MULTICHANNEL AR PARAMETER ESTIMATION FROM NOISY OBSERVATIONS AS AN ERRORS-IN-VARIABLES ISSUE

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ABSTRACT

In various applications from radar processing to mobile communication systems based on CDMA for instance, M-AR multichannel processes are often considered and may be combined with Kalman filtering. However, the estimations of the M-AR parameter matrices and the covariance matrices of the additive noise and the driving process from noisy observations are key issues to be addressed. In this paper, we propose to solve this problem as an errors-in-variables problem. Thus, the noisy observation autocorrelation matrix compensated by a specific diagonal block matrix and whose kernel is defined by the M-AR parameters matrices must be positive semi-definite. Hence, the parameter estimation consists of searching every diagonal block matrix that satisfies this property, of reiterating this search for a higher model order and then of extracting the solution that belongs to both sets. The proposed algorithm outperforms existing methods, especially for low signal-to-noise ratio and when the variances of the additive noise are not necessarily the same on each channel.

1. INTRODUCTION

Linear-model based approaches are very popular in various applications such as speech processing and biomedical. When dealing with scalar AutoRegressive (AR) process, the key issue is usually the selection of the model order and the estimations of the AR parameters from noisy observations. Indeed, to reduce the bias on the AR parameter estimation due to the additive measurement noise, one solution consists of using instrumental variable techniques such as the modified Yule-Walker (MYW) equations or mutually-interactive optimal filter based solutions [10]. Another approach is the 'noise-compensated' Yule-Walker equations which however require the estimation of the additive-noise variance [9]. To solve this dual estimation problem, several off-line approaches have been proposed[1] by Davila [5], Zheng [16], etc. In [6], Diversi et al. suggest viewing this joint estimation as an errors-in-variables issue. In theory, this solution has the advantage of blindly providing the AR parameters, the model order and the variances of the driving process and the additive noise. This method aims at studying the semi-definite positiveness of specific observation correlation matrices by using the so-called Frisch scheme[2] [2]. Meanwhile, we have analysed the relevance of the method for optimal filter-based speech enhancement using a single microphone [3].

Although scalar AR modelling is often used in various cases, a p^{th} order M-AR multichannel process $\underline{x}(n)$ is more suited when dealing with simultaneous processing of multiple correlated data channels. It is defined as follows:

$$\underline{x}(n)=\begin{bmatrix} x_1(n) \\ \vdots \\ x_M(n) \end{bmatrix} = -\sum_{l=1}^{p} A^{(l)}\,\underline{x}(n-l)+\underline{u}(n) \qquad (1)$$

where $\left\{A^{(l)}\right\}_{l=1,\dots,p}$ are the $M\times M$ AR parameter matrices and $\underline{u}(n)$ is a $M\times 1$ zero-mean white noise vector whose correlation matrix is denoted Σ_u and satisfies:

$$\Sigma_u = diag\left(\begin{bmatrix} \sigma^2_{u,1} & \cdots & \sigma^2_{u,M} \end{bmatrix}\right) \qquad (2)$$

It should be noted that the AR parameter matrices $\left\{A^{(l)}\right\}_{l=1,\dots,p}$ are constrained such that the roots of:

$$\det A_p(z)=0 \qquad (3)$$

[1] The reader is referred to [3] and [10] to have more information about the various methods that have been proposed to estimate the AR parameters from noisy observations.

[2] The Frisch scheme will be recalled in section 2.

lie inside the unit circle in the z-plane, where:

$$A_p(z) = I_M + A^{(1)}z^{-1} + A^{(2)}z^{-2} + ... + A^{(p)}z^{-p} \quad (4)$$

with I_M the $M \times M$ identity matrix and z^{-1} the backward shift operator.

Such arrays of signals are common in seismic data processing, in radar and sonar processing, mobile communication systems, etc.

