SPACE-TIME SPREADING-MULTIPLEXING FOR MIMO ANTENNA SYSTEMS WITH BLIND DETECTION USING THE PARATUCK-2 TENSOR DECOMPOSITION

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ABSTRACT

In this paper, we present a new space-time spreading-multiplexing model for Multiple-Input Multiple-Output (MIMO) wireless communication systems relying on a tensor modeling of the transmitted and received signals. At the transmitter, we exploit the core of a PARATUCK-2 tensor decomposition composed of a precoding matrix and two allocation matrices that allow to control the spreading-multiplexing of the data streams across the space dimension (transmit antennas) and time-dimension (time-slots). Consequently, different MIMO schemes combining space-time multiplexing and diversity can be derived from the proposed model. The algebraic structure of the PARATUCK-2 tensor model is then exploited at the receiver for a joint blind channel estimation and symbol detection. The bit-error-rate performance of different transmit schemes derived from the proposed model is evaluated by means of computer simulations.

1. INTRODUCTION

It is well known for some time that Multiple-Input Multiple-Output (MIMO) wireless communication systems employing multiple antennas at both the transmitter and receiver provide multiplexing gains [1] and/or diversity gains [2] to increase the data rate (i.e. higher spectral efficiencies) and/or the reliability of the transmission (i.e. lower error rates) without additional bandwidth. In order to provide multiple-accessing capabilities to MIMO systems, several approaches make use of Code-Division Multiple-Access (CDMA) technology by associating multiple transmit antennas and multiple user signals to orthogonal spreading transforms in different manners [3]– [4]. Optionally, when current channel state is known in advance at the transmitter, some form of precoding can also be used to improve system performance (see [5] and references therein).

The use of tensor decompositions for modeling multiple-antenna transmissions with blind receiver signal processing has been addressed in several recent works [6]– [7]. The approach of [6] relies on a PARAllel FACtor (PARAFAC) decomposition [8] of the third-order received signal tensor. Despite the variable diversity-multiplexing offered by the precoder structure, this multiple-antenna scheme relies on temporal-only spreading of each data stream. In other words, each symbol of a data stream is transmitted during multiple channel uses. The approach of [9] uses a different tensor model but is still limited to pure spatial multiplexing with temporal-only spreading. The model of [10] adds some transmission flexibility by allowing spatial spreading of the transmitted data streams in addition to temporal spreading (i.e. space-time spreading). This is achieved by means of a constrained tensor model which is exploited at the receiver for a joint blind detection and channel estimation.

More general space-time spreading structures were recently proposed relying on a third-order CONstrained FACtor (CONFAC) decomposition [7, 11]. The approach of [11] exploits two constraint matrices with variable 1's and 0's structure (therein referred to as stream and code "allocation matrices", respectively) to design transmit schemes with different spatial multiplexing/diversity and code multiplexing degrees for the data streams. [7] further generalizes [11] by including a third allocation matrix that defines the mapping of the precoded signals to the transmit antennas. In this case, the constrained structure of the CONFAC model is fully exploited at both the transmitter (to design sets of transmission schemes) and the receiver (for blind signal processing).

In this work, we present a novel tensor-based space-time spreading-multiplexing model. At the transmitter, we exploit the core of a PARATUCK-2 tensor decomposition to design different precoder structures combining space-time multiplexing and diversity. At the receiver, the multilinear algebraic structure of the resulting PARATUCK-2 model is used for a joint blind detection and channel estimation. Differently to the CONFAC-based model of [7, 11], two allocation matrices of the PARATUCK-2 core tensor jointly control the temporal coding/spreading of each data stream with respect to each transmit antenna. In this case, by varying the 1's and 0's of these matrices, the temporal allocation of data streams to transmit antennas is also varied, which provides the flexibility to accommodate/load data streams to time-slots in a different number of ways. Moreover, the number of channel uses associated with the transmission of each data stream may be different from one data stream to another, which is not possible with the existing tensor-based space-time transmission models.

The PARATUCK-2 decomposition can be viewed as a generalization of the PARAFAC one. It mixes the properties of both PARAFAC [8] and TUCKER-2 [12] decompositions. This decomposition has been studied in the psychometrics literature [13] and subsequently exploited in [14] to solve special data analysis problems in chemometrics. The first application of PARATUCK-2 in signal processing was proposed in [15] for the blind joint identification and equalization of Wiener-Hammerstein communication channels. The present paper shows that this decomposition is also useful to model transmitter and receiver signal processing in MIMO wireless systems.

The organization of the paper is as follows. Section 2 briefly presents the PARATUCK-2 decomposition of a third-order tensor. In Section 3, the proposed space-time multiplexing-spreading structure is presented and the associated tensor signal model is formulated. Section 4 discusses the identifiability issue and its link with the space-time multiplexing-spreading structure. In Section 5, we present a blind PARATUCK-2 based receiver for joint channel estimation and symbol detection. Some simulation results are presented in Section 6 and the paper is concluded in Section 7.

