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DOA ESTIMATION FOR POLARIZED SOURCES ON A VECTOR-SENSOR ARRAY BY PARAFAC DECOMPOSITION OF THE FOURTH-ORDER COVARIANCE TENSOR

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ABSTRACT

In this paper we propose a novel stochastic algorithm for direction of arrival (DOA) estimation of polarized electromagnetic sources impinging on a six-component vector-sensor array, based on the PARAFAC decomposition of the fourth-order covariance tensor of the polarized data. The Cramér-Rao bound is derived and the performance of the proposed method are compared to a prior trilinear deterministic version of the algorithm [1] and to MUSIC for polarized sources [2].

1. INTRODUCTION

Multilinear array analysis exhibits uniqueness properties under mild conditions, transforming it into a powerful tool in signal processing area. Several multilinear algorithms were proposed lately, mainly in telecommunication domain, using different diversity schemes such as code diversity [3], multi-array diversity [4] or time-block diversity [5]. A 3D multilinear model for array processing, using polarization as a third diversity, was first introduced in [6], and a PARAFAC-based algorithm for this model, was later proposed in [7]. In [1] we derived the identifiability conditions for this trilinear polarized model, and we showed that in terms of source separation, the performance of the proposed algorithm is similar to the classical non-blind techniques.

Nevertheless, the joint estimation of all the three parameters of the sources (DOA, polarization, and temporal sequence) is time-consuming, and it does not always have a practical interest, especially in array-processing applications. A novel stochastic algorithm for DOA estimation of polarized sources is introduced in this paper, allowing the estimation of only two source parameters (DOA and polarization), and thus presenting a smaller computational complexity than its trilinear version [1]. It is based on the PARAFAC decomposition of the fourth-order covariance tensor of the polarized data, using a quadrilinear alternating least squares (QALS) approach. Another significant advantage of the proposed algorithm resides in the fact that the methods based on statistical properties of the signals proved to outperform deterministic techniques [8], provided that the number of samples is sufficiently high (>600 or so).

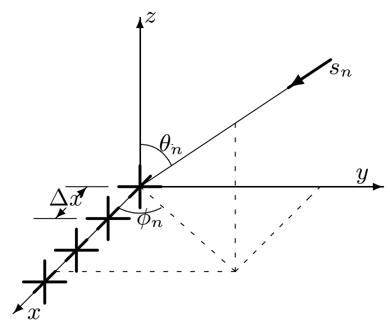

Figure 1: 2D-DOA on a vector-sensor array

The performance of the proposed algorithm is compared in simulations to the trilinear deterministic method introduced in [1] and to the CRB derived for this multilinear model. We show that the proposed algorithm, for low signal to noise ratio, provides better results than the deterministic one, while being more than four times faster. The simulation results show that it outperformes polarized MUSIC algorithm [2] for sources having close DOAs.

2. MODEL

We introduce in this section a quadrilinear model for electromagnetic sources covariance, recorded on a six-component vector-sensor array.

Consider a uniform array built up with M identical sensors spaced by Δx along the x-axis, collecting narrow-band signals emitted from N (N *a priori* known) spatially distinct far-field sources. For the nth incoming wave, the direction of arrival is determined by the elevation angle $\theta_n \in [0, \pi/2]$ (measured from $+z$-axis) and the azimuth angle $\phi_n \in [0, \pi)$ (measured from $+x$-axis)(Fig. 1).

Under the far-field assumption, the steering vector of the sensor array concerning the nth impinging wave can be modeled in a Vandermonde structure as

$$\boldsymbol{a}_n \triangleq \left[1,\ a_n,\ \cdots,\ a_n^{M-1}\right]^T, \tag{1}$$

where $a_n = \exp(jk_0 \Delta x \sin\theta_n \cos\phi_n)$ is the inter-sensor phase shift and k_0 is the wave number of the electromagnetic wave.

