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NEAR-END CROSSTALK MITIGATION USING WAVELETS

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ABSTRACT

A new method to mitigate near-end crosstalk (NEXT) in the wavelet transform domain is proposed. The method utilizes the fact that the received signal has greater energy in the lower end of the transmission bandwidth in comparison to the NEXT noise and therefore is more regular, or smooth, than the NEXT noise. The method entails a new technique of estimating the NEXT noise from the received signal so that appropriate denoising methods can be applied to reduce the NEXT noise. Experiment results have shown that the method is quite effective in reducing NEXT noise especially when the signal to crosstalk noise ratio is low (SCNR). Furthermore, the method has a low computational complexity which makes it fast, efficient, and well suited for high data-rate applications.

1. INTRODUCTION

As xDSL technology is improving, the data rate that can be achieved through twisted pairs is increasing. A major impairment that hinders the simultaneous transmission of signals through a bundle of twisted pairs is near-end crosstalk (NEXT). Several techniques to mitigate or cancel NEXT have appeared in the literature. One technique involves the spectral shaping of the transmitted signals [1]. Although the method has considerably improved interoperability, it does not always yield optimal results. Frequency division duplexing of the transmitted and received signals is another effective technique for reducing NEXT [2]. However, in a cable of mixed DSL sources the spectra may overlap making FDD ineffective. Another effective technique is to deploy adaptive filters to cancel the NEXT signals [3, 4, 5]. Although this technique is very effective, it can be computationally expensive for cables with a large number of twisted pairs; further, the technique requires the input signals from the crosstalk sources in order to work.

In this paper, a new crosstalk mitigation method that does not require the transmitted signals from the crosstalk sources to be available is proposed. In the method, the noisy signal is first transformed to the wavelet domain and then an appropriate threshold technique is applied to reduce the crosstalk noise. Crosstalk noise, i.e., NEXT or FEXT, has a spectral power that is typically confined to the higher end of the signal frequency band while the received signal is more concentrated at the lower end; as a consequence, the transmitted signal will have a Lipschitz regularity [6] that is much higher than that of crosstalk noise. The new threshold technique takes advantage of this difference in regularity to significantly improve the signal-to-crosstalk-noise ratio (SCNR) of the signal. Furthermore, the method can also be used to attenuate noises such as FEXT and background noise, that cannot be cancelled by adaptive filters since they do not have the reference signals available. Another advantage of the method over adaptive filters is that the computational complexity is independent of the number of crosstalk sources or, equivalently, the number of lines in the bundle, and this makes the method quite efficient for bundles with large number of twisted pairs. Moreover, unlike optimal linear estimators, such as the Wiener filter, where prior knowledge of the magnitude and exact statistics of the crosstalk is required for optimal performance, the proposed method does not require any prior knowledge about the crosstalk magnitude but estimates it as the algorithm progresses. And with the use of fast wavelet transform algorithms such as the one in [7], which have complexity of order $L\log_{10} L$ where L is the length of a block of data, the method can be very efficient and fast. The paper is organized as follows: Section 2 describes the characteristics of NEXT and FEXT, namely, their frequency distributions and Gaussian nature. Section 3 presents the various wavelet denoising techniques for NEXT reduction. Simulation results are presented and discussed in section 4 while conclusions are drawn in section 5.

2. GAUSSIAN NATURE OF CROSSTALK

In a transmission cable, the communication channel is subjected to crosstalk interference from multiple twisted pairs. Due to the presence of multiple crosstalk interferers, the NEXT tends to be a Gaussian random process [8]. Assuming that the transmitted signals in all the interfering pairs have the same power density spectrum, the NEXT PSD $S_N(f)$ for sufficiently long loops of 1000 feet or more can be expressed as

$$S_N(f) \approx \chi S(f) f^{3/2} \quad (1)$$

where χ is a Gaussian random variable which is a function of the disturbed pair under consideration and $S(f)$ is the PSD of the input signal of the interfering pair. Expression (1) has been found to be quite accurate for the kind of multi-interference NEXT that is observed in practice [9]. As such, the filter defined by (1) can be used to simulate NEXT by using white Gaussian noise as the input signal. Unlike the power of the NEXT signal, that of the received signal is more attenuated in the higher frequency regions of the transmission band. This is because the amplitude response of a twisted-pair channel usually decreases with frequency. Typically, the attenuation is approximately proportional to $\sqrt{f}$ [9].