∗ Supported by a CAPES/COFECUB postdoctoral fellowship.

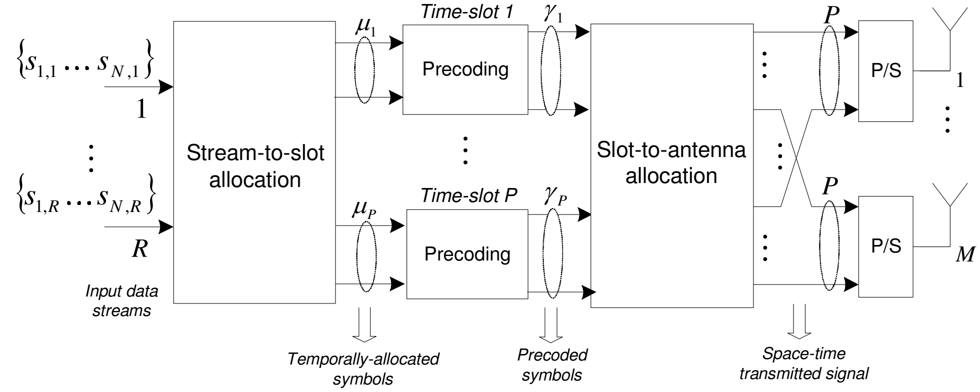

Figure 1: Proposed space-time spreading-multiplexing model as a cascade of 3 blocks: i) stream-to-slot allocation, ii) precoding, and iii) slot-to-antenna allocation.

2. PARATUCK-2 TENSOR DECOMPOSITION

The PARATUCK-2 decomposition of a third-order tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$ is given, in scalar form, by the following expression:

$$x_{i_1,i_2,i_3} = \sum_{r_1=1}^{R_1} \sum_{r_2=1}^{R_2} a_{i_1,r_1} b_{i_2,r_2} g_{r_1,r_2} c^A_{i_3,r_1} c^B_{i_3,r_2}, \quad (1)$$

where x_{i_1,i_2,i_3} is the (i_1,i_2,i_3)-th entry of tensor $\mathcal{X}$, $a_{i_1,r_1} = [\mathbf{A}]_{i_1,r_1}$, $b_{i_2,r_2} = [\mathbf{B}]_{i_2,r_2}$, $c^A_{i_3,r_1} = [\mathbf{C}^A]_{i_3,r_1}$, $c^B_{i_3,r_2} = [\mathbf{C}^B]_{i_3,r_2}$, and $g_{r_1,r_2} = [\mathbf{G}]_{r_1,r_2}$ are the entries of matrices $\mathbf{A} \in \mathbb{C}^{I_1 \times R_1}$, $\mathbf{B} \in \mathbb{C}^{I_2 \times R_2}$, $\mathbf{C}^A \in \mathbb{C}^{I_3 \times R_1}$, $\mathbf{C}^B \in \mathbb{C}^{I_3 \times R_2}$ and $\mathbf{G} \in \mathbb{C}^{R_1 \times R_2}$, respectively. The matrices $\mathbf{A}$ and $\mathbf{B}$ are the *factor matrices* of the decomposition. They are associated with the first and second dimensions of the tensor $\mathcal{X} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$. The matrices $\mathbf{C}^A$ and $\mathbf{C}^B$ are called *interaction matrices*. They define the linear combination profile between the R_1 columns of $\mathbf{A}$ and the R_2 columns of $\mathbf{B}$ along the third dimension of the tensor $\mathcal{X}$. The matrix $\mathbf{G}$ is the *core matrix* of the PARATUCK-2 decomposition. The element g_{r_1,r_2} of $\mathbf{G}$ defines the magnitude of the interaction between the r_1-th column of $\mathbf{A}$ and the r_2-th column of $\mathbf{B}$.

Let us define the matrix-slices $\mathbf{X}_{\cdot\cdot i_3} \in \mathbb{C}^{I_1 \times I_2}$, $i_3 = 1, \cdots, I_3$, obtained by "slicing" the tensor along its third dimension:

$$[\mathbf{X}_{\cdot\cdot i_3}]_{i_1,i_2} = x_{i_1,i_2,i_3}.$$

This matrix-slice can be written as:

$$\mathbf{X}_{\cdot\cdot i_3} = \mathbf{A} D_{i_3}(\mathbf{C}^A)\, \mathbf{G}\, D_{i_3}(\mathbf{C}^B) \mathbf{B}^T, \quad i_3 = 1, \cdots, I_3. \quad (2)$$

where $D_{i_3}(\mathbf{C}^A) \in \mathbb{C}^{R_1 \times R_1}$ and $D_{i_3}(\mathbf{C}^B) \in \mathbb{C}^{R_2 \times R_2}$ represent diagonal matrices that hold, respectively, the i_3-th row of $\mathbf{C}^A$ and $\mathbf{C}^B$ on the main diagonal.

