, receives stream i. In the case of hierarchical coding, each receiver subscribes to several multicast groups, $\{1, \dots, i\}$, and receives streams $\{1, \dots, i\}$.

The total number of receivers is J. Each receiver is connected to the sender over the network path P_j, $j \in \{1, \dots, J\}$, which consists of a number of network links. The link that has the lowest available transmission capacity, as governed by the total transmission capacity and the current link load, is denoted the bottleneck link. We define the reception capacity, c_j, of receiver j as the available capacity of the bottleneck link of network path P_j. The reception capacity is an upper bound on the (accumulated) rate that the receiver may subscribe to. We let c_j be a realization of a stochastic variable C_j with the probability density function $f_{C_j}(c)$, which in general is multimodal [3]. Further, we let $f_C(c)$ denote the underlying probability density of the reception capacities of all receivers, such that the reception capacity of any receiver is a realization of the stochastic variable C, distributed according to

$$f_C(c) = \frac{1}{J} \sum_{j=1}^{J} f_{C_j}(c). \tag{2}$$

Since the multicast paradigm is made possible by the fact that the paths from the sender to the receivers share common links, it is natural to assume that some of the receivers share their bottleneck links. Hence, $f_C(c)$ is in general a mixture of multimodal distributions, with component weights that are dependent on the number of receivers sharing the same bottleneck link.

Each receiver subscribes to a subset of the available multicast groups, receiving the (accumulated) assignment rate r_j. We let r_j be the output of the assignment function

$$r_j = \bar{a}(s, c_j), \tag{3}$$

where s denotes the set of all stream rates, $s = \{s_i\}_{i=1}^{I}$. In the case of replication, r_j is confined to the alphabet $\{0\} \cup \{s_i : i = 1, \dots, I\}$. In the case of embedded coding, r_j is confined to the alphabet $\{0\} \cup \{\sum_{n=1}^{i} s_i : i = 1, \dots, I\}$. Hence, for a given choice of stream rates, the assignment rates r_j can take $I + 1$ possible values from the set $r = \{0\} \cup \{r_i\}_{i=1}^{I}$. As r is a deterministic mapping of s, we introduce, for notational purposes, the equivalent assignment function

$$r_j = a(r, c_j). \tag{4}$$

The subset of streams assigned to receiver j is chosen such that the distortion for that receiver is minimized. Note that the cardinality of the subset is constrained to one if replication is used. Since the distortion-rate performance of the encoder is a monotonically decreasing function, the subset is chosen such that r_j is as large as possible. Hence,

$$\begin{aligned} r_j &= a(r, c_j) \\ &= \{r_i : r_i \le c_j < r_{i+1}, r_i \in r\}. \end{aligned} \tag{5}$$

The objective of the rate optimization problem is to minimize the expected distortion of a receiver by choosing the optimal stream rates s, or, equivalently, the optimal assignment rates r. We write the minimum mean distortion as

$$\begin{aligned} d^* &= \min_s E[d(\bar{a}(s, C))] \\ &= \min_r E[d(a(r, C))] \\ &= \min_r \sum_{i=0}^{I} \int_{r_i}^{r_{i+1}} d(r_i) f_C(c) \, \mathrm{d}c \\ &= \min_r \sum_{i=0}^{I} d_i, \end{aligned} \tag{6}$$

where we defined the cell distortion

$$d_i = \int_{r_i}^{r_{i+1}} d(r_i) f_C(c) \, \mathrm{d}c \tag{7}$$

and $r_{I+1} = \infty$.

3. ANALYTICAL RATE OPTIMIZATION

This section solves the rate optimization problem defined in Section 2. We begin by relating the rate optimization problem to quantization theory. Then, we use the common quantization theory assumption of small quantization cells to provide an analytical solution for the optimal assignment rates.

The optimization problem in (6) is equivalent to the optimization problem of scalar source quantization. Using quantization theory terminology, we recognize the source C, characterized by the probability density function $f_C(c)$, the quantization cell i with boundaries r_i and r_{i+1}, the reconstruction point r_i of cell i and the distortion function $d(\cdot)$.