Suppose the signals are completely polarized, and the propagation takes place in an isotropic, homogeneous medium. A 2×1 complex vector

$$\boldsymbol{g}_n = \begin{bmatrix} \cos\alpha_n & -\sin\alpha_n \\ \sin\alpha_n & \cos\alpha_n \end{bmatrix} \begin{bmatrix} j\sin\beta_n \\ \cos\beta_n \end{bmatrix}$$

is used to describe the polarization state of the nth signal in terms of the orientation angle $\alpha_n \in (-\pi/2, \pi/2]$ and ellipticity angle $\beta_n \in [-\pi/4, \pi/4]$, see [9]. If the nth incoming wave has unit power, the electric- and magnetic-field, $\boldsymbol{e}_n$ and $\boldsymbol{h}_n$, measured by each vector-sensor can be formulated by a 6×1 vector $\boldsymbol{b}_n$ [10] in Cartesian coordinates, which equals:

$$\boldsymbol{b}_n \triangleq \begin{bmatrix} \boldsymbol{e}_n \\ \boldsymbol{h}_n \end{bmatrix} = \begin{bmatrix} \cos\theta_n\cos\phi_n & -\sin\phi_n \\ \cos\theta_n\sin\phi_n & \cos\phi_n \\ -\sin\theta_n & 0 \\ -\sin\phi_n & -\cos\theta_n\cos\phi_n \\ \cos\phi_n & -\cos\theta_n\sin\phi_n \\ 0 & \sin\theta_n \end{bmatrix} \boldsymbol{g}_n. \tag{2}$$

Let p ($p = 1, 2, \cdots, 6$) index the six field components of the vector $\boldsymbol{b}_n$ respectively.

Define :

$$\mathbf{A} \triangleq [\boldsymbol{a}_1, \cdots, \boldsymbol{a}_N] \tag{3}$$

a $M \times N$ matrix containing the spatial responses of the array to the N sources,

$$\mathbf{B} \triangleq [\boldsymbol{b}_1, \cdots, \boldsymbol{b}_N], \tag{4}$$

a $6 \times N$ matrix containing the polarization responses and

$$\mathbf{S}(k) \triangleq \begin{bmatrix} s_1(k) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & s_N(k) \end{bmatrix} \tag{5}$$

a $N \times N$ diagonal containing the kth temporal samples of the N sources.

With these notations, the kth temporal samples collected at the output of the array can be organized as a $M \times 6$ matrix:

$$\mathbf{X}(k) = \mathbf{A}\mathbf{S}(k)\mathbf{B}^T + \mathbf{N}(k) \tag{6}$$

with $\mathbf{N}$ a $M \times 6$ matrix expressing the noise contribution on the antenna.

The following assumptions are made:

(A1) Sources are zero-mean, mutually uncorrelated;

(A2) The noise is i.i.d. centered, complex Gaussian process of variance σ^2, non-polarized and spatially white;

(A3) The sources have distinct DOAs.

Supposing that K temporal samples were recorded, we define a covariance estimate of the sources as the hyper-diagonal $N \times N \times N \times N$ tensor:

$$\hat{\mathcal{C}}_{SS} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{S}(k) \circ \mathbf{S}^*(k) \tag{7}$$

where $\circ$ is the tensor *outer product*[1] and * stands for the conjugate of a matrix.

We also compute the covariance of the received data as the $M \times 6 \times M \times 6$ tensor:

$$\hat{\mathcal{C}}_{XX} = \frac{1}{K}\sum_{k=1}^{K} \mathbf{X}(k) \circ \mathbf{X}^*(k) \tag{8}$$

From (6), (7), (8) and assumptions (A1) and (A2) the covariance tensor of the received data takes the following form:

$$\hat{\mathcal{C}}_{XX} = \hat{\mathcal{C}}_{SS} \bullet_1 \mathbf{A} \bullet_2 \mathbf{B} \bullet_3 \mathbf{A}^* \bullet_4 \mathbf{B}^* + \mathcal{N} \tag{9}$$

where $\mathcal{N}$ is a $M \times 6 \times M \times 6$ tensor containing the noise power on the sensors. The operator $\bullet_n$ stands for the *n-mode product*[2] of a tensor by a matrix.

