PACKET COMBINING FOR MULTI-LAYER HYBRID-ARQ OVER FREQUENCY-SELECTIVE FADING CHANNELS

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ABSTRACT

For reliable data transmission using single-carrier signaling over multi-path fading channels, multi-layer hybrid automatic repeat request (HARQ) protocols is a promising technique for high data throughput communication systems. In this paper¹, we propose a joint equalization scheme for multilayer packet retransmission over a frequency-selective fading channel. We first derive the equivalent MIMO channel model for multi-layer retransmission. Then, we present the corresponding receiver implementing a successive interference cancellation scheme with integrated joint equalization for each layer. We evaluate, by computer simulations, the frame error rate performance and the throughput performance of a retransmission based multi-layer HARQ protocol using the proposed combining scheme. Simulation results show a significant performance gain in comparison with post-combining methods based on separate detection and packet combining at the bit level.

1. INTRODUCTION

In data packet communication systems using single-carrier signaling over multipath fading channel, multi-layer transmission is an efficient way to improve data throughput if no channel state information (CSI) is available at the transmitter [1]. In multi-layer transmission, multiple coded packets, each of which is referred to as a layer, are simultaneously transmitted over the medium channel using a linear superposition of modulated symbols. Each layer is allocated a different power and transmission rate under the constraint of a fixed total power. The performance of the multi-layer system depends on the efficiency of the receiver in separating the different layers taking into account the channel effects on the transmitted signal. However, for single-carrier signaling over frequency-selective fading channels, the received signal suffers from channel fading and inter-symbol interference (ISI) resulting from multipath propagation. Therefore, advanced signal processing techniques are required at the receiver in order to limit the effect of the ISI on the system performance. In particular, turbo-equalization [2] is an efficient technique that combines signal detection and error correction in an iterative way leading to significant performance gains in comparison with systems using separated signal detection and decoding.

In data communication systems, HARQ protocols [3] are usually used in order to ensure data reliability. In [1], the performances of multi-layered HARQ protocols have been

investigated for a flat fading channel with infinite number of layers assuming perfect channel coding. In practice, only a finite number of layers can be used. In [4], the performance of a multi-level coded QAM modulation was investigated for a frequency-selective channel. As in [4], we consider a multi-layered HARQ system over a frequency-selective fading channel. However, we consider Chase combining retransmission mode where the same packet is retransmitted in case of receiving error. For multi-layer HARQ transmission, each packet has its own cyclic redundancy check (CRC) signature allowing a layer-wise retransmission control. In most previous works, like in [5], multi-layer HARQ was investigated under the incremental redundancy mode using code combining strategies. In the context of Chase combining mode, to the authors' knowledge, no previous work has been addressed the problem of the combining different multilayered signals. Among different combining strategies, input signals combining approaches, such as joint equalization [6], offer in general better performance than output combining at the bit level such as log-likelihood ratios (LLRs) packet combining [7]. In this paper, we present a packet combining scheme for Chase combining multi-layered HARQ protocols over a frequency-selective channel. The main idea in this paper is that performing a joint detection for one layer from multiple signals containing that layer, would enhance the detection performance of other layers, hence the overall system performance. We consider the multi-layer transmission as multiple-input single-output (MISO) transmission system with correlated channels. For multiple multi-layer transmissions, we derive the equivalent multiple-input multipleoutput (MIMO) channel model. We present the structure of the corresponding receiver. We compare the performance of the proposed combining scheme with other methods using separate detection per received signal and performing output packet combining at the bit level.

The remaining of this paper is organized as follows. Section 2 introduces the system model for multi-layer HARQ retransmission. In section 3, we derive the equivalent MIMO channel model for multiple multi-layer transmissions. In Section 4, we present the receiver structure with integrated joint equalization for each layer. In Section 5, we give some simulation results showing a performance comparison of the proposed scheme with post combining strategies. Finally, conclusions are given in Section 6.

