16th European Signal Processing Conference (EUSIPCO 2008), Lausanne, Switzerland, August 25-29, 2008, copyright by EURASIP

SENSITIVITY TO NOISE AND PERFORMANCE OF HRR-BASED ATR USING THE MAXIMUM POSITION ALIGNMENT METHOD

R. Gil-Pita, M. Rosa-Zurera, R. Vicen-Bueno, L. Álvarez and F. López-Ferreras

Teoría de la Señal y Comunicaciones, Universidad de Alcalá
28805 Alcalá de Henares - Madrid (SPAIN)
e-mail: roberto.gil@uah.es

ABSTRACT

This paper studies the alignment of noisy high resolution radar signals using the Maximum Position method in automatic target recognition. The relationship between the shift estimation, the performance of the classifier and the signal to noise ratio is analyzed. Several experiments are carried out in order to study the influence of the alignment in the performance of a classification system. These experiments allow us to improve the understanding of the sensitivity to noise of the Maximum Position alignment method.

1. INTRODUCTION

High Resolution Radar (HRR) uses broad-band step frequency waveforms to measure range profiles of targets [1]. These range profiles are obtained by the integration of the returned signal over range bins. Each range bin contains the total radar return for that time segment. If a range profile is measured with sufficient resolution, the parts of the aircraft that strongly reflect the radar energy are resolved. Therefore, range profiles provide information about the geometry and structure of the target, and so they are suitable features for automatic target recognition (ATR). This term originated in the early 1980s with the Low Altitude Navigation and Targeting Infra-Red for Night (LANTIRN) program. One of its objectives was to develop a system capable of distinguishing tanks from trucks, jeeps, and other less important targets. In the case of HRR-based ATR, the extreme variability in the HRR profile with minor changes in azimuth, elevation, and time makes this task very difficult. Furthermore, the conventional way to construct HRR profiles is through an inverse Fourier transform, which cause circular shifts of the received signal due to variations in the distance to the target [2], and most of the classification algorithms (neural networks, support vector machines, k-nearest neighbor, etc.) highly depend on shifts over the input signal. This fact makes the design of the alignment stage very important when implementing efficient classifiers.

The alignment of HRR signals is found several times in the literature. In [2] a new HRR aligning method is proposed which combines the relative Cross-correlation and the Zero Phase methods. The application of the Maximum Position method, the Cross-correlation method [3] and the Zero Phase method [4] in the alignment of HRR signals is studied in [5]. An analytical study of the Maximum Position method for aligning noisy HRR signals is included in [6]. This method determines the position of the maximum value of each vector and shifts the signal using this position so that the maximum values of all the shifted vectors are found in the same sample. This is the simplest method among those described in the literature, and its associated computational cost is very low. The comparison of this alignment method to other methods, like, for example, the Zero Phase method or Cross-correlation based methods, offers contradictory results. Although the sensitivity to noise of the Maximum Position method is not very low [7], it is, however, a good alternative when the global performance of the classification system is considered [5].

There are two choices in order to measure the performance of the alignment method in classification applications:

- *The study of the variations in the estimation of the shift.* Due to the presence of noise, the selected maximum position may be different from the one selected without noise. The study of the sensitivity to noise allows to make comparisons of the relationship of the performance in the alignment with the signal-to-noise ratio (SNR). For this purpose, the root mean square value of the difference in the shift estimation with and without noise is studied.
- *The global performance of the recognition system in terms of classification error rate.* In this case, the performance of a classification system designed using the patterns aligned by the Maximum Position method is analyzed. For this purpose, the Nearest Neighbor (NN) method, which has been frequently used for automatic target recognition purposes [8] due to the simplicity of its implementation, is selected to evaluate the classification performance of the alignment method. [9].

