# A CONCURRENT BLIND RECEIVER FOR STBC OVER DOUBLY DISPERSIVE CHANNELS

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#### **ABSTRACT**

Space-time block coding (STBC) achieves the maximum diversity gain of a flat fading multiple input multiple output (MIMO) transmission link. For frequency selective fading, only a few receivers can be found in the literature. These are mainly trained and block based, such as time reversal STBC (TRSTBC). A non-block based blind receiver for STBC over dispersive channels has been derived previously, adding a new term to the cost function of the constant modulus algorithm to orthogonalise the outputs. In this paper a decision directed (DD) equalizer is used concurrently with the CMA receiver to achieve faster convergence. Simulation results demonstrate the benefit of the proposed approach in terms of convergence speed and bit error ratio in a doubly-dispersive scenario.

**Keywords**-Space-time block coding, concurrent equalization, constant modulus, decision directed, blind equalization, selective fading.

# 1. INTRODUCTION

Wireless communication systems are becoming increasingly attractive due to the growing demand for data communications. In recent years, Multiple-Input Multiple-Output (MIMO) has received considerable attention due to its ability to increase the capacity of the transmission link. This can be used to either increase the throughput of the communication system or to improve the resilience to fading. For the latter, a technique which acheives the maximum improvement in flat fading is knwon as Space-Time Block Coding (STBC), first introduced by Alamouti in [1]. With two transmit and preceive antennas, STBC achieves the same diversity level as Maximal Ratio Receiver Combining (MRRC) system with one transmit and 2p receive antennas. In [2], The same idea was adopted to achieve the maximum diversity gain over channels with Inter-Symbol Interference (ISI), named Time-Reversal STBC (TRSTBC).

TRSTBC is a block-based technique, which assumes full Channel State Information (CSI) at the receiver. This usually implies the regular transmission of a training sequence, which adds to the redundancy introduced by the block transmission consequently reducing the overall throughput of the system. in [6], a tap constrained Constant Modulus Algorithm (CMA) was derived for the blind equalization of TRSTBC. However, the proposed system assumes stationarity of the channel over the duration of two blocks of data, which makes the algorithm inefficient at tracking fast varying channels.

In a previous paper by the authors, [3], a non-block based constant modulus receiver was designed for the equalization of STBC. The algorithm assumes the transmitted symbols have a single modulus, known at the receiver. The equalizer adapts its coefficients by forcing the outputs to have the same modulus. The proposed algorithm adds a new term to the cost function of the CMA to minimize the cross correlation between the outputs and prevent multiple extractions of the same source at more than one output.

The main drawback of the Constant Modulus Algorithm is its slow convergence. One possible solution to this is the use of Newton's search method, which requires the inverse of the input covariance matrix [4]. While this speeds up the convergence, computer simulations showed that its Bit-Error Rate (BER) performance is worse than that of the gradient descent method. In this paper, a concurrent CM and Decision Directed (DD) equalizer is used. The DD adaptation is known for its fast convergence but erroneous hard decisions result in an error propagation, thus reducing the performance. To overcome this, the algorithm at hand only enables the DD adaptation when the CM step does not lead to an alteration in the decision. Hence, the concurrent algorithm takes advantage of the robustness of CMA and the fast convergence of DD and supresses the drawbacks of both schemes.

Computer simulations are presented here to show how the concurrent receiver acheives faster convergence as well as a better BER than the standard STBC-CMA at a minimal increase in the complexity.