2. SYSTEM MODEL

We consider the single-carrier multi-layer transmission system shown in Fig. 1. The information data packets d, including CRC bits for error detection, are distributed over

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K branches which are referred to as layers. For each layer $k=1,\cdots,K$, the information data packet is first encoded by a foreword error correction (FEC) codes giving 2N encoded bits. A different code may be used for each layer, but we assume that the same FEC code is used for all layers. This simplifies the system complexity by using the same encoder and decoder for all layers. The coded bits are then bit-interleaved using a random interleaver Π_k in order to decorrelate the encoded bits. The obtained interleaved bits are mapped into N modulated symbols using Gray-mapped QPSK constellation with unit average power. The modulated symbols are multiplied by complex coefficients $a^{(k)} = \rho_k \exp(j\theta_k)$ where ρ_k is a scaling factor and θ_k is a phase-shift $(\theta_k \in [0, 2\pi))$. The obtained signals in all layers are added together to form the transmitted block \mathbf{x} ,

$$\mathbf{x} = \sum_{k=1}^{K} a^{(k)} \mathbf{s}^{(k)},$$

where s_k and x are complex vectors of dimension N. A cyclic-prefix (CP) of a sufficient length is inserted at the beginning of the transmitted block in order to avoid inter-block interference and to facilitate equalization at the receiver in the frequency domain. The scaling parameter ρ_k determine the allocated power to each layer and can be chosen under the constraint of a unit average transmitted power, i.e. $\sum_{k=1}^{K} \rho_k^2 = 1$. Whereas, the phase-shift angles θ_k determine the form of the combined signal constellation that affects the characteristics of the output signal such as the peak-toaverage power ratio [8]. The control of these parameters is particularly useful for adaptive transmission systems. Their optimization depends on CSI at the transmitter as in [9] and reference therein, but this is out the scope of this paper. The signal is transmitted through a multipath channel which is modeled by its equivalent discrete-time impulse response of length L denoted by $\mathbf{h} = (h(0), ..., h(L-1))$ and assumed constant during one block transmission but changes from one block to another. We assume that the channel tap coefficients are modeled as complex Gaussian random variables with zeros means and variance determined by the delay-power profile of the multipath channel response. According to this model, the received sequence samples corresponding to the t-th transmitted block are given by,

$$r_t(n) = \sum_{i=0}^{L-1} h_t(i) x_t(n-i) + w_t(n), \ n = 1 \cdots N,$$
 (1)

where $w_t(n)$ is an independent additive white Gaussian noise with variance σ_w^2 . The average signal to noise ratio (SNR) is defined by SNR= $E_s/N_0 = E_s/\sigma_w^2$. At the receiver side, we consider a turbo-equalizer for iterative detection and decoding with perfect channel state information detailed in Section 4.

After the decoding of the received signal, an ACK signal is returned to the transmitter for each correctly decoded layer through an error-free channel, whereas a NACK signal is returned for layers in errors. The transmitter responds by resending the same packets of layers in error and send new packets on successfully decoded layers. This operation continues for each layer until the correct reception of transmitted packets or a maximum number of retransmission $F_{\rm max}$ (in addition to the first transmission) has been reached. In this

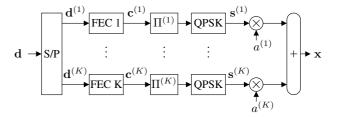


Figure 1: Transmitter scheme for multi-layer packet transmission.

case the packet in error is dropped out from the transmission buffer and an error is declared. In other words, the HARQ retransmission control sees the different layers as independent parallel channels. Of course, other retransmission strategies may be considered depending on the channel model. For example, for slowly time-varying channels where the channel remains the same, the acknowledge signals indicate implicitly a quantized value of the channel gain. In this case, the transmitter can decide to transmit only the layers that match the channel gain with appropriate power levels as explained in [1].

3. EQUIVALENT MIMO CHANNEL MODEL

Thanks to the cyclic-prefix insertion, the convolution in the time-domain in (1) becomes a simple multiplication in the frequency-domain. This allows a frequency-domain equalization of the received signal after removing the cyclic-prefix and applying a discrete Fourier transform (DFT). In the following, frequency-domain variables are denoted by capitals.