Both studies allow the sensitivity to noise of the alignment method and its suitability for classification tasks to be studied. The purpose of this paper is to establish a relationship between these two measurements in the Maximum Position alignment method, that allows to better understand the performance of this alignment method in ATR applications

2. VARIATIONS IN THE ESTIMATION OF THE SHIFT

In this section, the sensitivity of the Maximum Position alignment method to noise variations is analyzed. The objective is to establish both theoretical and empirical sensitivity curves, that allow to understand the behavior of the alignment method, and the variations in the estimation of the shift.

In order to carry out a theoretical analysis, the HRR signal must be statistically characterized. An extension of this analysis can be found in [6]. Considering P_n, $(n = 1, \cdots, L)$ is the probability of selecting the n-th sample of the HRR signal as maximum, the root mean square value of the dif-

This paper was supported by the Spanish Project TEC2006-13883-C04-04/TCM

ference between the estimation of the shift with and without noise (E_{MP}) can be calculated using equation (1).

$$E_{MP} = \sqrt{\sum_{n=1}^{L} P_n \cdot \left(\frac{L}{2} - \left| |n-p| - \frac{L}{2} \right| \right)^2} \qquad (1)$$

Where p is the position of the maximum sample of the vector without noise. So, the study of error in the estimation of the shift requires the characterization of the Probability Density Function (PDF) associated to the HRR signal. Let's consider that $\mathbf{y} = [y_1, y_2, ..., y_L]$ is the HRR signal we want to align. Considering the deterministic model of a static target described in [10], the random variable associated to y_n is Rice with a PDF $f_{Y_n}(y_n)$ given by equation (2) [11].

$$f_{Y_n}(y_n) = \frac{y_n}{\sigma^2} e^{-(y_n^2+|x_n|^2)/2\sigma^2} I_0\left(\frac{y_n|x_n|}{\sigma^2}\right) u(y_n) \qquad (2)$$

Where I_0 is the zero order modified Bessel function, x_n is the HRR signal without noise, σ^2 is the variance of the noise, and $u(y_n)$ is the unit step function so that $u(y_n) = 1$ for $y_n \geq 0$ and $u(y_n) = 0$ otherwise.

The Rice distribution is not analytically integrable and, in order to carry out an analytical analysis of the alignment methods, we propose the use of the log-sigmoidal function to approximate the Cumulative Distribution Function (CDF) of a Rice variable [6]. Therefore, CDF given by equation (3) is an approximation of the CDF of the random Rice variable y_n.

$$F_{Y_n}(y_n) \simeq \left(1 + e^{-\frac{\pi}{\sqrt{3}} \frac{y_n - \sqrt{|x_n|^2+\sigma^2}}{\sigma}} \right)^{-1} \qquad (3)$$

This function is analytically integrable and can be used to determine analytical approximations of the performance of the Maximum Position alignment method.

In the proposed model, the samples of the HRR signal are independent, and, therefore, the probability that sample p is the maximum value is expressed in equation (4)

$$P_p = \int_{-\infty}^{\infty} \Big(\prod_{\forall n \neq p} \int_{-\infty}^{y_p} f_{Y_n}(y_n) dy_n \Big) f_{Y_p}(y_p) dy_p \qquad (4)$$

Concerning the HRR signal without noise x_n, in this paper we approximate it by two Kronecker deltas, so that only the p-th and q-th samples have significant values, $|x_p| > |x_q|$, and the probability of a sample other than the one of these two samples being maximum is considered zero. Thus, the probability that sample p is the maximum can be approximated with (5).

$$P_p \approx P(y_p > y_q) = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{y_p} f_{Y_q}(y_q) dy_q \right) f_{Y_p}(y_p) dy_p \quad (5)$$

E_{MP} is therefore expressed using equation (6).

$$E_{MP} \simeq (L/2 - ||q-p| - L/2|) \sqrt{1 - P_p} \qquad (6)$$

Using the analytically integrable approach of the Rice distribution given by equation (3), the integrals of equation (5) are solved, obtaining equation (7).