3. CROSSTALK MITIGATION USING WAVELETS

Since the paper of Donoho et al. on wavelet denoising in 1993 [10], the use of wavelet techniques for the removal of noise from a signal has become increasingly popular. One reason is the flexibility in choosing the wavelets; depending upon the type of the signal, we can select the most appropriate wavelet that minimizes the magnitude of coefficients at the lower levels, or finer scales, for the more-regular received signal and thereby enable the less-regular noise at the lower levels to be more accurately estimated and removed [11]. Another reason is that it provides a quasi-optimal minmax estimate of a noisy piecewise-smooth signal on a wavelet basis, which is as good, or in some cases better, than linear estimators [12].

When denoising a signal, it is important to know the nature of the noise. Consider a noise-corrupted signal $x(n)$ which can be modelled as

$$x(n) = s(n) + w(n) \tag{2}$$

where $s(n)$ is the noise-free signal and $w(n)$ is the noise. If $w(n)$ is an additive white Gaussian noise, the wavelet transform of $w(n)$ will also be white Gaussian with the same magnitude across all levels. Hence, (2) can be expressed in the wavelet domain as

$$\kappa(n) = \delta(n) + \gamma(n) \tag{3}$$

where $\kappa(n)$ is the wavelet transform of $x(n)$, $\delta(n)$ is the wavelet transform of $s(n)$, and $\gamma(n)$ is the wavelet transform of $w(n)$. It should be noted that $\gamma(n)$, like $w(n)$, will also be white Gaussian with the same variance. The noise contribution in $\kappa(n)$ is suppressed by applying a soft threshold on the wavelet coefficients, given by

$$y_s(n) = \begin{cases} \text{sgn}[y(n)]\,(|y(n)| - \lambda) & \text{if } |y(n)| > \lambda \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where λ is a threshold that can be estimated from two methods. In the first method, λ is estimated using the so called universal threshold method [13]. The method assumes that the noise is Gaussian distributed and is given by

$$\lambda_{i_{univ}} = \sigma_i \sqrt{2 \log n_i} \tag{5}$$

where

$$\sigma_i = \frac{MAD(\kappa_i)}{0.6745}, \tag{6}$$

MAD is the median absolute deviation of the coefficients, and κ_i is a vector whose elements are the wavelet coefficients for level i. The second method is based on Stein's unbiased risk estimate (SURE). As described in [14], the SURE estimate is unbiased not only for white Gaussian noise but also for correlated Gaussian noise and is given by

$$\lambda_{i_{SURE}} = \sigma_i t(w_i/\sigma_i) \tag{7}$$

where

$$t(x) = \text{argmin}_{\ 0 \le t \le \sqrt{2 \log d}}\ U(t) \tag{8}$$

and

$$U(t) = n_i + \sum_k \left\{ \min(x_k^2, t^2) - 2I(|x| \le t) \right\} \tag{9}$$

where I is the indicator function ($I(\cdot) = 1$ if $|x_k| \le t$ and $I(\cdot) = 0$ if $|x_k| > t$), and n_i is the number of coefficients at level i.

3.1 Estimation of the crosstalk noise across the wavelet levels

Since the NEXT noise has greater power towards the higher end of the frequency spectrum, the magnitude of the wavelet coefficients at the higher wavelet levels will be predominantly due to NEXT. However, there are instances where the magnitude of the NEXT noise in a twisted pair is small or the received signal is large. In such cases, the magnitude of the wavelet coefficients at the higher levels will also have significant contributions from the received signal. As such, using the lowest level wavelet coefficients to estimate the NEXT noise will not always be accurate, especially, when the SCNR of the received signal is high.

To get a fairly accurate estimate of the magnitude of the NEXT noise in each wavelet level, we take advantage of the fact that the larger wavelet coefficients of the NEXT signal are confined to the higher levels, while for the received signal they are confined to the lower levels. Therefore, first, the typical distributions of the NEXT magnitudes across the different wavelet levels are estimated. This is done by passing white Gaussian noise through a filter with NEXT PSD as given in (1), taking its wavelet transform, and then using the wavelet-transformed signal to evaluate the NEXT distribution ratio α_i, given by

$$\alpha_i = \frac{\text{MAD}(\gamma_i)}{\text{MAD}(\gamma_L)} \tag{10}$$

where γ_i is a vector whose elements are the wavelet coefficients of the wavelet-transformed crosstalk noise for level i, and L the finest (highest) level. Next, the distributions of the magnitude of the received signal across the wavelet levels are estimated. This is done by taking the wavelet transform of the received signal and evaluating the ratio β_i given by

$$\beta_i = \frac{\text{MAD}(\delta_i)}{\text{MAD}(\delta_L)} \tag{11}$$

where δ_i is a vector whose elements are the wavelet coefficients of the wavelet-transformed received signal for level i. Thus, by using the ratios α_i and β_i it is possible to estimate the powers of the NEXT signal and the received signal at a particular wavelet level.