- In radar processing based on antenna arrays, the Space-Time Adaptive Processing (STAP) algorithm, which makes it possible to weaken the influence of the additive measurement noise, the clutter and the jamming signals to determine the presence of a target, provides significant results especially for slowly moving targets [1]. However the corresponding computation cost is high. To reduce it, options known as the Parametric Adaptive Matched Filter (PAMF) and the Space-Time Auto-Regressive Filter (STAR) consist of modelling the clutter and the interference of various range/azimuth cells as a *M*-AR process [14].
- In mobile communication systems based on CDMA for instance, the fading-channel processes can be estimated or predicted by using a Kalman algorithm for instance and by modelling the channels by a *M*-AR multichannel process [4] [11].

In the above applications, the estimations of the *M*-AR matrices and the noise covariance matrices from noisy observations are key problems to be addressed, but few papers really deal with this issue. When noise-free observations are available, the multichannel Yule-Walker equation, its fast version -namely the multichannel Levinson algorithm-, and alternative such as the Nuttall-Strand method [12] can be used to estimate the *M*-AR parameter matrices. A Maximum Likelihood (ML) estimation has been also proposed by Pham et al. [13]. More recently, Schlögl [15] points out the relevance of the Nuttall-Strand method by carrying out a comparative study between standard approaches and the so-called "Algorithm 808-ARfit" proposed by Schneider and Neumaier (See [15] for more details about the references).

However, when the multichannel process is disturbed by an additive white noise, the estimation of the *M*-AR parameter matrices becomes biased [12]. In [8], Hasan presents an iterative approach which alternately estimates the noise variances on each channel by means of the Newton–Raphson gradient search technique and the *M*-AR parameter matrices by solving the corresponding noise-compensated Yule-Walker equation. Nevertheless, this method is no longer reliable when the signal-to-noise ratio (SNR) becomes low.

In this paper, a new application of errors-in-variables approach based on the Frisch scheme [6] is presented. To estimate the *M*-AR parameter matrices, the noisy observation autocorrelation matrix compensated by a specific diagonal block matrix must be semi-definite positive. The algorithm consists of:

- searching every diagonal block matrix that satisfies this property,
- reiterating this search for a higher model order,
- extracting the solution that belongs to both sets.

It extends to the *M*-AR process case the method presented in [3] for the scalar AR process. It should be noted that, like Hasan's approach [8], the variances of the additive noise are not necessarily equal on each channel. In addition, our approach in theory also provides the *M*-AR model order and the covariance matrix of the driving process. As we will see, the proposed algorithm outperforms Hasan's method [8], in practical case especially for low SNR.

The remainder of the paper is organized as follows. In section 2, the estimation approach is detailed. In section 3, to illustrate the relevance of our approach, we carry out comparative study with existing approaches such as [8], the multichannel Yule-Walker equation and the Nutall-Strand method.

2. PROBLEM STATEMENT

2.1 General case

Let the *M*-AR process $\underline{x}(n)$ be disturbed by an additive zero-mean white noise vector $\underline{b}(n)$ uncorrelated with $\underline{u}(n)$ and with correlation matrix $\Sigma_b = diag\,([\sigma_{b,1}^2 \cdots \sigma_{b,M}^2])$.

$$\underline{y}(n) = \underline{x}(n) + \underline{b}(n) \quad (5)$$

The purpose of our method is to estimate the *M*-AR parameter matrices and the autocorrelation matrix Σ_u and Σ_b from the autocorrelation matrix of $\underline{y}(n)$. Indeed, (1) can be rewritten as follows:

$$\underline{x}(n) - \underline{u}(n) + \sum_{l=1}^{p} A^{(l)} \underline{x}(n-l) = 0 \quad (6)$$

or equivalently in a matrix form:

$$\begin{bmatrix} (\underline{x}(n)-\underline{u}(n))^T & \underline{x}^T(n-1) & \cdots & \underline{x}^T(n-p) \end{bmatrix} \begin{bmatrix} I_M \\ A^{(1)^T} \\ \vdots \\ A^{(p)^T} \end{bmatrix} = 0 \quad (7)$$