3. MODEL IDENTIFIABILITY AND PARAMETER ESTIMATION

As the tensor $\hat{\mathcal{C}}_{SS}$ is hyper-diagonal, (9) yields the PARAFAC decompositions of $\hat{\mathcal{C}}_{XX}$. We state next the conditions under which this multilinear decomposition is unique (up to a scaling factor and a permutation indeterminacy), meaning that the model is identifiable.

3.1 Identifiability

In order to derive the identifiability conditions, we suppose the absence of noise in the recorded signals, meaning that $\mathcal{N}$ in (9) has only zero entries. The uniqueness of the PARAFAC decomposition of $\hat{\mathcal{C}}_{XX}$ is ensured (up to permutation and scaling of columns) if (*Kruskal's Condition*) [11]:

$$k_{\mathbf{A}} + k_{\mathbf{B}} + k_{\mathbf{A}^*} + k_{\mathbf{B}^*} \geq 2N + 3 \tag{10}$$

which equals:

$$k_{\mathbf{A}} + k_{\mathbf{B}} \geq N + 1.5 \tag{11}$$

In equations (10) and (11), $k_{\mathbf{A}}$ and $k_{\mathbf{B}}$ represent the *Kruskal-rank*[3] of matrices $\mathbf{A}$ and $\mathbf{B}$.

Eq.(1) along with the assumption (A3) guarantee that matrix $\mathbf{A}$ is full column rank, i.e. $k_{\mathbf{A}} = N$. This means that the PARAFAC decomposition is unique if the following constraint is fulfilled:

$$k_{\mathbf{B}} \geq 1.5 \tag{12}$$

We have proved in [1] that if (A3) is verified then $k_{\mathbf{B}} \geq 2$, which ensures identifiability of (9). This means that as long as the N sources have distinct DOAs, the PARAFAC decomposition of $\hat{\mathcal{C}}_{XX}$ is unique, polarization plays an auxiliary role (see [1]).

[1] The *outer product* of two matrices $\mathbf{A}(I_1 \times I_2)$ and $\mathbf{B}(J_1 \times J_2)$ is a fourth-order tensor $\mathcal{C}(I_1 \times I_2 \times J_1 \times J_2)$ defined by $c_{i_1 i_2 j_1 j_2} \triangleq a_{i_1 i_2} b_{j_1 j_2}$.

[2] The *n-mode product* of a $(I_1 \times \ldots \times I_N)$ tensor $\mathcal{C}$ and a $(J_n \times I_n)$ matrix $\mathbf{A}$, is a $(I_1 \times \ldots \times J_n \times \ldots \times I_N)$ tensor given by: $(\mathcal{C} \bullet_n \mathbf{A})_{i_1 \ldots j_n \ldots i_N} \triangleq \sum_{i_n} c_{i_1 \ldots i_n \ldots i_N}\, a_{j_n i_n}$

[3] Given a matrix $\mathbf{A} \in \mathbb{C}^{I \times J}$, if every linear combination of l columns has full column rank, but this condition does not hold for $l+1$, then the *Kruskal-rank* of $\mathbf{A}$ is l, written as $k_{\mathbf{A}} = l$.

3.2 Parameter estimation

We show next that the parameter matrices $\mathbf{A}$ and $\mathbf{B}$ can be estimated via the *Quadrilinear Alternative Least Squares (QALS)* algorithm [12].

Denote by $\hat{\mathbf{C}}_{pq} = \hat{\mathcal{C}}_{XX}(:,p,:,q)$ the (p,q)th matrix slice $(M \times M)$ of the covariance tensor $\hat{\mathcal{C}}_{XX}$. Also note $\mathrm{D}_p(\cdot)$ the operator that builds a diagonal matrix from the pth row of another and $\Delta = \mathrm{diag}\left(E\|s_1\|^2, \ldots, E\|s_N\|^2\right)$, the diagonal matrix containing the powers of the sources. The matrices $\mathbf{A}$ and $\mathbf{B}$ can then be identified by minimizing the Least Squares (LS) criterion:

$$\phi(\sigma, \Delta, \mathbf{A}, \mathbf{B}) = \sum_{p,q=1}^{6} \left\| \hat{\mathbf{C}}_{pq} - \mathbf{A}\Delta \mathrm{D}_p(\mathbf{B})\mathrm{D}_q(\mathbf{B}^*)\mathbf{A}^H - \sigma^2 \delta_{pq}\mathbf{I}_M \right\|_F^2 \quad (13)$$

that equals:

$$\phi(\sigma, \Delta, \mathbf{A}, \mathbf{B}) = \sum_{p,q} \left\| \hat{\mathbf{C}}_{pq} - \mathbf{A}\Delta \mathrm{D}_p(\mathbf{B})\mathrm{D}_q(\mathbf{B}^*)\mathbf{A}^H \right\|_F^2 - 2\sigma^2 \sum_p \Re\left\{ \mathrm{tr}\left(\hat{\mathbf{C}}_{pp} - \mathbf{A}\Delta \mathrm{D}_p(\mathbf{B})\mathrm{D}_p(\mathbf{B}^*)\mathbf{A}^H \right) \right\} + 6M\sigma^4 \quad (14)$$

where $\mathrm{tr}(\cdot)$ computes the trace of a matrix and $\Re(\cdot)$ denotes the real part of a quantity.

With the columns of $\mathbf{A}$ and $\mathbf{B}$ ($\boldsymbol{a}_n$ and $\boldsymbol{b}_n$) given by (1) and (2), the criterion (14) becomes :

$$\phi(\sigma, \Delta, \mathbf{A}, \mathbf{B}) = \sum_{p,q} \left\| \hat{\mathbf{C}}_{pq} - \mathbf{A}\Delta \mathrm{D}_p(\mathbf{B})\mathrm{D}_q(\mathbf{B}^*)\mathbf{A}^H \right\|_F^2 \quad (15a)$$

$$- 2\sigma^2 \sum_p \Re\left\{ \mathrm{tr}\left(\hat{\mathbf{C}}_{pp} - 2M\Delta \right) \right\} \quad (15b)$$

$$+ 6M\sigma^4 \quad (15c)$$

Thus, finding $\mathbf{A}$ and $\mathbf{B}$ is equivalent to the minimization of (15a) with respect to $\mathbf{A}$ and $\mathbf{B}$, i.e.:

$$\{\hat{\mathbf{A}}, \hat{\mathbf{B}}\} = \min_{\mathbf{A},\mathbf{B}} \omega(\Delta, \mathbf{A}, \mathbf{B}) \quad (16)$$

subject to $\|\boldsymbol{a}_n\|^2 = M$ and $\|\boldsymbol{b}_n\|^2 = 2$, where:

$$\omega(\Delta, \mathbf{A}, \mathbf{B}) = \sum_{p,q} \left\| \hat{\mathbf{C}}_{pq} - \mathbf{A}\Delta \mathrm{D}_p(\mathbf{B})\mathrm{D}_q(\mathbf{B}^*)\mathbf{A}^H \right\|_F^2 \quad (17)$$

The optimization process in (16) can be implemented using the quadrilinear alternative least squares (QALS) algorithm.

Once the $\hat{\mathbf{A}}$, $\hat{\mathbf{B}}$ are estimated, the DOAs of the N sources are found via the minimization of the following LS criterion :

$$\{\theta_n, \phi_n\} = \min_{\theta,\phi} \left(\|\boldsymbol{a}(\theta,\phi) - \hat{\boldsymbol{a}}_n\| + \|\boldsymbol{p}(\theta,\phi) - \hat{\boldsymbol{p}}_n\| \right) \quad (18)$$

with $n = 1 \ldots N$. In (18) $\hat{\boldsymbol{a}}_n$ is the nth column of $\hat{\mathbf{A}}$. $\hat{\boldsymbol{p}}_n$ is the Poynting vector for the nth source [13] obtained from the nth column of $\hat{\mathbf{B}}$ (see eq. (2)) as:

$$\hat{\boldsymbol{p}}_n = \hat{\boldsymbol{e}}_n \times \hat{\boldsymbol{h}}_n^* \quad (19)$$