# 2. SIGNAL MODEL

Consider the 2-transmit and 2-receive antenna configuration shown in Figure 1. The transmitted data is encoded in space and time according to STBC, as in [1]. At times n and n+1, two symbols,  $a_1$  and  $a_2$ , arrive at the encoder, which are drawn from a PSK constellation set. The transmitted symbols are calculated as,

$$\begin{bmatrix} s_1[n] & s_1[n+1] \\ s_2[n] & s_2[n+1] \end{bmatrix} = \begin{bmatrix} a_1 & -a_2^* \\ a_2 & a_1^* \end{bmatrix}.$$
 (1)

where  $(\cdot)^*$  dentoes complex conjugation. The received signals over a window of L samples is given by

$$\mathbf{R}_{n} = \begin{bmatrix} \mathbf{r}_{1}^{T} \\ \mathbf{r}_{2}^{T} \end{bmatrix}$$
$$= \sum_{v=0}^{N-1} \mathbf{H}_{v} \mathbf{S}_{n-v} + \mathbf{V}_{n}$$
(2)

where  $\left(\cdot\right)^T$  denotes transposition and  $\mathbf{S}_n$  is the matrix of transmitted data

$$\mathbf{S}_{n} = \begin{bmatrix} s_{1}[n] & \cdots & s_{1}[n-L+1] \\ s_{2}[n] & \cdots & s_{2}[n-L+1] \end{bmatrix}, \tag{3}$$

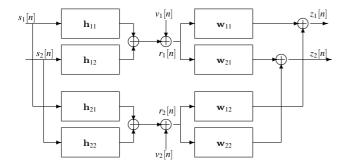


Figure 1: Channels and equalizers for a 2-by-2 MIMO system

and  $\mathbf{V}_n$  is Additive White Gaussian Noise (AWGN) with zero mean and  $\mathscr{E}\{\{\}\mathbf{V}_n\mathbf{V}_n^H\} = L\sigma_r\mathbf{I}_{2\times 2}$  corrupting the received signal. The matrix  $\mathbf{H}_v$  is the  $v^{th}$  time slice of the channel transfer function and is defined as

$$\mathbf{H}_{\mathbf{v}} = \begin{bmatrix} h_{1,1}[\mathbf{v}] & h_{1,2}[\mathbf{v}] \\ h_{2,1}[\mathbf{v}] & h_{2,2}[\mathbf{v}] \end{bmatrix}, \tag{4}$$

where  $\mathbf{h}_{i,j}$  is the frequency selective channel from the  $j^{th}$  transmit antenna to the  $i^{th}$  receive antenna. The channels are assumed to be of the same length, N. If the length of the channels varies, N is the length of the longest channel and the remaining channels are appended with zeros. The transfer function of the dispersive MIMO channel can be written as

$$\mathbf{H}(z) = \sum_{\nu=0}^{N-1} \mathbf{H}_{\nu} z^{-\nu}.$$
 (5)

#### 3. CONSTANT MODULUS RECEIVER

As shown in Fig. 1, two space-time equalizers are used, each with two subequalizers. The outputs of the two space-time equalizers are collected over two consecutive symbol periods, n and n+1, and are given by

$$\begin{bmatrix} z_1[n] & z_1[n+1] \\ z_2[n] & z_2[n+1] \end{bmatrix} = \begin{bmatrix} \mathbf{w}_1^H[n] \\ \mathbf{w}_2^H[n] \end{bmatrix} \cdot [\mathbf{r}_n \ \mathbf{r}_{n+1}], \quad (6)$$

where  $(\cdot)^H$  denotes the conjugate transpose and  $\mathbf{r}_n$  is the the transversal delay line vector

$$\mathbf{r}_n = [r_1[n] \cdots r_1[n-L+1] \ r_2[n] \cdots r_2[n-L+1]]^T,$$
(7)

the vector  $\begin{bmatrix} r_1[n] \\ r_2[n] \end{bmatrix}$  is a column of the matrix  $\mathbf{R}_n$ , and  $\mathbf{w}_j[n]$  is the  $j^{th}$  equalizer coefficients vector

$$\mathbf{w}_{j}[n] = \left[w_{j1}^{*}[0] \cdots w_{j1}^{*}[L-1] w_{j2}^{*}[0] \cdots w_{j2}^{*}[L-1]\right]^{T}.$$
(8)