Pre-multiplying (7) by $\begin{bmatrix} (\underline{x}(n)-\underline{u}(n)) \\ \underline{x}(n-1) \\ \vdots \\ \underline{x}(n-p) \end{bmatrix}^*$ and taking the expectation leads to:

$$\begin{bmatrix} R_x(0)-\Sigma_u & R_x(1) & \cdots & R_x(p) \\ R_x(-1) & R_x(0) & \ddots & \vdots \\ \vdots & \ddots & \ddots & R_x(1) \\ R_x(-p) & \cdots & R_x(-1) & R_x(0) \end{bmatrix}^* \times \begin{bmatrix} I_M \\ A^{(1)^T} \\ \vdots \\ A^{(p)^T} \end{bmatrix} = 0 \quad (8)$$

where $R_x(m) = E\left[\underline{x}(n)\underline{x}^{T*}(n-m)\right]$.

Thus, one has:

$$R_{x,u}^{(p+1)^*}\begin{bmatrix} I_M \\ A^{(1)^T} \\ \vdots \\ A^{(p)^T}\end{bmatrix} = R_{x,u}^{(p+1)^*}\,\theta_{p+1} = 0 \qquad (9)$$

with

$$R_{x,u}^{(p+1)} = \begin{bmatrix} R_x(0)-\Sigma_u & R_x(1) & \cdots & R_x(p) \\ R_x(-1) & R_x(0) & \ddots & \vdots \\ \vdots & \ddots & \ddots & R_x(1) \\ R_x(-p) & \cdots & R_x(-1) & R_x(0) \end{bmatrix} \qquad (10)$$

or equivalently:

$$R_{x,u}^{(p+1)} = R_x^{(p+1)} - diag([\Sigma_u \quad \underbrace{0 \ \cdots \ 0}_{Mp}]) \qquad (11)$$

Due to (9), $R_{x,u}^{(p+1)}$ is positive semi-definite and the *M*-AR parameter matrices span the kernel of $R_{x,u}^{(p+1)}$.

However, in practical case, $R_{x,u}^{(p+1)}$ is not directly available and only the positive definite autocorrelation matrix of the noisy observations $R_y^{(p+1)}$ can be considered. It satisfies:

$$R_y^{(p+1)} = R_x^{(p+1)} + diag([\Sigma_b \quad \Sigma_b \quad \cdots \quad \Sigma_b]) \qquad (12)$$

Combining (11) and (12) leads to the semi-definite positiveness property of the following matrix:

$$R_{x,u}^{(p+1)} = R_y^{(p+1)} - diag\left(\left[\Sigma_u + \Sigma_b \quad \underbrace{\Sigma_b \ldots \Sigma_b}_{p}\right]\right) \geq 0 \qquad (13)$$

where $\Sigma_u + \Sigma_b = diag\left(\left[\sigma_{u,1}^2 + \sigma_{b,1}^2 \ \ldots \ \sigma_{u,M}^2 + \sigma_{b,M}^2\right]\right)$.

Therefore, searching for the autocorrelation matrix Σ_u and Σ_b consists of finding the matrices that make $R_y^{(p+1)} - diag\left(\left[\Sigma_u + \Sigma_b \quad \underbrace{\Sigma_b \ldots \Sigma_b}_{p}\right]\right)$ positive semi-definite. At that stage, the *M*-AR parameter matrices can be obtained by solving the noise-compensated Yule Walker equation.