The steering vector $\boldsymbol{a}(\theta,\phi)$ for a source of DOA $\{\theta,\phi\}$ is given by eq.(1) and $\boldsymbol{p}(\theta,\phi)$ is defined as:

$$\boldsymbol{p} \triangleq \boldsymbol{e} \times \boldsymbol{h}^* = \begin{bmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{bmatrix} \quad (20)$$

Thus, a set of two DOA parameters is obtained for each of the N sources. The polarization parameters can be obtained in a similar way from $\hat{\mathbf{B}}$, but for space reasons we will not address the polarization estimation problem in the present paper.

4. PERFORMANCE ISSUES

4.1 Cramér-Rao bound

In this section the Cramér-Rao bound on the covariance tensor of the data is derived, based on the results of Stoica and Nehorai presented in [14]. Define $\mathbf{x}(k)$ as the vector obtained by column-wise unfolding of $\mathbf{X}(k)$, $k = 1, \ldots, K$. Also, define $\mathbf{S} \triangleq [\mathbf{s}_1 \cdots \mathbf{s}_N]$ as the $(K \times N)$ matrix containing column-wise the K temporal samples of the N sources. Under the assumption (A2), the observation $\mathbf{x}(k)$ satisfies

$$\mathbf{x}(k) \sim \mathcal{N}\left(\boldsymbol{\mu}(k), \sigma^2\mathbf{I}_{6M}\right), \; k = 1, \ldots, K$$

where $\boldsymbol{\mu}(k) = (\mathbf{B} \odot \mathbf{A})\mathbf{S}(k,:)^T$. If we define the parameter vector $\boldsymbol{\theta}$ by rearranging all the unknown parameters in a long vector :

$$\boldsymbol{\theta} = [\theta_1, \ldots, \theta_N, \phi_1, \ldots, \phi_N, \alpha_1, \ldots, \alpha_N, \beta_1, \ldots, \beta_N]^T,$$

the likelihood function of $\mathbf{X} \triangleq [\mathbf{x}(1) \cdots \mathbf{x}(K)]$ can be written as:

$$L(\mathbf{X} \mid \boldsymbol{\theta}) = \frac{1}{(\pi\sigma^2)^{6MK}} \exp\left\{ -\frac{1}{\sigma^2} \left\| \mathbf{X} - (\mathbf{B} \odot \mathbf{A})\mathbf{S}^T \right\|_F^2 \right\}. \quad (21)$$

Then the log-likelihood function of the data equals

$$f(\boldsymbol{\theta}) = -6MK\ln(\pi\sigma^2) - \frac{1}{\sigma^2} \sum_k \left\| \mathbf{x}(k) - (\mathbf{B} \odot \mathbf{A})\mathbf{S}(k,:)^T \right\|_F^2 \quad (22)$$

If we note $\mathbf{D} \triangleq \mathbf{B} \odot \mathbf{A}$, by extension of the work in [14] to the vector-sensor case, the CRB for $\boldsymbol{\theta}$ is obtained as :

$$\mathrm{CRB}(\boldsymbol{\theta}) = \frac{\sigma^2}{2} \left\{ \sum_{k=1}^{K} \Re\left\{ \mathbf{G}_k^H \mathbf{W}^H \left[\mathbf{I} - \mathbf{D}(\mathbf{D}^H\mathbf{D})^{-1}\mathbf{D}^H \right] \mathbf{W}\mathbf{G}_k \right\} \right\}^{-1} \quad (23)$$

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where

$$\mathbf{W} = \left[\frac{\partial \mathbf{d}_1}{\partial \theta_1} \cdots \frac{\partial \mathbf{d}_N}{\partial \theta_N}\ \frac{\partial \mathbf{d}_1}{\partial \phi_1} \cdots \frac{\partial \mathbf{d}_N}{\partial \phi_N}\ \frac{\partial \mathbf{d}_1}{\partial \alpha_1} \cdots \frac{\partial \mathbf{d}_N}{\partial \alpha_N}\ \frac{\partial \mathbf{d}_1}{\partial \beta_1} \cdots \frac{\partial \mathbf{d}_N}{\partial \beta_N}\right]$$

with $\mathbf{d}_n$ denoting the nth column of $\mathbf{D}$, and $\mathbf{G}_k = \mathbf{I}_4 \otimes \text{diag}(\mathbf{S}(k,:))$ (the operator $\otimes$ denotes the Kronecker product). In section 5, the CRB is illustrated for the DOA parameters θ and ϕ.