Suppose a received signal $x(n)$ corrupted by NEXT noise is transformed such that the wavelet coefficients for level i are represented by vector κ_i; in such a case, the MAD estimate σ_{xi} is given by

$$\sigma_{xi}^2 = \text{MAD}(\kappa_i) \tag{12}$$

If σ_{Ni}^2 and σ_{Si}^2 are the NEXT signal power and the received signal power, respectively, for level i, then σ_{xi}^2 will be the sum of σ_{Ni}^2 and σ_{Si}^2, i.e.,

$$\sigma_{xi}^2 = \sigma_{Ni}^2 + \sigma_{Si}^2 \tag{13}$$

By using (10), (11), and (13), we have the following two equations for the ith and Lth levels:

$$\sigma_{NL}^2 + \sigma_{SL}^2 = \sigma_{xL}^2 \tag{14}$$

$$\alpha_i^2 \sigma_{NL}^2 + \beta_i^2 \sigma_{SL}^2 = \sigma_{xi}^2 \tag{15}$$

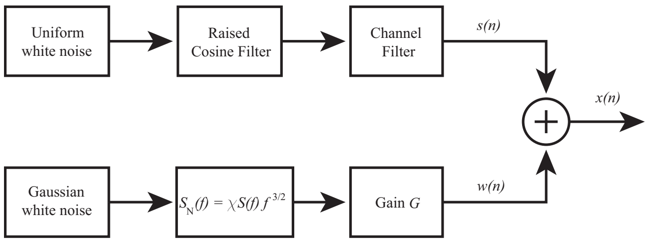


Figure 1: Block diagram for generating NEXT-interfered received signals.

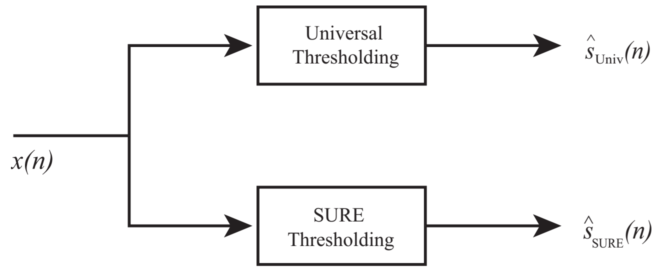


Figure 2: Simulation setup for comparing the performance between the universal and SURE estimates in reducing NEXT.

Solving (14) and (15), the crosstalk noise power estimate for level i is given by

$$\sigma_{Ni}^2 = \alpha_i^2 \sigma_{NL}^2 = \alpha_i^2 \frac{\beta_i^2 \sigma_{xi}^2 - \sigma_{xL}^2}{\beta_i^2 - \alpha_i^2} \quad (16)$$

Once the powers of the crosstalk noise at the different levels have been estimated, the threshold values for the various levels are computed. Both the universal and the SURE threshold estimates, given in (5) and (7), respectively, can be used. However, in situations of extreme sparcity in the wavelet coefficients of the desired signal, which happens when the SNR of the received signal is low, the SURE principle described in (7) has a serious drawback [14]: the noise contributed to the SURE profile by the many coordinates at which the signal is zero swamps the information contributed to the SURE profile by the few coordinates where the signal is nonzero. Hence, in such cases the universal threshold given in (5) would be a more accurate threshold estimate. Consequently, a hybrid scheme similar to the one proposed in [14] can be adopted; that is, when the SNR of the signal is below a certain threshold the universal threshold estimate is used, and when the SNR is above the threshold the SURE estimate is used. In equation form, this is given by

$$\lambda_i = \begin{cases} \lambda_{i_{SURE}} & \text{if SNR} > \Gamma \\ \lambda_{i_{univ}} & \text{if SNR} \leq \Gamma \end{cases} \quad (17)$$

where Γ is an appropriately selected threshold. Since the SNR of the received signal is not readily available, an approximate estimate can be obtained by solving equations (14) and (15) for σ_{Ni}^2 and σ_{Si}^2, respectively, and taking their ratios; that is,

$$\text{SNR} \approx k \frac{\sigma_{SL}^2}{\sigma_{NL}^2} \quad (18)$$

where k is some constant which can be determined empirically.