2.2 Formulation with two channels

In the following, let us assume that $M = 2$ for the sake of clarity and simplicity. In that case, property (13) only depends on $\sigma_{u,1}^2, \sigma_{u,2}^2, \sigma_{b,1}^2$ and $\sigma_{b,2}^2$. The idea is then to retrieve the values $\beta_1^2, \beta_2^2, \alpha_1^2$ and α_2^2 that makes the noise compensated matrix positive semi-definite.

$$R_{x,u}^{(p+1)} = R_y^{(p+1)} - diag\left(\left[\mathrm{B} \ \ \underbrace{\mathrm{A} \ \ldots \ \mathrm{A}}_{p}\right]\right) \geq 0 \qquad (14)$$

with $\mathrm{B} = diag\left(\left[\beta_1^2 \ \beta_2^2\right]\right)$ and $\mathrm{A} = diag\left(\left[\alpha_1^2 \ \alpha_2^2\right]\right)$.

For the order p, the locus of admissible points that satisfy the above condition is a hypersurface denoted by $S(R_y^{(p+1)})$ and whose concavity faces the origin $O(0,0,0,0)$ in the orthant of R^4. When increasing the order to q, another set $S(R_y^{(q)})$, with $q > p$, can be obtained.

In theory, the solution we search belong to both sets. In practical case, however, no common point exist and criteria have to be considered like those proposed in the very last years in the area. For more information, the reader is referred to [2], [3] and [6].

In this paper, two criteria are presented and lead to two algorithms explained in the next paragraphs.

<u>*Remark:*</u> a point of $S(R_y^{(p+1)})$ can be expressed [7] as follows:

Let $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)$ in the orthant of R^4 and r a straight line from the origin through the point ε. Its intersection with $S(R_y^{(p+1)})$ is defined by the point P. Therefore, $\overrightarrow{OP}$ and $\overrightarrow{O\varepsilon}$ are collinear, i.e. one has:

$$\left(\beta_1^2, \beta_2^2, \alpha_1^2, \alpha_1^2\right) = \frac{(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)}{\lambda} \qquad (15)$$

where λ is largest eigenvalue of the matrix $\left(R_y^{(p+1)}\right)^{-1} diag\left(\left[\varepsilon_1 \ \varepsilon_2 \ \underbrace{\varepsilon_3 \ \varepsilon_4 \ \ldots \ \varepsilon_3 \ \varepsilon_4}_{2p}\right]\right)$.

2.3 Algorithm 1

1/ Start with a generic point $\varepsilon = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)$ in the first orthant of R^4.

2/ Determine the point $\mathrm{P}_{p+1} = \frac{1}{\lambda_{p+1}}(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)$ on the hypersurface $S(R_y^{(p+1)})$ and $\mathrm{P}_{q+1} = \frac{1}{\lambda_{q+1}}(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)$ on $S(R_y^{(q+1)})$, where λ_{p+1} and λ_{q+1} are respectively the largest eigenvalues of the matrix $\left(R_y^{(p+1)}\right)^{-1} diag\left(\left[\varepsilon_1 \ \varepsilon_2 \ \underbrace{\varepsilon_3 \ \varepsilon_4 \ \ldots \ \varepsilon_3 \ \varepsilon_4}_{2p}\right]\right)$ and of $\left(R_y^{(q+1)}\right)^{-1} diag\left(\left[\varepsilon_1 \ \varepsilon_2 \ \underbrace{\varepsilon_3 \ \varepsilon_4 \ \ldots \ \varepsilon_3 \ \varepsilon_4}_{2q}\right]\right)$, respectively.

3/ Calculate the M-AR parameter matrices by solving the noise compensated Yule-Walker equation associated to the noise variances that define P_{p+1}. It gives $\theta_{p+1}(P_{p+1})$.

Calculate the criterion $\left\| R^{(q+1)} \begin{bmatrix} \theta_{p+1}(P_{p+1}) \\ 0_{(q-p)M\times M} \end{bmatrix} \right\|_2^2$ and minimize the criterion by studying the locus of solutions defining $R^{(p+1)}(P_{p+1})$.