4.2 Complexity of the algorithm

In this subsection we compare the complexity of the proposed algorithm, based on a fourth-order tensor decomposition, with the complexity of the 3-way PARAFAC algorithm introduced in [1].

Generally, for an N-way array of size $I_1 \times I_2 \times \cdots \times I_N$, the complexity of its PARAFAC decomposition in a sum of F rank-1 tensors, using ALS algorithm is $\mathcal{O}(F \prod_{n=1}^{N} I_n)$ [15], for each iteration. Hence, as normally $K \gg 6M$, the number of computations involved in the $M \times 6 \times M \times 6$ covariance array decomposition is of order $\mathcal{O}(6^2 FM^2)$, quite smaller compared to $\mathcal{O}(6FMK)$ involved in the direct decomposition of the $M \times 6 \times K$ array as it is the case in [1]. The number of iterations required before the decomposition reaches its convergence, is not determined only by the data size, which makes an exact theoretical analysis of the algorithms complexity rather difficult. Table 1 below lists respectively the average running time for 500 Monte Carlo trials ($M = 7$ sensors, $K = 1000$ temporal samples, SNR = 10dB) for QALS and the COMFAC[4] algorithm [3], used for the PARAFAC decomposition of the three-way array in [1].

	COMFAC	QALS
Average running time (sec)	1.3749	0.3080

Table 1: Comparison of computational times

One can see that, in our case, QALS is approximately four times faster than COMFAC. The explanation is that for the fourth-order covariance tensor, only two matrices ($\mathbf{A}$ and $\mathbf{B}$) are in fact estimated while the three-way algorithm estimates the three matrices ($\mathbf{A}$, $\mathbf{B}$ and $\mathbf{S}$). Thus, the convergence is much faster in the first case.

5. SIMULATIONS AND RESULTS

In this section, we compare in numerical simulations the performance of the proposed algorithm, with the three-way deterministic COMFAC-based approach [1] and the polarized MUSIC algorithm [2]. The simulation results are compared to the CRB derived in subsection 4.1.

First, a performance analysis is performed with respect to the SNR. A number of $M = 7$ identical sensors are used to build the uniform array, on which two adjacent sensors are set half a wavelength apart. A mixture of $N = 2$ uncorrelated sources is recorded at the receiver; both sources are realizations of zero-mean Gaussian processes of equal variance. The observations are taken from $K = 1000$ independent snapshots and the source DOAs and polarization parameters as listed in Table 2. The DOA parameters for the two sources were

[4] COMFAC is a fast implementation of trilinear ALS working with a compressed version of the data.