$$P_p \simeq \frac{\frac{\pi(\mu_q-\mu_p)}{\sqrt{3}\sigma} e^{\frac{\pi(\mu_q+\mu_p)}{\sqrt{3}\sigma}}}{\left(e^{\frac{\pi\mu_q}{\sqrt{3}\sigma}} - e^{\frac{\pi\mu_p}{\sqrt{3}\sigma}}\right)^2} - \frac{e^{\frac{\pi\mu_p}{\sqrt{3}\sigma}}}{e^{\frac{\pi\mu_q}{\sqrt{3}\sigma}} - e^{\frac{\pi\mu_p}{\sqrt{3}\sigma}}} \qquad (7)$$

Where $\mu_p = \sqrt{|x_p|^2 + \sigma^2}$, and $\mu_q = \sqrt{|x_q|^2 + \sigma^2}$. The objective of this study is to determine the sensitivity of the Maximum Position to the SNR. Due to the spatial localization of the HRR signals, the SNR is defined using the peak energy of the signal, given by equation (8).

$$SNR(dB) = 10 \log \left(\frac{\max\{|\mathbf{x}|\}^2}{\sigma^2} \right) = 10 \log \left(\frac{|x_p|^2}{\sigma^2} \right) \quad (8)$$

So, equation (7) is an estimation of the probability P_p as function of μ_p/σ and μ_q/σ, which are both function of the SNR:

$$\frac{\mu_p}{\sigma} = \sqrt{\frac{|x_p|^2}{\sigma^2} + 1} = \sqrt{10^{\frac{SNR(dB)}{10}} + 1} \qquad (9)$$

$$\frac{\mu_q}{\sigma} = \sqrt{\frac{|x_q|^2}{|x_p|^2} \frac{|x_p|^2}{\sigma^2} + 1} = \sqrt{\frac{|x_q|^2}{|x_p|^2} 10^{\frac{SNR(dB)}{10}} + 1} \qquad (10)$$

So, the application of (7) in (6) allows to obtain an estimation of the performance of the maximum position method, using the two highest samples of the vector. It is function of $|q-p|$, the absolute difference between the position of the two highest samples of the signal, and P_p, which is function of SNR and $|x_q|/|x_p|$, as stated before. In this case, the effects of the samples apart from the p-th and q-th ones are ignored. Therefore, the model is not accurate with signals with high noise levels, because in those cases the probability of the samples apart from the p-th and q-th ones being maximum must be considered.

In order to evaluate the sensitivity to noise of the Maximum Position method for aligning HRR profiles, a database containing the HRR profiles of six types of aircraft has been used. These signals have been generated using a state of the art electromagnetic modeling program, and they are the same signals used to generate the results presented in [12]. The assumed target position is head-on with an azimuth range of 25^o and elevations of -20^o to 0^o. The database contains 3810 HRR signals, and the length of each one is 128. Each profile in the database has been randomly shifted in order to study the capabilities of the alignment method. Therefore, the original data has been shifted using a uniform random integer variable from 0 to 127, which represents a complete misalignment of the profiles. The Rice PDF was used to generate noise, and the SNR was a parameter of the study. The SNR varies from 1 dB to 50 dB in increments of 1 dB. Figure 1 shows HRR signals for two targets in three different orientations, extracted from the available database. Each plot represents the amplitudes of the 128 samples of the HRR signal.