The problem with MIMO-CMA is that the outputs may end up identifying the same source more than once. This may reduce the diversity gain of the system. However a constraint can be placed on the outputs to minimize the crosscorrelation. Consider the following cost function

$$\xi_{CM} = \mathcal{E}\left\{\sum_{i=1}^{2} \sum_{\tau=0}^{1} (|z_i[n+\tau]|^2 - 1)^2 + \mathbf{a}_n^{\mathsf{H}} \mathbf{a}_n\right\}, \quad (9)$$

with

$$\mathbf{a} = \begin{bmatrix} z_1[n] & - & z_2^*[n+1] \\ z_2[n] & + & z_1^*[n+1] \end{bmatrix}, \tag{10}$$

where the first term of the cost function represents the CM criterion over two consecutive symbol periods. The new term minimizes the vector a which forces the two outputs, $z_1[n]$  and  $z_2[n]$ , to have an STBC structure and consequently minimizes the cross-correlation between them.

Similar to the derivation of CMA in [7], the gradient of the first term of  $\xi_{CM}$  can be given by:

$$\frac{\partial}{\partial \mathbf{w}_{i}^{*}} (z_{k}[n+\tau]z_{k}^{*}[n+\tau]-1)^{2} = \begin{cases}
2(z_{k}[n+\tau]z_{k}^{*}[n+\tau]-1)z_{k}^{*}[n+\tau]\mathbf{r}_{n+\tau} & k=i \\
\underline{0} & k \neq i
\end{cases}$$
(11)

The cross-correlation term of the cost function requires closer evaluation. It can be shown that the gradient of the second term is

$$\frac{\partial}{\partial \mathbf{w}_{1}^{*}} \mathbf{a}_{n}^{H} \mathbf{a}_{n} = (z_{1}^{*}[n] - z_{2}[n+1]) \mathbf{r}_{n} + (z_{2}[n] + z_{1}^{*}[n+1]) \mathbf{r}_{n+1} 
\frac{\partial}{\partial \mathbf{w}_{2}^{*}} \mathbf{a}_{n}^{H} \mathbf{a}_{n} = (z_{2}^{*}[n] + z_{1}[n+1]) \mathbf{r}_{n} + (z_{2}^{*}[n+1] - z_{1}[n]) \mathbf{r}_{n+1}.$$
(12)

Using the stochastic gradient descent search method, the update equations for the two space-time equalizers can be given by

$$\mathbf{w}_{1}[n+2] = \mathbf{w}_{1}[n] \\ -\mu \left( 2(z_{1}[n]z_{1}^{*}[n] - \frac{1}{2})z_{1}^{*}[n] - z_{2}[n+1] \right) \mathbf{r}_{n} \\ -\mu \left( 2(z_{1}[n+1]z_{1}^{*}[n+1] - \frac{1}{2})z_{1}^{*}[n+1] + z_{2}[n] \right) \mathbf{r}_{n+1}$$

$$\mathbf{w}_{2}[n+2] = \mathbf{w}_{2}[n] - \mu \left( 2(z_{2}[n]z_{2}^{*}[n] - \frac{1}{2})z_{2}^{*}[n] + z_{1}[n+1] \right) \mathbf{r}_{n} - \mu \left( 2(z_{2}[n+1]z_{2}^{*}[n+1] - \frac{1}{2})z_{2}^{*}[n+1] - z_{1}[n] \right) \mathbf{r}_{n+1}$$
(13)

The derived algorithm is suitable for the spatio-temporal equalization of constant modulus STBC signals. A windowed estimate of the cost function,  $\xi$ , can be used instead of the instantaneous estimate for better convergence. The derived equations can be easily extended to an R-receive antenna configuration for better diversity gains.