4. SIMULATION RESULTS

The NEXT-corrupted received signal for the simulations was generated as shown in Fig. 1. As can be seen in the figure, the received signal was generated by passing a zero-mean uniformly distributed random sequence through a channel filter that models a twisted pair channel of length 7600 feet and gauge 26 AWG. The NEXT signal was generated by passing white Gaussian noise through a filter with PSD as defined in (1). The term $S(f)$ in (1) represents the combined PSD of the HDSL2 mask, the transmit filter, and the receive filter. The NEXT-interfered received signal was obtained by adding the NEXT noise, $w(n)$, with the clean signal, $s(n)$. By adjusting the gain G, the magnitude of the NEXT noise (or correspondingly the SNR of the received signal) can be adjusted.

The NEXT noise distribution ratio α_i and the received signal distribution ratio β_i were evaluated for a particular wavelet using (10) and (11), respectively. The two ratios were evaluated beforehand, and were required in order to estimate the crosstalk noise power at the various wavelet levels.

To study the effectiveness of the method, two simulation experiments, Experiments A and B, were conducted as detailed below.

Experiment A

In Experiment A, two threshold methods, namely, the universal and SURE threshold methods were tested. The setup used is shown in Fig. 2. In both methods, the noisy signal was divided into blocks, each of length 2048, and by using WAVELAB[1] the discrete wavelet transform was applied on each block. The NEXT-noise powers for levels 7, 8, 9 and 10 were then estimated using (16). Once the NEXT-noise powers at the different levels were estimated, the threshold values were next evaluated using two threshold methods, namely, the universal and SURE threshold methods. To reduce the wavelet coefficients for attenuating the NEXT noise the soft thresholding method in (4) was used. Tables 1 and 2 show a comparison of the two threshold methods for the Battle-Lemarie wavelet and the Daubechies wavelet of order 10, respectively. On comparing the SNRs in Tables 1 and 2, it can be seen that the Battle-Lemarie wavelet gives better SNR improvement (by a few dBs) than the Daubechies wavelet of order 10. It is also observed that at high SNRs, the SURE threshold method gives better SNR improvement whereas at very low SNRs the universal threshold method outperforms the SURE method. Hence, by adopting the hybrid technique in (17), the best of the two threshold methods can be utilized.

Experiment B

In Experiment B, the SNR was evaluated by passing both the noisy and denoised signals through a matched filter, that is, a filter that maximizes the SNR of the received

[1] Available from the Stanford Statistics Department, courtesy of D. L. Donoho and I. M. Johnstone

Table 1: Comparison between the universal and SURE estimates, using the Battle-Lemarie wavelet.

SNR$_{x(n)}$ (dB)	SNR$_{\hat{s}_{univ}(n)}$ (dB)	SNR$_{\hat{s}_{SURE}(n)}$ (dB)
-25	13.9	-3
-5	19.8	15.8
0.03	21.2	19.36
10	23.36	24.3
20	26.2	31.8

Table 2: Comparison between the universal and SURE estimates, using the Daubechies wavelet of order 10.

SNR$_{x(n)}$ (dB)	SNR$_{\hat{s}_{univ}(n)}$ (dB)	SNR$_{\hat{s}_{SURE}(n)}$ (dB)
-25	14.02	-3.3
-5	18.8	15.29
0.03	20.37	18.48
10	22.53	23.95
20	25	28.68

signal. A block diagram of the simulation setup used is shown in Fig. 3. In our simulations, the causal matched filter was realized by using the expression

$$f(t) = h(C - t) \tag{19}$$

where C is some constant and $h(t)$ is the received pulse shape of finite length of a single data symbol. To select the threshold, the hybrid estimate as given in (17) was used. The measured SNRs of the received signal are given in Table 3. From the table, it can be seen that for low SNRs, the wavelet-denoised signal $\hat{s}_{mh}(n)$ has significantly better SNR than the noisy signal $\hat{s}_m(n)$. However, as the SNR of the received signal increases, the SNR improvement obtained by using the wavelet-denoising method decreases. And when the SNR becomes relatively high as in row 5 of Table 3, the denoised signal has a much lower SNR than the noisy signal. The power spectra of $\hat{s}_m(n)$ and $\hat{s}_{mh}(n)$ are compared in Figs. 4(a) and 4(b) for two SNRs of the noisy signal $x(n)$; the power spectra of a clean (NEXT-free) received signal is also shown in both the figures to serve as a reference. The SNR of $x(n)$ in Fig. 4(a) is -5 dB while in Fig. 4(b) it is 10 dB. From the two figures, it can be seen that at a low SNR of -5 dB there is greater spectral improvement in using the wavelet technique