Constrained PARATUCK-2: We are interested in a special PARATUCK-2 decomposition, where $\mathbf{C}^A$ and $\mathbf{C}^B$ are constrained to have only 1's and 0's entries. For instance, $c^A_{i_3,r_1} = c^B_{i_3,r_2} = 1$ means that the r_1-th column of $\mathbf{A}$ interacts with the r_2-th column of $\mathbf{B}$ in the generation of the i_3-th matrix-slice $\mathbf{X}_{\cdot\cdot i_3}$, the magnitude of this interaction being determined by the entry g_{r_1,r_2} of the core matrix $\mathbf{G}$. Otherwise if $c^A_{i_3,r_1} = c^B_{i_3,r_2} = 0$, it means that there is no interaction between the corresponding columns of $\mathbf{A}$ and $\mathbf{B}$. This work exploits this concept to design different space-time spreading-multiplexing schemes for MIMO antenna systems.

3. PROPOSED SPACE-TIME SPREADING-MULTIPLEXING MODEL

Let us consider a MIMO wireless communication system with M transmit antennas and K receive antennas. At the transmitter, the serial input stream is parsed into R data streams composed of N symbols each. The proposed space-time spreading-multiplexing model consists in jointly multiplexing/allocating the R data streams across *space* and *time* dimensions, i.e. across M transmit antennas and P time-slots. Each time-slot corresponds to one channel use (N symbol periods) for transmitting R data streams. Figure 1 illustrates the proposed space-time spreading-multiplexing model. The *stream-to-slot allocation* block determines the mapping of the R data streams across the P time-slots. Likewise, the *slot-to-antenna allocation* block determines the mapping of the P time-slots to the M transmit antennas. We call attention to the fact that the same data stream and antenna can be allocated to (i.e. repeated over) more than one time-slot.

Define $\mu_p \in [1, R]$ and $\gamma_p \in [1, M]$ as the number of data streams and transmit antennas allocated to the p-th time-slot, respectively, $p = 1, \ldots, P$. The spatial precoder combines the μ_p data streams to generate γ_p precoded streams which are then transmitted by a subset of γ_p transmit antennas at the p-th time-slot. After precoding over P time-slots, the resulting data streams are properly organized at each transmit antenna and then parallel-to-serial converted before being transmitted. The wireless channel is characterized by rich-scattering Rayleigh flat-fading propagation and is assumed constant during N symbol periods. The data streams are transmitted with equal powers and the total transmitted power is normalized at any channel use and is independent on the number of data streams and transmit antennas.

3.1 Allocation structure

Let us define a *stream-to-slot allocation matrix* $\mathbf{\Psi} \in \mathbb{C}^{P \times R}$ and a *slot-to-antenna allocation matrix* $\mathbf{\Phi} \in \mathbb{C}^{P \times M}$, which are composed uniquely of 1's and 0's. These matrices are known to both the transmitter and receiver, and are the core of the space-time precoder. Let $\psi_{p,r}$ and $\phi_{p,m}$ be the entries of $\mathbf{\Psi}$ and $\mathbf{\Phi}$, respectively. We have:

$$\mu_p = \sum_{r=1}^{R} \psi_{p,r} = \mathbf{\Psi}_{p\cdot}(\mathbf{\Psi}_{p\cdot})^T,$$
$$\gamma_p = \sum_{m=1}^{M} \phi_{p,m} = \mathbf{\Phi}_{p\cdot}(\mathbf{\Phi}_{p\cdot})^T. \quad (3)$$

The p-th row $\mathbf{\Psi}_{p\cdot} \in \mathbb{C}^{1 \times R}$ of $\mathbf{\Psi}$ determines which μ_p data streams are allocated to the p-th slot. Likewise, the p-th row $\mathbf{\Phi}_{p\cdot} \in \mathbb{C}^{1 \times M}$ of $\mathbf{\Phi}$ determines which γ_p transmit antennas are allocated to the p-th slot. For example, suppose that $\mu_p = 2$ and $\gamma_p = 3$ with $\mathbf{\Psi}_{p\cdot} = [1\ 1\ 0]$ and $\mathbf{\Phi}_{p\cdot} = [1\ 0\ 1\ 1]$. This means that the first and the second streams will be transmitted by the first, third and fourth transmit antennas at the p-th time-slot. Since each time-slot has is own stream-to-antenna allocation, different levels of space-time multiplexing and diversity are possible by varying the pattern of 1's and 0's of $\mathbf{\Psi}$ and $\mathbf{\Phi}$.