# 4. CONCURRENT CM AND DECISION DIRECTED RECEIVER

In this section, an extension of the concurrent equalizer in [5] is derived for use with a  $2 \times 2$  STBC system. The algorithm uses the modified cost function from the previous section in conjunction with a DD equalizer. The idea behind the concurrent equalizer is that the CM part of the coefficient vectors, denoted  $\mathbf{w}_i^{(c)}$ , is updated for every iteration whereas the DD part, denoted  $\mathbf{w}_i^{(d)}$ , is only updated when the previous CMA adaptation is assumed to be successful. The cost function for the DD equalizer can be given by:

$$\xi_{DD} = \mathscr{E}\left\{\sum_{i=1}^{2} \sum_{\tau=0}^{1} |q(z_i[n+\tau]) - z_i[n+\tau]|^2\right\},\tag{14}$$

Table 1: Concurrent CMA and Decision Directed Algorithm.

1: Update 
$$\mathbf{r}_{n}$$
 and  $\mathbf{r}_{n+1}$ .

2: Calculate  $z_{i}[n]$  and  $z_{i}[n+1]$ , for  $i=1,2$ .

3:  $\mathbf{e}_{i}^{(c)} = \frac{\partial}{\partial \mathbf{w}_{i}^{c}} \hat{\xi}_{CM}$ , for  $i=1,2$ .

4:  $\mathbf{w}_{i}^{c}[n+2] = \mathbf{w}_{i}^{c}[n] - \mu_{CM} * \mathbf{e}_{i}^{(c)}$ , for  $i=1,2$ .

5:  $\tilde{\mathbf{w}}_{i}[n] = \mathbf{w}_{i}^{c}[n+2] + \mathbf{w}_{i}^{d}[n]$ , for  $i=1,2$ .

6: Calculate outputs  $\tilde{z}_{i}[n+\tau]$  using  $\tilde{\mathbf{w}}_{i}[n]$ , for  $\tau=0,1$ .

7:  $\mathbf{e}_{i}^{(d)}[n+\tau] = (z_{i}[n+\tau] - q(z_{i}[n+\tau]))^{*} * \mathbf{r}_{n}$ .

8:  $\delta_{i} = \prod_{\tau=0}^{1} \delta(q(z_{i}[n+\tau]) - q(\tilde{z}_{i}[n+\tau]))$ 

9:  $\mathbf{w}_{i}^{(d)}[n+2] = \mathbf{w}_{i}^{(d)}[n] - \mu_{DD} * \delta_{i} * \left(\mathbf{e}_{i}^{(d)}[n] + \mathbf{e}_{i}^{(d)}[n+1]\right)$ 

10:  $\mathbf{w}_{i}[n+2] = \mathbf{w}_{i}^{(c)}[n+2] + \mathbf{w}_{i}^{(d)}[n+2]$ , for  $i=1,2$ .

where the function q(.) represents a decision device that maps its input to the closest transmit constellation point.

The concurrent algorithm can be described by the following steps:

- 1. The outputs  $z_1[n+\tau]$  and  $z_2[n+\tau]$  are calculated for  $\tau=0,1$  as in (6).
- 2. Identical to (13), the Constant Modulus parts of the coefficient vectors are calculated as follows:

$$\begin{split} &\mathbf{w}_{1}^{(c)}[n+2] = \mathbf{w}_{1}^{(c)}[n] \\ &-\mu_{CM} \left( 2(z_{1}[n]z_{1}^{*}[n] - \frac{1}{2})z_{1}^{*}[n] - z_{2}[n+1] \right) \mathbf{r}_{n} \\ &-\mu_{CM} \left( 2(z_{1}[n+1]z_{1}^{*}[n+1] - \frac{1}{2})z_{1}^{*}[n+1] + z_{2}[n] \right) \mathbf{r}_{n+1} \end{split}$$

$$\mathbf{w}_{2}^{(c)}[n+2] = \mathbf{w}_{2}^{(c)}[n] -\mu_{CM} \left( 2(z_{2}[n]z_{2}^{*}[n] - \frac{1}{2})z_{2}^{*}[n] + z_{1}[n+1] \right) \mathbf{r}_{n} -\mu_{CM} \left( 2(z_{2}[n+1]z_{2}^{*}[n+1] - \frac{1}{2})z_{2}^{*}[n+1] - z_{1}[n] \right) \mathbf{r}_{n+1},$$
(15)

where  $\mu_{CM}$  is the CMA step size.