2.3 Algorithm 2

Given (1), the autocorrelation matrices of the noise free M-AR process and the noisy observations satisfy, for $\tau \geq p+1$:

$$R_x^*(\tau) = R_y^*(\tau) = -\sum_{l=1}^{p} A^{(l)} R_x^*(\tau - l) = -\sum_{l=1}^{p} A^{(l)} R_y^*(\tau - l) \quad (16)$$

or equivalently where $q \geq p$:

$$\theta_{p+1}^T(P_{p+1}) \begin{bmatrix} R_y(p+1) & \cdots & R_y(p+q) \\ \vdots & & \vdots \\ R_y(1) & \cdots & R_y(q) \end{bmatrix} = 0 \quad (17)$$

Therefore, a second criterion can be derived by looking for the points P_{p+1} that minimizes:

$$\left\| \theta_{p+1}^T(P_{p+1}) \begin{bmatrix} R_y(p+1) & \cdots & R_y(p+q) \\ \vdots & & \vdots \\ R_y(1) & \cdots & R_y(q) \end{bmatrix} \right\|_2^2$$

In the next section, we propose to compare the relevance of both algorithms with Hasan's method proposed in [8] and the standard multichannel Yule–Walker equations and the Nutall-Strand method.

3. SIMULATION RESULTS

We have carried out various simulation tests. In this paper, let us consider a 2nd order ($p = 2$) two-channel ($M = 2$) AR process:

$$\underline{x}(n) = -A^{(1)}\underline{x}(n-1) - A^{(2)}\underline{x}(n-2) + \underline{u}(n) \quad (18)$$

$$\underline{y}(n) = \underline{x}(n) + \underline{b}(n) \quad (19)$$

where the AR coefficient matrices are those defined by Hasan in [8]:

$$A^{(1)} = \begin{pmatrix} -0.71 & 0.32 \\ -0.88 & -0.24 \end{pmatrix} \text{ and } A^{(2)} = \begin{pmatrix} 0.57 & -0.15 \\ -0.49 & -0.30 \end{pmatrix}$$

In addition, $\underline{u}(n)$ is the two-channel stationary Gaussian white noise, uncorrelated between channels and with unit variance on each channel. The additive noise $\underline{b}(n)$ is also a two-channel stationary Gaussian white noise, uncorrelated with $\underline{u}(n)$.

Hence, various scenarios can be studied depending on the SNR on each channel. In the experiments below, 4000 data samples for each channel are first used.

In addition, let us first consider that the SNR on the first and the second channel are respectively equal to 10 dB and 5dB, as suggested by Hasan in [8].

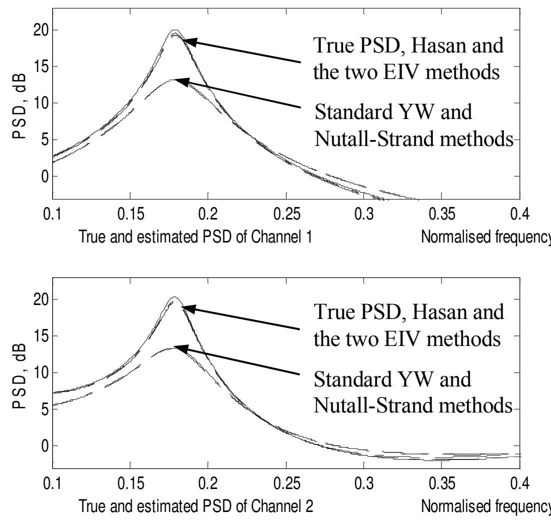

Figure 1 – True and estimated PSD of the two channels in the 1st test

As pointed out in Fig 1, Hasan's approach and ours make it possible to retrieve the true spectra of the AR process from noisy observations. For the sake of space, we do not present the estimated cross-PSD between channel 1 and 2 in the first test, but the methods provide similar results.