Note that R data streams pass through the channel during P time-slots of duration N symbol periods. Therefore, the rate of the space-time transmission is given by:

$$\text{Rate} = \left(\frac{R}{P}\right) \log_2(\nu) \text{ bits per channel use,}$$

where ν is the modulation cardinality.

3.2 Tensor modeling of the received signal

Let us define $\mathbf{S} \in \mathbb{C}^{N\times R}$ as a symbol matrix collecting the N symbols of the R data streams, where $s_{n,r} = [\mathbf{S}]_{n,r}$ denotes the n-th transmitted symbol of the r-th data-stream. The MIMO channel is defined by $\mathbf{H} \in \mathbb{C}^{K\times M}$, where $h_{k,m} = [\mathbf{H}]_{k,m}$ is the complex coefficient of the channel associating the m-th transmit antenna with the k-th receive antenna. Define also a spatial precoding matrix $\mathbf{W} \in \mathbb{C}^{M\times R}$ that combines R data streams with M transmit antennas. The structure of $\mathbf{W}$ will be discussed later. The transmitted space-time signal is given by:

$$u_{m,n,p} = \sum_{r=1}^{R} w_{m,r} s_{n,r} \phi_{p,m} \psi_{p,r}, \tag{4}$$

where $u_{m,n,p}$ is the signal transmitted by the m-th transmit antenna at the n-th symbol period of the p-th time-slot, i.e. the (m,p,n)-th element of the tensor $\mathcal{U} \in \mathbb{C}^{M\times N\times P}$. In absence of noise, the discrete-time baseband version of the received signal tensor is given by:

$$\begin{aligned} x_{k,n,p} &= \sum_{m=1}^{M} h_{k,m} u_{m,n,p} \\ &= \sum_{m=1}^{M}\sum_{r=1}^{R} h_{k,m} s_{n,r} w_{m,r} \phi_{p,m} \psi_{p,r}, \end{aligned} \tag{5}$$

where $x_{k,n,p}$ is the received signal associated with the k-th receive antenna, n-th symbol period and p-th time-slot. It is the (k,n,p)-th element of the tensor $\mathcal{X} \in \mathbb{C}^{K\times N\times P}$. Note that (5) follows a PARATUCK-2 decomposition, and the correspondences between (1) and (5) are:

$$\begin{aligned} &(I_1, I_2, I_3, R_1, R_2) \rightarrow (K, N, P, M, R) \\ &(\mathbf{A}, \mathbf{B}, \mathbf{G}, \mathbf{C}^A, \mathbf{C}^B) \rightarrow (\mathbf{H}, \mathbf{S}, \mathbf{W}, \mathbf{\Phi}, \mathbf{\Psi}). \end{aligned} \tag{6}$$

Let us define $\mathbf{X}_{\cdot\cdot p} \in \mathbb{C}^{K\times N}$ as the p-th matrix "slice" obtained by slicing $\mathcal{X} \in \mathbb{C}^{K\times N\times P}$ along its third dimension. This matrix can be factored as:

$$\begin{aligned} \mathbf{X}_{\cdot\cdot p} &= \mathbf{H} D_p(\mathbf{\Phi}) \mathbf{W} D_p(\mathbf{\Psi}) \mathbf{S}^T \\ &= \mathbf{H}\mathbf{F}_{\cdot\cdot p}\mathbf{S}^T, \end{aligned} \tag{7}$$

where

$$\mathbf{F}_{\cdot\cdot p} = D_p(\mathbf{\Phi}) \mathbf{W} D_p(\mathbf{\Psi}) \in \mathbb{C}^{M\times R} \tag{8}$$

is the p-th slice of the overall space-time precoder tensor $\mathcal{F} \in \mathbb{C}^{M\times R\times P}$. This slice associates the R data streams to the M transmit antennas at the p-th time-slot. Figure 2 illustrates the factorization of the p-th slice $\mathbf{X}_{\cdot\cdot p}$ of the received signal tensor as a function of the system parameters.