Hence, the rate optimization problem in multicast is equivalent to the problem of scalar quantization of the stochastic source C, optimized for minimum distortion. Throughout this section, we confine ourselves, for convenience, to using terminology from quantization theory.

Let us denote the quantization cell width by Δ_i,

$$\Delta_i = r_{i+1} - r_i. \tag{8}$$

The assumption that the quantization cell width is small, such that the probability density within the cell can be assumed constant, was first introduced by Bennett in his classic paper from 1948 [16]. This assumption is often referred to as the high-rate assumption in quantization theory. Using the high-rate assumption, we motivate approximations that make the problem analytically tractable.

The distortion within a cell can be approximated with a function that, instead of being dependent of the reconstruction point of the cell, i.e., r_i, is dependent of the quantization variable c and the cell width Δ_i. This is done by assuming that $f_C(c)$ is constant and $d(\cdot)$ is linear within a small neighborhood of the cell, an assumption that is true asymptotically in the number of streams. For these assumptions, we show in Appendix A that the distortion of cell i can be written as

$$d_i \approx \int_{r_i}^{r_{i+1}} d(c - \frac{1}{2}\Delta_i) f_C(c) \, \mathrm{d}c. \tag{9}$$

The quantization cell width is approximated as a function, $\Delta(c)$, of the quantization variable c, such that

$$\Delta(r_i) = \Delta_i. \tag{10}$$

The inverse of $\Delta(c)$ then represents the density of reconstruction points in a unit interval. We denote this density by

$$g_R(c) = 1/\Delta(c). \tag{11}$$

The total number of non-zero reconstruction points, I, puts a constraint on the reconstruction point density

$$\int_{\mathbb{R}^+} g_R(c) \, \mathrm{d}c = I. \tag{12}$$

Using the approximations of (9), (10) and (11), and the definition of (1), we rewrite the problem formulation to an optimization problem of the reconstruction point density $g_R(c)$, yielding

$$\begin{aligned} d^* &= \min_r \sum_{i=0}^{I} \int_{r_i}^{r_{i+1}} d\left(c - \frac{1}{2}\Delta_i\right) f_C(c) \, \mathrm{d}c \\ &= \min_{g_R(c)} \int_{\mathbb{R}^+} d\left(c - \frac{1}{2} g_R^{-1}(c)\right) f_C(c) \, \mathrm{d}c \\ &= \min_{g_R(c)} \int_{\mathbb{R}^+} \left(\frac{\theta}{c - \frac{1}{2} g_R^{-1}(c) - R_0} + D_0 \right) f_C(c) \, \mathrm{d}c. \end{aligned} \tag{13}$$

The minimum distortion in (13), under the constraint (12), can be found by applying the method of Lagrange multipliers. The Lagrangian that is to be minimized becomes

$$\Lambda = \int_{\mathbb{R}^+} \left(\frac{\theta}{c - \frac{1}{2} g_R^{-1}(c) - R_0} f_C(c) + \lambda g_R(c) \right) \mathrm{d}c, \tag{14}$$

where λ is a strictly positive Lagrange multiplier. The Euler-Lagrange equation gives us the implicit solution. We set the partial derivative of the integrand of (14) with respect to $g_R(c)$ equal to zero and solve for $g_R(c)$. This yields the optimal reconstruction point density

$$g_R^*(c) = \frac{\frac{1}{2} \pm \sqrt{\theta f_C(c)/2\lambda}}{c - R_0}, \tag{15}$$

where λ is chosen such that (12) is satisfied.

To obtain the optimal reconstruction points r^* from the optimal reconstruction point density $g_R^*(c)$, we use the companding approach, first introduced for scalar quantization in [16]. Companding is the implementation of non-uniform quantization using the setup of a non-linear transform followed by a uniform quantizer. For the case of scalar quantization, companding does not incur any loss of generality [17].