Figure 2 represents the root mean square value of E_{MP} applying the theoretical approach of equation (6) and the experimental results using the available database, applying a Monte Carlo study. To obtain the results of the theoretical approaches, the parameters (SNR, $|q-p|$, and $|x_q|/|x_p|$) are selected from each original HRR signal without noise. As

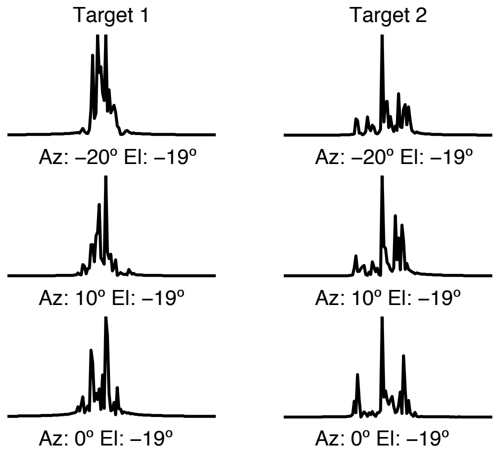

Figure 1: HRR signals for two targets in three different orientations, extracted from the available database.

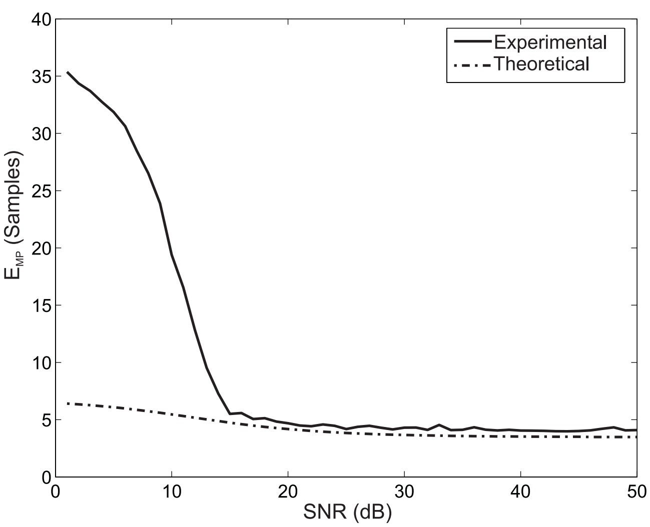

Figure 2: Root mean square value of E_{MP}, defined in section 2

can be seen in the Figure, experimental behavior is approximated by the theoretical approach for medium and high SNR values (over 15 dB).

For both the experimental case and the theoretical approach, the deviation in the estimation of the shift does not converge to zero. This behaviour can be explained by the complexity of range profiles, which causes the Maximum Position method to fail in the alignment of the profiles. This variability is caused by the existence of several signals in the database that have at least two high peaks with similar amplitudes (see the first HRR signal in Figure 1). In these cases, the position of the maximum is very sensitive to the presence of noise, causing the maximum sample to be selected with considerably low probability. The theoretical approach only considers the two highest values of the HRR signal and, therefore, does not properly model the sensitivity of the method for very low SNR values. The experimental results show the usefulness of the theoretical study carried out in this paper when a database of HRR signals is available. In those cases in which there is not a database to draw parameters from, expression (6) still allows to study the relationship between E_{MP} and the SNR for different values of $|q-p|$ and $|x_q|/|x_p|$.

3. VARIATIONS IN THE PERFORMANCE OF THE CLASSIFIER

In order to evaluate the performance of the alignment method from the point of view of a classification error standpoint, a NN classifier is implemented. This method is a distance-based classification technique in that has been applied in many different real classification scenarios. Given a set of pre-classified patterns denominated training patterns, it consists of determining the pattern of the training set with less distance to the input pattern, the pattern to classify. So, the class of the input pattern is assumed to be the same as the class of its “nearest neighbor”.

One important issue in the study of the NN classifiers is the selection of the metric for determining distances. There are many different metrics that are more or less suitable for specific applications. In the literature, the Euclidean distance uses to be the commonest choice, and it has been selected for the experiments of this paper.

In order to study the influence of the classification performance with the alignment of the HRR profiles, in this paper we will measure the performance of the NN classifier under three different conditions:

1. In first place, we consider the case in which the patterns are not misaligned. It allows to study the influence of the misalignment in the performance of the classifier.
2. In second place, the case in which the patterns are completely misaligned. This study helps to understand the utility of the alignment stage.
3. At last, the case in which the misaligned patterns are aligned using the Maximum Position method, previously to the classifier stage. The objective of this last group of experiments is to better understand the relationship between the sensitivity to noise of the alignment method and the classification performance.