Define $\mathbf{X}_1 = \left[\text{vec}(\mathbf{X}_{\cdot\cdot 1}), \ldots, \text{vec}(\mathbf{X}_{\cdot\cdot P})\right] \in \mathbb{C}^{KN\times P}$ collecting the received signal over the P time-slots. From (7) and (8), it can be shown that $\mathbf{X}_1$ admits the following factorization:

$$\begin{aligned} \mathbf{X}_1 &= (\mathbf{S}\otimes\mathbf{H})\left[\text{vec}(\mathbf{F}_{\cdot\cdot 1}), \ldots, \text{vec}(\mathbf{F}_{\cdot\cdot P})\right] \\ &= (\mathbf{S}\otimes\mathbf{H})\mathbf{F}_1, \end{aligned} \tag{9}$$

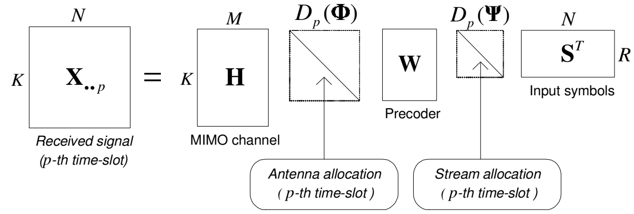

Figure 2: Visualization of the PARATUCK-2 decomposition of the p-th slice of the received signal tensor.

where

$$\mathbf{F}_1 = \left[\text{vec}(\mathbf{F}_{\cdot\cdot 1}), \ldots, \text{vec}(\mathbf{F}_{\cdot\cdot P})\right] \in \mathbb{C}^{MR\times P}, \tag{10}$$

and $\otimes$ denotes the Kronecker product.

The matrix $\mathbf{X}_1$ defined in (9) can be viewed as a matrix "unfolding" of the received signal tensor $\mathcal{X} \in \mathbb{C}^{K\times N\times P}$ stacking column-wise its first and second dimensions (K and N), each column being associated with a given time-slot. We can also define two other unfoldings $\mathbf{X}_2 \in \mathbb{C}^{PK\times N}$ and $\mathbf{X}_3 \in \mathbb{C}^{PN\times K}$ from the set of slices $\{\mathbf{X}_{\cdot\cdot 1}, \ldots, \mathbf{X}_{\cdot\cdot P}\}$, in the following manner:

$$\begin{aligned} \mathbf{X}_2 &= \begin{bmatrix} \mathbf{X}_{\cdot\cdot 1} \\ \vdots \\ \mathbf{X}_{\cdot\cdot P} \end{bmatrix} = \begin{bmatrix} \mathbf{H}\mathbf{F}_{\cdot\cdot 1} \\ \vdots \\ \mathbf{H}\mathbf{F}_{\cdot\cdot P} \end{bmatrix} \mathbf{S}^T \\ &= (\mathbf{I}_P \otimes \mathbf{H})\mathbf{F}_2\mathbf{S}^T, \end{aligned} \tag{11}$$

and

$$\begin{aligned} \mathbf{X}_3 &= \begin{bmatrix} \mathbf{X}^T_{\cdot\cdot 1} \\ \vdots \\ \mathbf{X}^T_{\cdot\cdot P} \end{bmatrix} = \begin{bmatrix} \mathbf{S}\mathbf{F}^T_{\cdot\cdot 1} \\ \vdots \\ \mathbf{S}\mathbf{F}^T_{\cdot\cdot P} \end{bmatrix} \mathbf{H}^T \\ &= (\mathbf{I}_P \otimes \mathbf{S})\mathbf{F}_3\mathbf{H}^T, \end{aligned} \tag{12}$$

where

$$\mathbf{F}_2 = \begin{bmatrix} \mathbf{F}_{\cdot\cdot 1} \\ \vdots \\ \mathbf{F}_{\cdot\cdot P} \end{bmatrix} \in \mathbb{C}^{PM\times R}, \; \mathbf{F}_3 = \begin{bmatrix} \mathbf{F}^T_{\cdot\cdot 1} \\ \vdots \\ \mathbf{F}^T_{\cdot\cdot P} \end{bmatrix} \in \mathbb{C}^{PR\times M},$$

are the two corresponding unfoldings of the overall precoder tensor constructed from the set of precoder slices $\{\mathbf{F}_{\cdot\cdot 1}, \ldots, \mathbf{F}_{\cdot\cdot P}\}$.

4. IDENTIFIABILITY ISSUE: DESIGN OF $\mathbf{\Psi}$ AND $\mathbf{\Phi}$

The identifiability of the underlying PARATUCK-2 model for the received signal is important since we are interested in a blind *joint* estimation of the channel and the symbols. Identifiability of $\mathbf{H}$ and $\mathbf{S}$ in the Least Squares (LS) sense is not only linked to the relation between the three dimensions K, N and P of the received signal tensor (c.f. Section 5) but also depends on the joint structure of the allocation matrices $\mathbf{\Psi}$ and $\mathbf{\Phi}$. We begin with two assumptions that are satisfied by the allocation matrices:

(*A1*) Both $\mathbf{\Psi}$ *and* $\mathbf{\Phi}$ *have no all-zero row*; each time-slot transmits at least one data stream and uses at least one transmit antenna;

(*A2*) Both $\mathbf{\Psi}$ *and* $\mathbf{\Phi}$ *have no all-zero column*; every data stream and transmit antenna is allocated at least once during the P time-slots.