Let us define a compressor h as a monotonically increasing function

$$\gamma = h(c), \tag{16}$$

that maps c to γ in the interval $[0, 1]$. The compressor is such that the optimal quantization cells in the γ domain are all of equal width. Using the relation between a random variable and its transform by a monotonic function, we rewrite the reconstruction point density as

$$\begin{aligned} g_R(c) &= g_\Gamma\big(h(c)\big) \Big| \frac{\partial h(c)}{\partial c} \Big| \\ &= I \Big| \frac{\partial h(c)}{\partial c} \Big|, \end{aligned} \tag{17}$$

where we used that $g_\Gamma(\gamma) = I$. Reevaluating the Euler-Lagrange equation for $g_R(c) = I \big| \frac{\partial h(c)}{\partial c} \big|$, and solving for $\big| \frac{\partial h(c)}{\partial c} \big|$, we get the following relation between the optimal reconstruction point density $g_R^*(c)$ and the optimal compressor $h^*(c)$

$$h^*(c) = I^{-1} \int_{-\infty}^{c} g_R^*(c) \, \mathrm{d}c. \tag{18}$$

The optimal quantization cells are found by expanding the optimal quantization cells in the companded domain with the inverse of (18). The optimal reconstruction points are the points for which the distortion within the cells is minimized and the quantization cell widths are not altered. For cells $i \in \{2, \ldots, I\}$, we set the reconstruction points to the lower boundary, since choosing any other point would result in non-optimal quantization cell boundaries according to the assignment function (5). This is not the case for the first cell. Namely, choosing any reconstruction point within the cell effectively divides the cell into two subcells. The first subcell has the reconstruction point at rate zero, $r_0 = 0$ and all receivers assigned to r_0 are excluded from the multicast. The second subcell has the reconstruction point, r_1, at its lower boundary. We set r_1 to the value that minimizes the expected distortion within the first cell, i.e.,

$$r_1 = \underset{r_1}{\operatorname{argmin}} \int_0^{r_1} d(0) f_C(c) \, \mathrm{d}c + \int_{r_1}^{r_2} d(r_1) f_C(c) \, \mathrm{d}c. \tag{19}$$

4. SIMULATION RESULTS

In this section, we present the results from conducted simulations. We begin by providing the asymptotically optimal rates r^* to one specific multicast scenario. Then, we evaluate the performance of the competing solution of Yang et al. and compare it to the asymptotically optimal solution for three different multicast scenarios, characterized by the receiver reception capacity distribution. The solutions are evaluated for $I \in \{2, \ldots, 10\}$ multicast groups.

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All simulations were performed for the *foreman* CIF video source at 30 frames per second. We used the H.264 reference software, version JM 12.2, for which the model parameters of (1) were found to be $\theta = 7086$, $R_0 = 18.8$ and $D_0 = 2.8$.

As described earlier, cf. Equation (2), the underlying distribution of receiver reception capacities C can be assumed multimodal. For the purpose of our result illustration, we chose to let C be distributed according to a mixture of Gaussians with two components. More specifically, we chose

$$f_C(c) = \frac{1}{2}\mathcal{N}(400, 2000) + \frac{1}{2}\mathcal{N}(700, 2000), \qquad (20)$$

where the unit of the variable is kbps. This distribution of C is plotted in Figure 1, along with the optimal reconstruction point density $g_R^*(c)$ and compressor $h^*(c)$ for $I = 4$ streams. As can be seen, the optimal reconstruction point