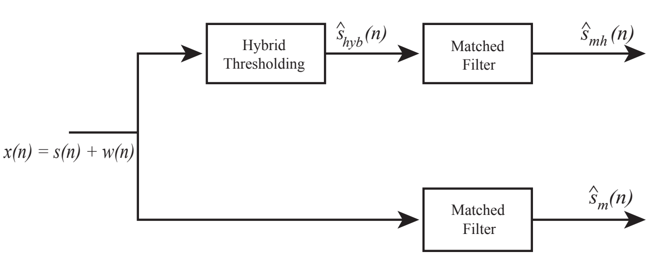
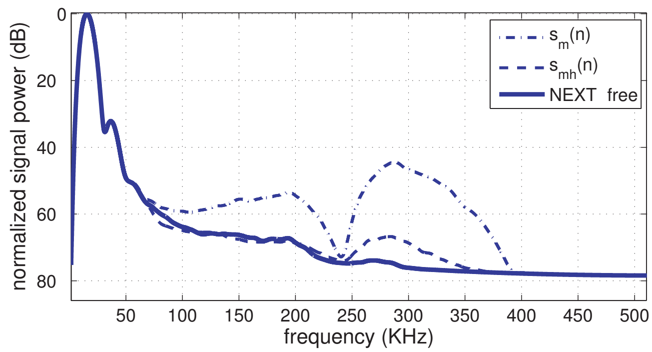


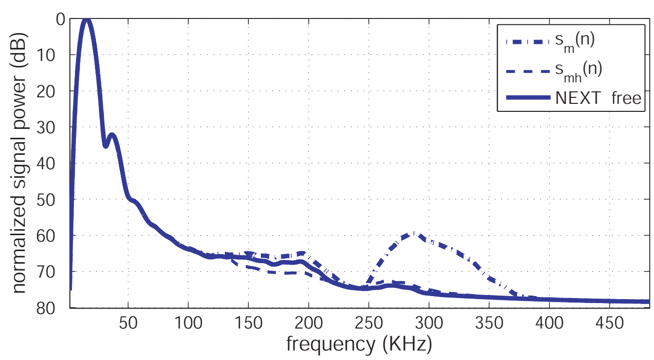
Figure 3: Block diagram of the simulation setup for comparing the SNR performance between the noisy signal and the denoised signal (the Battle-Lemarie wavelet was used).

Table 3: Effectiveness of the wavelet denoising technique in reducing NEXT.

SNR$_{x(n)}$ (dB)	SNR$_{\hat{s}_{hyb}(n)}$ (dB)	SNR$_{\hat{s}_{mh}}(n)$ (dB)	SNR$_{\hat{s}_m}(n)$ (dB)
-25	13.9	27.37	17
-5	15.8	43.7	37
0.03	19.36	46.53	42.03
10	24.3	52.41	52.04
20	31.8	57.47	62



(a)



(b)

Figure 4: PSD of the noisy, denoised, and crosstalk-free signals after passing through the matched filter: (a) the SNR of the noisy signal is -5 dB (b) the SNR of the noisy signal is 10 dB.

than at a higher SNR of 10 dB, in agreement with the results in Table 3.

5. CONCLUSIONS

A new method to mitigate near-end crosstalk (NEXT) in the wavelet transform domain was proposed. The method utilized the fact that the receive signal is more regular than the NEXT noise and therefore has larger wavelet coefficients in the lower levels while NEXT noise has larger coefficients in the higher levels. The method entailed a new technique of estimating the NEXT noise from the received signal, so that appropriate denoising methods can be applied to reduce the NEXT noise. Experiment results have shown that the method is quite effective in reducing NEXT noise especially when the SCNR is low. Furthermore, the method has a low computational complexity which makes it fast, efficient, and well suited for high data-rate applications.

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