Theorem 1: Supposing that $\mathbf{S}$ and $\mathbf{H}$ are nonsingular matrices, their identifiability in the LS sense requires that:

$$P \geq \max\left(\left\lceil\frac{R}{M}\right\rceil, \left\lceil\frac{M}{R}\right\rceil, 2\right). \tag{13}$$

Proof: Let us rewrite the two expressions (11) and (12) as $\mathbf{X}_2 = \mathbf{Z}_2\mathbf{S}^T$ and $\mathbf{X}_3 = \mathbf{Z}_3\mathbf{H}^T$, where $\mathbf{Z}_2 = (\mathbf{I}_P \otimes \mathbf{H})\mathbf{F}_2 \in \mathbb{C}^{PK\times R}$ and $\mathbf{Z}_3 = (\mathbf{I}_P \otimes \mathbf{S})\mathbf{F}_3 \in \mathbb{C}^{PN\times M}$. Identifiability of $\mathbf{S}$ and $\mathbf{H}$ in the LS sense requires that $\mathbf{Z}_2$ and $\mathbf{Z}_3$ be full column-rank to be left-invertible. Note that $(\mathbf{I}_P \otimes \mathbf{H})$ and $(\mathbf{I}_P \otimes \mathbf{S})$ are also nonsingular since $\mathbf{S}$ and $\mathbf{H}$ are assumed to be nonsingular. Consequently, rank($\mathbf{Z}_2$)=rank($\mathbf{F}_2$) and rank($\mathbf{Z}_3$)=rank($\mathbf{F}_3$), which means that $\mathbf{F}_2 \in \mathbb{C}^{PM\times R}$ and $\mathbf{F}_3 \in \mathbb{C}^{PR\times M}$ must be full column-rank to ensure the identifiability of $\mathbf{S}$ and $\mathbf{H}$. Therefore, we must have $PM \geq R$ and $PR \geq M$ or, equivalently, $P \geq \lceil \frac{R}{M} \rceil$ and $P \geq \lceil \frac{M}{R} \rceil$. It is also necessary to have $P \geq 2$ in order that $\mathcal{X}$ be a third-order tensor. Since these three conditions must be simultaneously satisfied, we arrive at (13). ■

Theorem 1 is useful when we are interested in quickly eliminating the space-time spreading-multiplexing configurations that lead to a nonidentifiable model. However, this condition is not sufficient for identifiability of $\mathbf{S}$ and $\mathbf{H}$. Note that rank($\mathbf{F}_2$) and rank($\mathbf{F}_3$) depend on the joint structure of the allocation matrices $\mathbf{\Psi} \in \mathbb{C}^{P\times R}$ and $\mathbf{\Phi} \in \mathbb{C}^{P\times M}$ and, more specifically, on their pattern of 1's and 0's.

Theorem 2: Suppose that $\mathbf{W} \in \mathbb{C}^{M\times R}$ is nonsingular and has no zero elements and that $\mathbf{\Psi} \in \mathbb{C}^{P\times R}$ and $\mathbf{\Phi} \in \mathbb{C}^{P\times M}$ satisfy assumptions (*A1*)-(*A2*) with $\mu_p = \mathbf{\Psi}_{p\cdot}(\mathbf{\Psi}_{p\cdot})^T \in [1, R]$ and $\gamma_p = \mathbf{\Phi}_{p\cdot}(\mathbf{\Phi}_{p\cdot})^T \in [1, M]$, $p = 1, \ldots, P$. If

$$R \leq \sum_{p=1}^{P} \gamma_p \quad \text{and} \quad M \leq \sum_{p=1}^{P} \mu_p, \tag{14}$$

then $\mathbf{S}$ and $\mathbf{H}$ are identifiable from (11) and (12). The proof of this theorem is not provided here due to a lack of space. It consists in studying the rank of $\mathbf{F}_2$ and $\mathbf{F}_3$ by rewriting these matrices in terms of the slices $\{\mathbf{F}_{\cdot\cdot p}\}$, $p = 1, \ldots, P$, and by taking the factorization (8) into account.