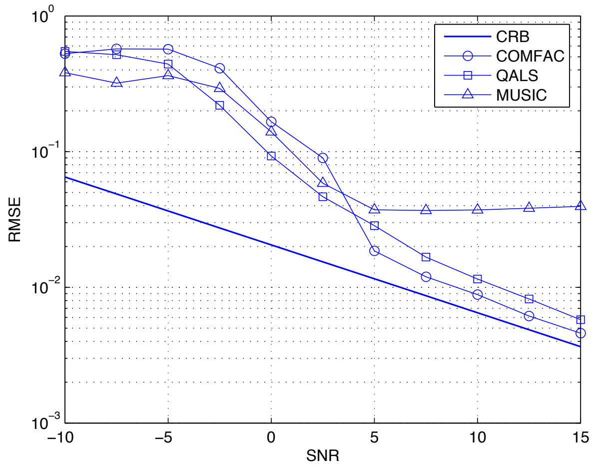

Figure 2: RMSE of Elevation Angle θ Estimation vs. SNR

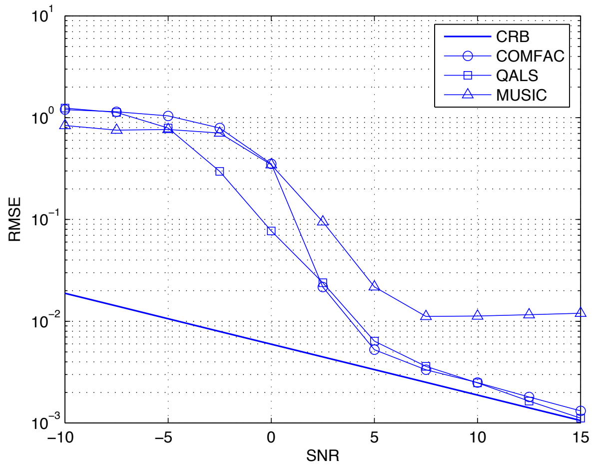

Figure 3: RMSE of Azimuth Angle ϕ Estimation vs. SNR

set very close, to test the algorithms performance in adverse situations.

The algorithm performances are compared in terms of the root mean square error (RMSE) of the estimates, as given by: $\text{RMSE}(\hat{\xi}) = \sqrt{\frac{1}{LN}\sum_{l=1}^{L}\sum_{n=1}^{N}\left\|\hat{\xi}_n^l - \xi_n\right\|^2}$, $\xi = \theta$ or ϕ, where $\hat{\xi}_n^l$ is the estimate of ξ obtained for the nth source at the lth trial. $L = 500$ independent trials contribute to each data point on the figures.

Figures 2 and 3 illustrate the average RMS error of the elevation angle and the azimuth angle estimation, respectively, for different SNRs. The QALS algorithm presents better performance than COMFAC for low SNR (SNR $\leq$ 0dB), and a similar behavior for high SNR. Both multilinear approaches present comparable performance to the polarized MUSIC estimator at low SNR, but largely outperform the latter as SNR > 5dB.

Next, we compare the performance of the algorithms

Angular parameters	Elevation (θ)	Azimuth (ϕ)	Auxiliary (α)	Ellipticity (β)
Source 1	$87.73°$	$88.62°$	$-14.36°$	$-44.40°$
Source 2	$83.54°$	$89.89°$	$-62.80°$	$-39.62°$

Table 2: DOA and Polarization Parameters of the Sources

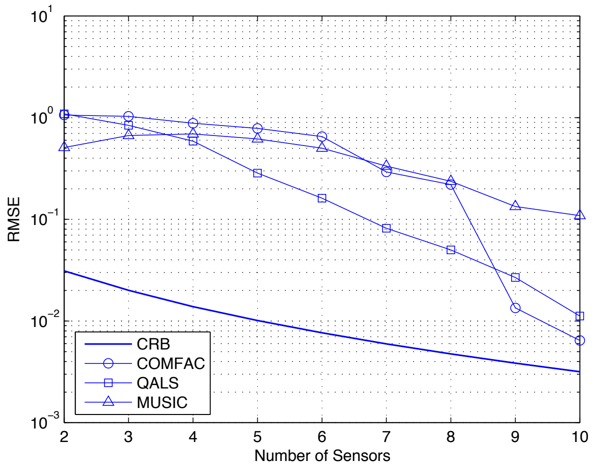

Figure 4: RMSE of Azimuth Angle ϕ Estimation vs. the Number of Sensors

as the number of sensors increases. Figure 4 illustrates the RMS error of the azimuth angle estimation for the proposed approach, comparing to that of MUSIC and COMFAC, as the number of sensors M gradually grows. The results are similar for the elevation angle θ. The SNR = 0dB, $K = 1000$ and the same source parameters from Table 2 are used. Consistent with the results given on Fig. 3, the proposed method yields more accurate estimations comparing to the other algorithms. This advantage compared to COMFAC fades for $M > 8$, but both multilinear approaches outperform MUSIC for the given source configuration.