Again, the SNR has been a parameter of the experiments, varying it from 5 dB to 50 dB in steps of 5 dB. In order to avoid the loss of generalization in the obtained results, the database has been divided in two subsets:

- The training set, totaling 1920 profiles randomly selected from the original data set, which is composed of those patterns that are used to design the NN classifier.
- The test set, composed of 1710 profiles, which is used to evaluate the performance of the NN classifier, specified as the *probability of misclassification* or *error rate*, which expresses the percentage of overall classification errors. This second set has not been used in the design of the classifiers.

Table 1 includes the classification error obtained by a NN classifier applied to correctly aligned noisy HRR signals, to misaligned noisy HRR signals, and to HRR signals aligned using the position of the maximum value. This Table shows the relationship between the SNR and the classification error measured with the test set. The results show how misalignment affects the classifier performance (the error rate achieved with misaligned patterns is up to four times greater than the one achieved with correctly aligned patterns). Furthermore, the Maximum Position alignment drastically reduces the error rate, but its performance is about 1.3 times the

Table 1: Classification error (%) using the NN method with misaligned patterns aligned using the Maximum Position method, and patterns without misalignment

SNR	Correctly aligned	Completely misaligned	Maximum Position alignment
5 dB	82.63%	83.51%	83.27%
10 dB	72.51%	82.69%	81.58%
15 dB	43.22%	71.11%	54.68%
20 dB	15.15%	50.12%	19.77%
25 dB	7.72%	42.28%	10.06%
30 dB	5.15%	35.96%	7.31%
35 dB	4.62%	34.91%	6.32%
40 dB	4.56%	34.62%	6.43%
45 dB	4.27%	34.56%	5.61%
50 dB	4.33%	34.33%	5.50%

Table 2: Confusion matrix between classes, showing the join probability (%) of each decision D_i and each hypothesis H_j for a SNR of 20 dB, and for the Maximum Position alignment method.

	H_1	H_2	H_3	H_4	H_5	H_6
D_1	15.15	0.06	0.70	0.35	0.58	0.70
D_2	0.23	11.87	0.29	4.74	0.53	0.53
D_3	0.29	0.18	14.09	0.18	0.06	0.58
D_4	0.18	3.33	0.41	9.94	0.23	0.12
D_5	0.70	0.94	0.88	0.94	15.20	0.76
D_6	0.12	0.29	0.29	0.53	0.06	13.98

one achieved by the NN applied to patterns correctly aligned, for SNRs equal or greater than 15 dB.

In order to understand if the error are concentred on one class or distributed equivalently for all the classes, tables 2 and 3 show the confusion matrixes for the Maximum Position alignment method for 20 and 50 dB, respectively. These tables show the probability of each decision and each class. So, D_i implies the classifier decides the pattern belongs to the i-th class, and H_j means the pattern belongs to the j-th class. As we can see, in both cases there is a higher probability of confusing patterns from classes 2 and 4, and the probability of the other kind of errors are quite similar.

Table 3: Confusion matrix between classes, showing the join probability (%) of each decision D_i and each hypothesis H_j for a SNR of 50 dB, and for the Maximum Position alignment method.