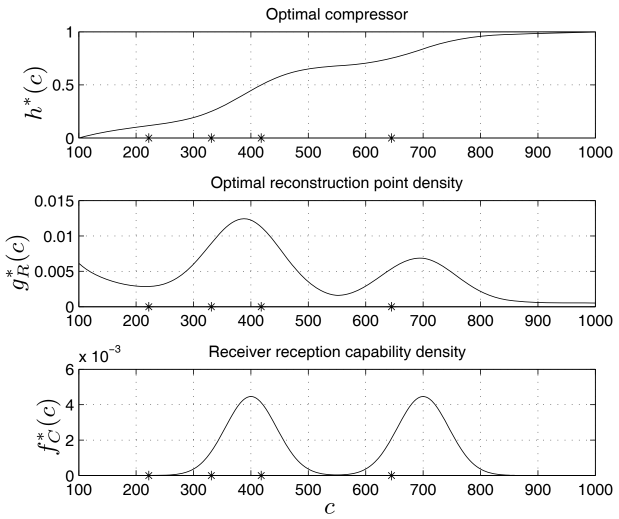


Figure 1: The distribution of receiver reception capacities from (20), the optimal reconstruction point density and the optimal compressor. Reconstruction points are illustrated with stars.

distribution follows the distribution of receiver reception capacities. However, due to the fact that the distortion function is proportional to the inverse of the reception capacity, most reconstruction points should be placed at low reception capacities. The optimal compressor is used to find the reconstruction points, which are illustrated with stars.

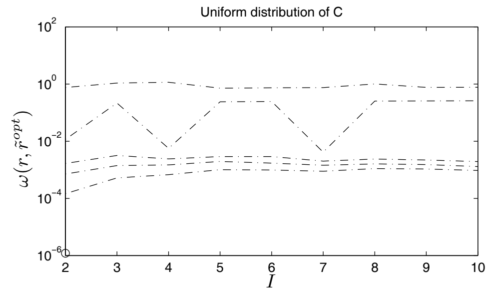
To compare competing methods for multicast rate allocation, we define the distortion overhead as

$$\omega(r, r^{opt}) = \frac{E[d(a(r, C)) - d(a(r^{opt}, C))]}{|E[d(a(r^{opt}, C))]|}. \qquad (21)$$

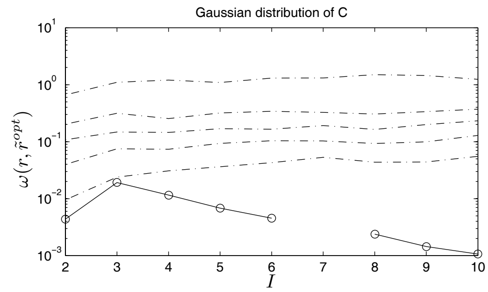
Here, r is the vector with assignment rates obtained from the specific multicast rate allocation method, r^{opt} is the vector with optimal stream rates and the expectations are with respect to C.

We used the distortion overhead definition to evaluate the performance of two rate allocation methods. The first was the asymptotically optimal rate allocation proposed in this paper. The second was the rate allocation algorithm proposed by Yang et al. in [4]. We denote the solutions of the two methods with r^* and r^{Yang}, respectively. As the optimal rates r^{opt} are not known in general, we evaluated the distortion overhead for $r^{opt} = \tilde{r}^{opt}$, the stream rates resulting from any of the two methods in our simulations, that evaluated to the least distortion.

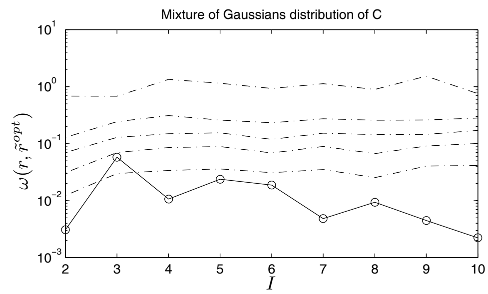
We performed the simulations for three multicast scenarios, characterized by the probability density functions of the receiver reception capacities C. These probability density functions used were the uniform, the Gaussian and a bimodal mixture of Gaussians. For all distributions, we limited the precision of realizations and the rate assignment to 1 kbps. Since r^{Yang} depends on the realization of receiver reception capacities, we ran the algorithm 200 times per distribution and number of multicast groups, for 1000 receivers each time. The number of multicast groups was 10 at its largest, due to the high computational complexity of the algorithm of Yang et al. Simulation results are shown in Figure 2.