When the SNR are lower, i.e. 5dB and 3dB for instance on the 1st and 2nd channel respectively, our approach outperforms Hasan's method. See fig. 2, 3. To confirm this result, Tables 1 and 2 provide:

1/ the mean square error (MSE) on the modulus of the roots of $\det \hat{A}_p(z)$, based on 100 realizations.

2/ the rate $T_{instability}$ of realizations, for which an estimated root of $\det \hat{A}_p(z)$ is outside the unit circle in the z-plane.

To test the limit of our methods, the same tests are then carried out with only 200 samples. See Table 3.

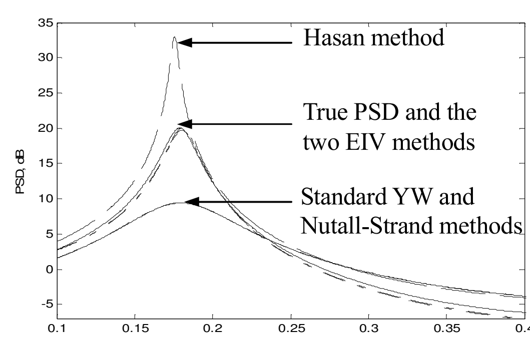

Figure 2 – True and estimated PSD of channel 1 in second test

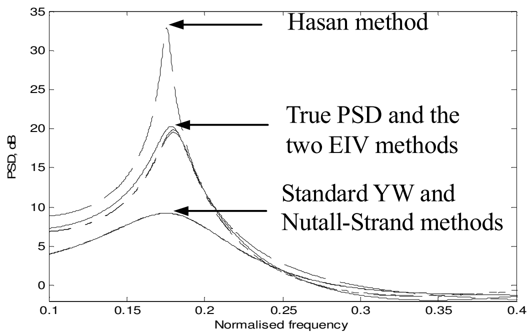

Figure 3 – True and estimated PSD of channel 2 in second test

True AR coefficients	Estimated AR coefficients		
	Hasan method	EIV (Algo 1)	EIV (Algo 2)
$a_{11}^{(1)} = -0.71$	-1.090	-0.780	-0.796
$a_{12}^{(1)} = 0.32$	-1.730	0.164	0.126
$a_{21}^{(1)} = -0.88$	-1.216	-0.798	-0.777
$a_{22}^{(1)} = -0.24$	-1.282	-0.169	-0.155
$a_{11}^{(2)} = 0.57$	2.609	0.733	0.774
$a_{12}^{(2)} = -0.15$	1.004	-0.102	-0.086
$a_{21}^{(2)} = -0.49$	0.610	-0.504	-0.521
$a_{22}^{(2)} = -0.30$	0.146	-0.264	-0.248

Table 1: Comparison of estimated AR coefficients

MSE of absolute value (10^{-3})	Hasan method	EIV (Algo 1)	EIV (Algo 2)
Pole 1	29.111	14.488	14.864
Pole 2	24.216	5.911	16.392
Pole 3	0.309	0.117	0.140
Pole 4	0.345	0.117	0.201
$T_{instability}$	19%	0%	0%

Table 2: MSE of the modulus of the roots of $\det A_p(z)$ with 4000 samples

MSE of absolute value (10^{-3})	Hasan method	EIV (Algo 1)	EIV (Algo 2)
Pole 1	691.0	82.8	67.8
Pole 2	612.7	36.4	42.2
Pole 3	367.2	3.2	2.2
Pole 4	368.8	3.2	2.2
$T_{instability}$	24%	0%	0%

Table 3: MSE of the modulus of the roots of $\det A_p(z)$ with 200 samples

4. CONCLUSIONS AND PERSPECTIVES

We have proposed a new method to blindly estimate the *M*-AR parameter matrices and the covariance matrices of the additive noise and the driving process from noisy observations. Our method is reliable for a SNR higher or equal to 3dB on each channel and with a number of data samples larger than 200. To reduce the computational cost, a recursive approach could be developed. We are currently analysing its relevance in radar processing.

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