Essential uniqueness: Identifiability in the LS sense is linked to the identifiability of the subspaces of $\mathbf{S}$ and $\mathbf{H}$ from $\mathbf{X}_2$ and $\mathbf{X}_3$, respectively. This means that the estimated symbol and channel matrices, denoted by $\widehat{\mathbf{S}}$ and $\widehat{\mathbf{H}}$, are related to the true matrices $\mathbf{S}$ and $\mathbf{H}$ by $\widehat{\mathbf{S}} = \mathbf{S}\mathbf{T}_s$ and $\widehat{\mathbf{H}} = \mathbf{H}\mathbf{T}_h$, where $\mathbf{T}_s \in \mathbb{C}^{R\times R}$ and $\mathbf{T}_h \in \mathbb{C}^{M\times M}$ are two arbitrary nonsingular transformation matrices. Therefore, apart from ensuring model identifiability, it is also necessary to fix $\mathbf{T}_s$ and/or $\mathbf{T}_h$ for ensuring the *essential uniqueness* of $\mathbf{S}$ and/or $\mathbf{H}$ (up to scaling and permutation ambiguities). In order to ensure the essential uniqueness, additional constraints must be imposed on $\mathbf{\Psi}$ and $\mathbf{\Phi}$. It can be shown that $\mathbf{S}$ is essentially unique if $\mathbf{\Psi}$ has orthogonal columns (which requires $R \leq P$). Likewise, $\mathbf{H}$ is essentially unique if $\mathbf{\Phi}$ has orthogonal columns (which requires $M \leq P$). We are mostly interested in blindly estimating the transmitted symbols rather than estimating the channel, therefore the essential uniqueness of $\mathbf{H}$ is not relevant.

Design examples: We present some design examples for the allocation matrices $\mathbf{\Psi}$ and $\mathbf{\Phi}$ in order to show that the proposed space-time spreading-multiplexing model has the flexibility of covering different multiple-antenna signaling schemes with multiplexing-spreading tradeoffs.

Example 1 ($R = 2, M = 3, P = 2$)*:* Consider the transmission of 2 data streams using 3 antennas and 2 time-slots with the following allocation structure:

$$\mathbf{\Psi} = \mathbf{I}_2, \quad \mathbf{\Phi} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

The first data stream is allocated to the first and third transmit antennas at the first time-slot, while the second data stream is allocated to the second and third transmit antennas at the second time-slot. Note that only two transmit antennas are used at each time-slot. While antennas 1 and 3 share the first time-slot, antennas 2 and 3 share the second one. Both data streams have a spatial transmit diversity gain of order two. In this case, uniqueness of $\mathbf{H}$ is not guaranteed since $\mathbf{\Phi}$ is not column-wise orthogonal.

Example 2 ($R = 2, M = 2, P = 3$)*:* Consider the transmission of 2 data streams using 2 antennas and 3 time-slots with the following allocation structure:

$$\mathbf{\Psi} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{\Phi} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

In this case, each time-slot is allocated to only one transmit antenna. The first data stream is transmitted at the first and second time-slots using antennas 1 and 2, respectively. The second data stream is allocated to the third time-slot and transmitted by antenna 1. Note that the first data stream has some spatial transmit diversity gain since it is spread across two transmit antennas, which is not the case for the second data stream.

Structure of W: The role of $\mathbf{W}$ is to combine/multiplex the R data streams across the M transmit antennas. We choose $\mathbf{W}$ as the following Vandermonde matrix:

$$[\mathbf{W}]_{m,r} \doteq e^{j2\pi(r-1)(m-1)/M}. \tag{15}$$

With this choice, we ensure that $\mathbf{W}$ is full-rank and has no zeros, as required for the identifiability of the proposed PARATUCK-2 model under the conditions previously described. It is to be noted, however, that this choice is not necessarily optimal. When using closed-loop transmission based on *a priori* channel knowledge at the transmitter, different choices for $\mathbf{W}$ exist depending on the chosen design criterion. The optimized design of $\mathbf{W}$ is not the focus of this work and will be addressed in a future contribution.

5. BLIND RECEIVER

We propose a blind receiver for a joint channel estimation and symbol detection based on the Alternating Least Squares (ALS) algorithm [14]. The ALS algorithm exploits the PARATUCK-2 tensor structure of the received signal by means of the two matrix factorizations (11) and (12). The algorithm consists in alternating between the estimation of the channel and symbol matrices in the LS sense. Define $\tilde{\mathbf{X}}_i = \mathbf{X}_i + \mathbf{V}_i$, $i = 2, 3$, as the noisy versions of $\mathbf{X}_i$, where $\mathbf{V}_i$ is an additive complex-valued white gaussian noise matrix. Recall that $\mathbf{F}_2$ and $\mathbf{F}_3$ are known. The receiver algorithm consists of the following steps:

Initialization: Set $i = 0$; Randomly initialize $\widehat{\mathbf{H}}_{(i=0)}$;

Alternating LS updates:

(1) $i = i + 1$;

(2) From $\tilde{\mathbf{X}}_2$ and using $\widehat{\mathbf{H}}_{(i-1)}$, calculate an LS estimate of $\mathbf{S}$:

$$\widehat{\mathbf{S}}^T_{(i)} = \left[(\mathbf{I}_P \otimes \widehat{\mathbf{H}}_{(i-1)})\mathbf{F}_2\right]^{\dagger} \tilde{\mathbf{X}}_2;$$

(3) From $\tilde{\mathbf{X}}_3$ and using $\widehat{\mathbf{S}}_{(i)}$, calculate an LS estimate of $\mathbf{H}$:

$$\widehat{\mathbf{H}}^T_{(i)} = \left[(\mathbf{I}_P \otimes \widehat{\mathbf{S}}_{(i)})\mathbf{F}_3\right]^{\dagger} \tilde{\mathbf{X}}_3;$$

(4) Repeat steps (1)-(3) until convergence.