(a) $f_C(c) = \mathcal{U}[200, 900]$



(b) $f_C(c) = \mathcal{N}(550, 5000)$



(c) $f_C(c) = \frac{1}{2}\mathcal{N}(400, 2000) + \frac{1}{2}\mathcal{N}(700, 2000)$

Figure 2: The distortion overhead, plotted as a function of the number of multicast streams, for the three multicast scenarios. Results for the asymptotically optimal rates r^* are given by the solid curves with circles. Zero distortion overhead, as for the Gaussian distribution and $I = 7$ streams, is indicated by the lack of a circle. The dashed curves represent the first through fifth quintile of the distortion overhead for r^{Yang}, from bottom to top, respectively.

The distortion overhead is, for the asymptotically optimal rates r^*, given by the solid line with circles. Zero distortion overhead is indicated by the lack of a circle. The distortion overhead for r^{Yang} is in the plots illustrated by the first through fifth quintile. Hence, the distortion overhead was in 20%, 40%, 60%, 80% and 100% of the 200 runs smaller than indicated by the dashed curves, from bottom to top, respectively.

We see that the distortion overhead, $\omega(r^*, \tilde{r}^{opt})$, for the asymptotically optimal rates r^* was smallest for the uniform distribution of C. This is consistent with the fact that $f_C(c)$ in this case is constant within the cells as was assumed in the derivations. As expected, $\omega(r^*, \tilde{r}^{opt})$ decreased with an increasing number of streams. The maximum value of the distortion overhead for r^* was always well below 10%.

For the solution proposed by Yang et al., we see that the largest distortion overhead, $\omega(r^{Yang}, \tilde{r}^{opt})$, obtained in the simulations was larger than 100%. This confirms our hypothesis that optimization for a realization of receiver reception capacities can incur large distortion overhead. Compared to our method, for over 80% of the runs $\omega(r^*, \tilde{r}^{opt}) < \omega(r^{Yang}, \tilde{r}^{opt})$. Naturally, since r^* is asymptotically optimal, this figure increases with the number of multicast groups, as seen in the figure.

5. CONCLUSIONS

We have developed a theory for analytically finding the stream rates for video multicast. The theory is based on the notion, confirmed by work in networking literature, that the receiver reception capacities should be modeled as realizations of a continuous stochastic variable. The rates are optimal asymptotically in the number of multicast groups. Simulation results show that the distortion overhead is low when this assumption is not valid and that it decreases with the number of multicast groups. The fact that the rates are obtained analytically makes the solution well suited for practical multicast scenarios, which is not the case for prior art. Further, we have compared our method to the state of the art, which is iterative optimization for a specific realization of receiver reception capacities. Our solution performs better in over 80% of the cases for small numbers of multicast groups. This figure approaches 100% with an increasing number of groups.

APPENDIX A

As described earlier, the high-rate assumption implies that, asymptotically with increasing rate, the source distribution $f_C(c)$ is constant and that the distortion $d(c)$ is linear within a neighborhood of each cell. For cell i and $c \in [r_i - \frac{1}{2}\Delta_i, r_{i+1}]$, we write

$$f_C(c) = f_C(r_i) \tag{22}$$

and

$$d(c) = m_i + k_i c. \tag{23}$$

Then, it follows that the cell distortion can be written as

$$\begin{aligned} d_i &= \int_{r_i}^{r_{i+1}} d(r_i) f_C(c)\ \mathrm{d}c \\ &\approx f_C(r_i)\,(m_i + k_i r_i)\,\Delta_i \\ &= f_C(r_i)\left(m_i\Delta_i + \frac{1}{2}k_i(r_{i+1}^2 - r_i^2) - \frac{1}{2}k_i\Delta_i^2\right) \\ &= f_C(r_i)\int_{r_i}^{r_{i+1}}\left(m_i + k_i(c - \frac{1}{2}\Delta_i)\right)\ \mathrm{d}c \\ &\approx \int_{r_i}^{r_{i+1}} d(c - \frac{1}{2}\Delta_i) f_C(c)\ \mathrm{d}c, \end{aligned} \tag{24}$$

where we used $\Delta_i = r_{i_1} - r_i$.

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