3. In order to evaluate the success of the previous step taken by the CM equalizer, the receiver calculates the intermediate outputs using the new CM vector and the previous DD vector:

$$\begin{bmatrix} \tilde{z}_1[n] & \tilde{z}_1[n+1] \\ \tilde{z}_2[n] & \tilde{z}_2[n+1] \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{w}}_1^H \\ \tilde{\mathbf{w}}_2^H \end{bmatrix} \cdot \begin{bmatrix} \mathbf{r}_n & \mathbf{r}_{n+1} \end{bmatrix}, \quad (16)$$

where

$$\tilde{\mathbf{w}}_i = \mathbf{w}_i^{(c)}[n+2] + \mathbf{w}_i^{(d)}[n].$$
 (17)

4. To avoid taking a large step in the wrong direction a decision must be made on the correctness of the previous CM step. The work in [5] suggests that if the equaliser's hard decisions before and after the CM adaptation are the same then the decision is likely to be correct. Hence, the DD coefficient vector w<sub>i</sub><sup>(d)</sup> should only be updated when the following is equal to 1:

$$\delta_i = \prod_{\tau=0}^{1} \delta\left(q\left(z_i[n+\tau]\right) - q\left(\tilde{z}_i[n+\tau]\right)\right) \tag{18}$$

where

$$\delta(k) = \begin{cases} 1 & k = 0 \\ 0 & k \neq 0 \end{cases}$$
 (19)

A summary of the algorithm is given in Table 1.

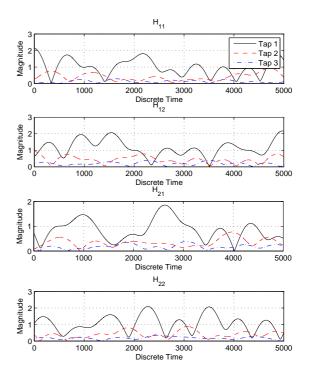


Figure 2: Time-frequency dispersive Rayleigh distributed channel matrix.

# 5. SIMULATION RESULTS

Computer simulation results are presented in this section to provide an insight into the proposed scheme and to demonstrate the performance of the concurrent algorithm. A  $2 \times 2$  MIMO model as indicated in Figure 1 is used in the simulations. QPSK modulation with a modulus equal to unity is performed on the source bits. At the receiver, signals are corrupted by AWGN. The length of the subequalizers is set to L=15. In the following, we first demonstrate the convergence behaviour and the steady state MSE given a fixed SNR value, and thereafter characterise the error performance of the two algorithms with regards to SNR.

#### 5.1 Mean Square Error

The step sizes for the CM and the DD terms are initialized to  $\mu_{CM}=3*10^{-3}$  and  $\mu_{DD}=5*10^{-2}$ . The Signal-to-Noise Ratio (SNR) is set to 20dB. The equalizer coefficient vectors are initialized having only two non zero elements equal to unity at entries 8 and L+8, respectively. The Mean Square Error (MSE) is averaged over 50 channels drawn from a Rayleigh distribution with a given power profile. Figure 3 shows the MSE curves for the STBC-CMA and the concurrent equalizer. It can be clearly observed that the concurrent receiver acheives faster convergence than the standard STBC-CMA, which makes it better in terms of tracking fast varying channels

#### 5.2 Quasi-Stationary Channel

This experiment is designed to evaluate the convergence behaviour of the standard STBC-CMA and the proposed con-

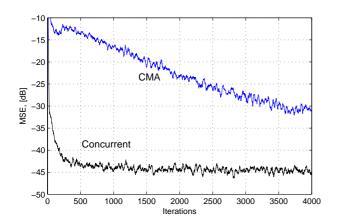


Figure 3: Mean Square Error curves.