6. CONCLUSIONS

In this paper we introduced a stochastic algorithm for DOA estimation of the polarized sources, based on a QALS decomposition of the covariance tensor of the data. The algorithm presents an inferior complexity compared to its deterministic trilinear version, and it proved to be four time faster, in simulations. Also, it showed better performance for low SNR (< 5dB), and outperformed polarized MUSIC for close DOA estimation.

REFERENCES

[1] X. Guo, S. Miron, and D. Brie, "Identifiability of the PARAFAC model for polarized source mixture on a vector sensor array," in *Proc. IEEE ICASSP 2008*, Las Vegas, USA, Mar. 30- Apr. 4, 2008.

[2] M.D. Zoltowski and K.T. Wong, "Closed-form eigenstructure-based direction finding using arbitrary but identical subarrays on a sparse uniform cartesian array grid," *IEEE Trans. Signal Processing*, vol. 48, no. 8, pp. 2205–2210, Aug. 2000.

[3] N.D. Sidiropoulos, G.B. Giannakis, and R. Bro, "Blind PARAFAC receivers for DS-CDMA systems," *IEEE Trans. Signal Processing*, vol. 48, no. 3, pp. 810–823, Mar. 2000.

[4] N.D. Sidiropoulos, R. Bro, and G.B. Giannakis, "Parallel factor analysis in sensor array processing," *IEEE Trans. Signal Processing*, vol. 48, no. 8, pp. 2377–2388, Aug. 2000.

[5] Y. Rong, S.A. Vorobyov, A.B. Gershman, and N.D. Sidiropoulos, "Blind spatial signature estimation via time-varying user power loading and parallel factor analysis," *IEEE Trans. Signal Processing*, vol. 53, no. 5, pp. 1697–1710, May 2005.

[6] S. Miron, N. Le Bihan, and J. I. Mars, "Vector-sensor MUSIC for polarized seismic sources localisation," *EURASIP Journal on Applied Signal Processing*, vol. 2005, no. 1, pp. 74–84, 2005.

[7] X. Zhang and D. Xu, "Blind PARAFAC signal detection for polarization sensitive array," *EURASIP Journal on Advances in Signal Processing*, vol. 2007, pp. Article ID 12025, 7 pages, 2007.

[8] A. Swindlehurst, M. Goris, and B. Ottersten, "Some experiments with array data collected in actual urban and suburban environments," in *IEEE Workshop on Signal Proc. Adv. in Wireless Comm.*, Paris, France, 1997, pp. 301–304.

[9] R.T. Compton, "The tripole antenna: an adaptive array with full polarization flexibility," *IEEE Trans. Antennas Propagat.*, vol. AP-29, no. 6, pp. 944–952, Nov. 1981.

[10] A. Nehorai, K.-C. Ho, and B.T.G. Tan, "Minimum-noise-variance beamformer with an electromagnetic vector sensor," *IEEE Trans. Signal Processing*, vol. 47, no. 3, pp. 601–618, Mar. 1999.

[11] N. D. Sidiropoulos and R. Bro, "On the uniqueness of multilinear decomposition of N-way arrays," *Journal of chemometrics*, , no. 14, pp. 229–239, 2000.

[12] R. Bro, *Multi-way Analysis in the Food Industry - Models, Algorithms, and Applications*, Ph.D dissertation, Royal Veterinary and Agricultural University, Denmark, 1998.

[13] A. Nehorai and E. Paldi, "Vector-sensor array processing for electromagnetic source localisation," *IEEE Trans. Signal Processing*, vol. 42, no. 2, pp. 376–398, Feb. 1994.

[14] P. Stoica and A. Nehorai, "MUSIC, maximum likelihood, and Cramér-Rao bound," *IEEE Trans. Acoust., Speech and Signal Processing*, vol. 37, no. 5, pp. 720–741, 1989.

[15] M. Rajih and P. Comon, "Enhanced line search: A novel method to accelerate PARAFAC," in *Proc. EUSIPCO 2005*, Antalya, Turkey, Sept. 2005.