The operator $(\cdot)^\dagger$ denotes the matrix pseudo-inverse. We decide the convergence of the algorithm at the i-th iteration when the error between the received signal tensor and its reconstructed version from the estimated channel and symbol matrices does not significantly change between iterations i and $i + 1$.

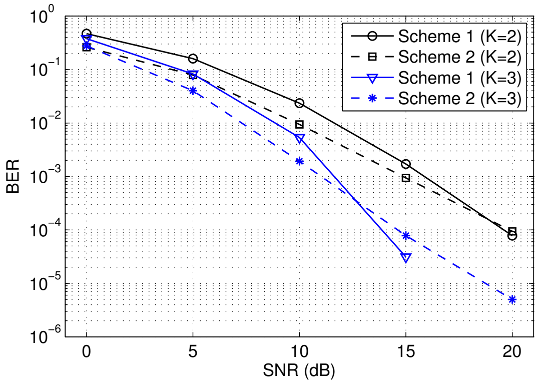

Figure 3: BER vs. SNR.

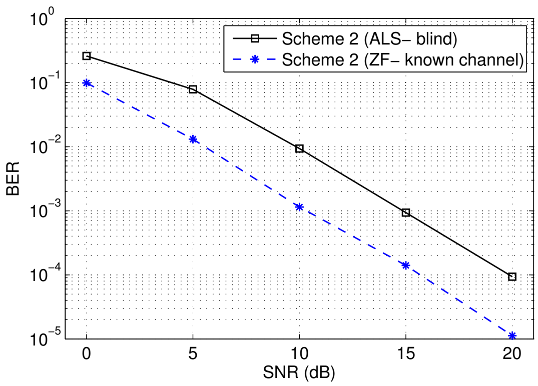

Figure 4: ALS (blind) vs. ZF (perfect channel knowledge).

6. SIMULATION RESULTS

We present some simulation results for evaluating the Bit-Error-Rate (BER) performance of a MIMO system using the proposed space-time spreading-multiplexing model along with the ALS-based blind detection. Each BER curve is an average of one thousand Monte Carlo runs. Each run represents one realization of the flat-fading channel the coefficients of which are drawn from an i.i.d. complex-valued Gaussian generator. At each run, the transmitted symbols are drawn from a QPSK sequence. The BER curves represent the performance averaged on the $R = 2$ data streams. In all cases, we consider a very short data stream of N=5 symbols, which is a challenging assumption for a blind receiver. We simulate the two space-time schemes of Examples 1 and 2 of Section 4 (Schemes 1 and 2, respectively). The results are depicted in Fig. 3 for K=2 and K=3. Solid lines are associated with the Scheme 1 and dashed lines with the Scheme 2. We can observe that the second scheme outperforms the first one at low-to-medium SNR values. Such a gain comes from the fact that Scheme 1 has no temporal coding gain for the data streams which are only spread in the spatial domain and not repeated over multiple time-slots. Scheme 2 has some temporal coding gain for the first data stream in addition to spatial transmit diversity. Note, however, that Scheme 1 tends to be better for higher SNRs due to the higher spatial diversity gain (both data streams are spatially-spread). In a second experiment, we compare the proposed receiver with a Zero Forcing (ZF) receiver that assumes perfect knowledge of the channel matrix, for Scheme 2 and K=2. Both receivers present the same BER vs. SNR slope. The performance degradation when using the ALS-based blind receiver is around 5dB for a BER=10^{-3}.

7. CONCLUSION AND PERSPECTIVES

We have proposed a new tensor modeling approach to space-time spreading-multiplexing for MIMO antenna systems with joint blind channel estimation and detection. The core of the proposed PARATUCK-2 model is composed of a precoding matrix and two allocation matrices $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$ that allow to control the spreading-multiplexing of the data symbols across multiple transmit antennas and time-slots. The PARATUCK-2 space-time transmission structure has the flexibility to accommodate data streams to time-slots in a different number of ways. Identifiability has been discussed and linked to the design of the allocation matrices. We have also derived a blind ALS receiver based on the PARATUCK-2 tensor structure. Perspectives of this work include an extension of the PARATUCK-2 modeling approach to multicarrier systems with space-time-frequency transmission.

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