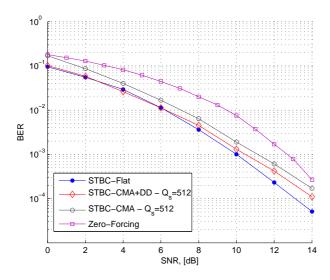


Figure 4: Bit-Error Rate (BER) curves for the quasi-stationary case.

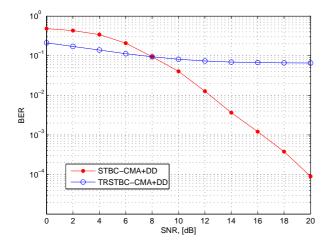


Figure 5: BER curves for the smoothly time-varying case.

current receiver in the presense of noise. The step sizes are initialized to  $\mu_{CM} = 8 * 10^{-4}$  and  $\mu_{DD} = 8 * 10^{-3}$  and the

equalizer vectors  $\mathbf{w}_1$  and  $\mathbf{w}_2$  are intialized in the same way as in the first experiment. The channels are assumed to be stationary over a window of 496 symbols. Channel realisations are obtained through a sample and hold process of the Rayleigh distributions shown in Figure 2. Figure 4 shows the BER curves of the two algorithms in comparison along with that of the full channel information flat fading case proposed by Alamouti, [1]. The concurrent receiver clearly outperforms the STBC-CMA BER-wise. This is achieved at a minimal increase in system complexity as it has been shown that the implementation of the DD equalizer is similar to that of the Least Mean Square (LMS) algorithm. The figure also shows the BER when a Zero-Forcing (ZF) equalizer is used at the receiver. The inverse of the polynomial matrix H can be obtained in frequency domain as shown in [9]. Both the STBC-CMA and the concurrent equalizer achieve a lower BER when the SNR is lower than 14dB due to the noise amplification inherent in the ZF equalizer.

#### 5.3 Smoothly-Varying Rayleigh Channel

In this experiment, we assess the tracking capability of the concurrent receiver. The channels are assumed to be smoothly time-varying and frequency selective. The coefficients are drawn from a Rayleigh process with a vehicular speed of 55km/h. The same transmission and equalisation parameters are used as in experiment 2. The performance of the proposed equaliser is benchmarked against the TRSTBC-CMA scheme in [6]. Figure 5 shows the BER curves over a range of SNR values. The TRSTBC-CMA clearly gives very poor results, which can be attributed to its block-based nature. The proposed algorithm exhibits a coding loss compared to the quasi-stationary case, but still manages to track the channel and achieve a BER of  $10^{-4}$  at SNR = 20dB.

## 6. CONCLUSION

In this paper, a concurrent algorithm was derived for the equalization of STBC over frequency selective channels. The algorithm takes advantage of the robustness of the Constant Modulus Algorithm and the fast convergence of the Decision Directed equalization. It can be divided into two main steps. In the first step, the receiver updates CM parts of the coefficient vector. In the second step, the receiver makes a decision on the success of the first step. If the step is deemed correct then the DD criterion is used to update the second part of the coefficient vectors.

Simulation results showed that the concurrent equalizer acheives considerably faster convergence than the standard STBC-CMA. The new receiver also exhibited better performance in terms of Bit Error Rate over a quasi-stationary channel. This confirmed the convergence behaviour demonstrated by the MSE curves. The tracking capability of the concurrent receiver was also assessed and benchmarked against the block-based TRSTBC-CMA. The proposed receiver was able to blindly track the channel, assuming a vehicular speed of 55km/h.

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