	H_1	H_2	H_3	H_4	H_5	H_6
D_1	16.37	0.00	0.06	0.12	0.00	0.35
D_2	0.00	14.85	0.12	1.58	0.06	0.06
D_3	0.23	0.06	16.20	0.06	0.00	0.06
D_4	0.00	1.52	0.18	14.50	0.23	0.00
D_5	0.06	0.06	0.06	0.23	16.37	0.00
D_6	0.00	0.18	0.06	0.18	0.00	16.20

4. CONCLUSIONS

In this paper we study the performance of the Maximum Position method in the alignment of noisy one-dimensional signals shifted in time. In a first approach, the root mean square value of the difference in the shift estimation with and without noise was studied using a database of HRR profiles. Different SNR values ranging from 1 to 50 dBs were also considered. For comparison purposes, results were compared to those obtained by an analytical expression of the error in the shift estimation, which only considered the two most important samples of the HRR signal, discarding the remaining samples. This analytical approximation was only valid for SNR values higher than 15 dB.

From the results, the following conclusions can be drawn:

- The effects of the noisy samples on alignment performance seem to be only significant for very low SNR values.
- The use of the two most relevant samples seems to be useful to determine the sensitivity of the method for medium and high SNR values.
- The highest sensitivity to noise of the Maximum Position method occurs when the amplitudes of the two highest samples of the HRR signal are similar.

On the other hand, the paper also includes a set of experiments designed in order to study the effects of the alignment in the performance of a classifier. For this purpose, the variations in the classification error rate of a NN classifier were evaluated in three different alignment scenarios. Results demonstrate the importance of the alignment stage in the design of HRR-based ATR systems. These experiments allow us to better understand the relationship of the alignment performance and the classification error rate, and better alignment methods should be designed that improve the global performance of the ATR systems in terms of both sensitivity to noise and error rate.

REFERENCES

[1] C.R. Smith and P.M. Goggans, "Radar Target Identification", *IEEE Antennas and Prop. Magazine*, vol. 35, no. 2, pp. 27-37, April 1993.

[2] J. P. Zwart, R. van der Heiden, S. Gelsema, F. Groen, "Fast translation invariant classification of HRR range profiles in a zero phase representation", *IEE Proceedings Radar, Sonar and Navigation*, vol. 150, no. 6, pp. 411-418, December 2003.

[3] K.J. Coakley and P. Hale, "Alignment of noisy signals", *IEEE Trans. on Instrumentation and Measurement*, vol. 50, pp. 141-149, 2001.

[4] T. Padjla and V. Alavac, "Zero phase representation of panoramic images for image based location", Proceedings of the 8th International Conference on Computer Analysis of Images and Patterns, pp. 550-557, 1999.

[5] R. Gil Pita, M. Rosa Zurera, R. Vicen Bueno, F. López-Ferreras and M. Utrilla Manso, "Study of the performance of different alignment methods in pattern recognition", European Signal Proccesing Conference EUSIPCO 2006, Florence, Italy.

[6] R. Gil Pita, M. Rosa Zurera, R. Vicen Bueno, and F. López-Ferreras, "Maximum position alignment method for noisy high resolution radar target classification",

IEEE Signal Processing Letters, vol. 15, pp. 120-123, 2008.

[7] R. Gil Pita, M. Rosa Zurera, R. Vicen Bueno and M. Utrilla Manso, "Improving alignment of Noisy Signals Using an Iterative Zero Phase Method", European Signal Proccesing Conference EUSIPCO 2005, Antalya, Turkey.

[8] A.A. Kisienski, Y.-T. Lin and L.J. White, "Low-frequency approach to target identification", *Proceedings of the IEEE*, vol. 63, pp. 1651-1659, 1975.

[9] C.M. Bishop, *Neural networks for pattern recognition*, New York: Oxford University Press Inc., 1995.

[10] S. P. Jacobs and J. A. O'Sullivan "High Resolution Radar Models for Joint Tracking and Recognition", Proceedings of the IEEE National Radar Conference, 1997.

[11] J.G. Proakis, *Digital communications*, New York: McGraw-Hill, 1995.

[12] D. E. Nelson, J. A. Starzyk and D. D. Ensley, "Iterated wavelet transformation and signal discrimination for HRR radar target recognition", IEEE Trans. on Sys., Man and Cyb. part A, vol. 33, no. 1, pp. 52-57, 2003.