One can see the multi-layer transmission as a MISO system with K transmitting antennas where each antenna corresponds to one layer. At the time t, expressed in number of transmitted blocks, the received signal can be written in the form

$$R_t = \begin{bmatrix} a^{(1)}H_t & \cdots & a^{(K)}H_t \end{bmatrix} \begin{bmatrix} S_t^{(1)} \\ \vdots \\ S_t^{(K)} \end{bmatrix} + W_t, \quad (2)$$

where the modulation coefficients $\{a^{(k)}\}$ are considered as a part of the channel. We denote the equivalent channel for the k-th layer by $\tilde{H}_t^{(k)} = a^{(k)}H_t$. The received signal is initially stored in a buffer, and the buffered signal is referred to by the variable \underline{R}_t . According to the considered HARQ protocols, undecoded layers in the previous transmissions are retransmitted in the current transmission as long as the maximum number of retransmission F_{max} was not reached. Let $F_{t,k}$ be the number of retransmission of the k-th layer at the instant t. Naturally, for new transmitted layers $F_{t,k} = 0$ and $S_t^{(k)} = S_{t-i}^{(k)}$ for $i = 0, \dots, F_{t,k}$.

The main idea in this paper is to use the previous $F_{t,k}$ received signals in order to detect the k-th layer. Since $F_{t,k} \le F_{\max}$, the receiving buffer size is chosen to be $F_{\max} + 1$. In the following, we derive the equivalent MIMO channel model between the transmitted layers and the buffered signals. After a successful decoding of one layer, as it will be explained later in this section, the buffer is updated by removing the

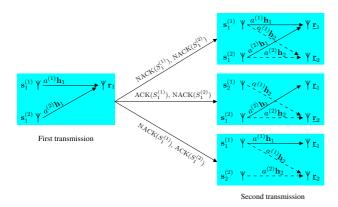


Figure 2: Equivalent MIMO model for multi-layer packet retransmission. Dashed lines denote the new transmitted signal

contribution of the decoded layer from all buffered signals

$$\underline{R}_{t-i} = \underline{R}_{t-i} - \tilde{H}_{t-i}^{(k)} S_t^{(k)}, \ i = 0 \cdots F_{t,k}. \tag{3}$$

In this way, the buffered signals contain the non decoded layers only.

For better understanding, we start by a simple example of two layers K=2 and a maximum number of retransmission $F_{\rm max}=1$. The general case is presented later in this section. After an initial transmission,

$$\underline{R}_1 = R_1 = \tilde{H}_1^{(1)} S_1^{(1)} + \tilde{H}_1^{(2)} S_1^{(2)} + W_1.$$

Assuming that the first layer was correctly decoded, \underline{R}_1 is updated as follows

$$\underline{R}_1 = \underline{R}_1 - \tilde{H}_1^{(1)} S_1^{(1)} = \tilde{H}_1^{(2)} S_1^{(2)} + W_1. \tag{4}$$

The transmitter responds by sending a new packet on the first layer and the same packet on the second layer. The corresponding received signal is

$$\underline{R}_2 = R_2 = \tilde{H}_2^{(1)} S_2^{(1)} + \tilde{H}_2^{(2)} S_2^{(2)} + W_2, \tag{5}$$

with $S_1^{(2)} = S_2^{(2)}$. By combining (4) and (5), the two buffered signals can be viewed as the output of a 2×2 MIMO channel expressed as

$$\left[\begin{array}{c}\underline{R}_1\\\underline{R}_2\end{array}\right] = \left[\begin{array}{cc}0 & \tilde{H}_1^{(2)}\\\tilde{H}_2^{(1)} & \tilde{H}_2^{(2)}\end{array}\right] \left[\begin{array}{c}S_2^{(1)}\\S_2^{(2)}\end{array}\right] + \left[\begin{array}{c}W_1\\W_2\end{array}\right].$$

For other cases, similar results is obtained with a different MIMO channel response for each case as illustrated in Fig. 2.

In the general case, the received buffered signal contains the contribution of all undetected layers including active layers for which $F_{t,k} \leq F_{\max}$, but also layers which have been dropped out after achieving the maximum number of allowable transmissions. The dropped layers are necessarily present in the oldest signals in the buffer, i.e. $\underline{R}_{I-F_{\max}}$. Therefore, the equivalent MIMO model for the buffered signals can be written as

$$\underline{\mathbf{R}}_{t} = \mathbf{H}_{t} \mathbf{S}_{t} + \overline{\mathbf{H}}_{t} \mathbf{S}_{t-F_{\text{max}}} + \mathbf{W}_{t}, \tag{6}$$

where $\mathbf{S}_t = [S_t^{(1)}, \cdots, S_t^{(K)}]^T$, $\mathbf{R}_t = [\underline{R}_t, \cdots, \underline{R}_{t-F_{\max}}]^T$, $\mathbf{W}_t = [W_t, \cdots, W_{t-F_{\max}}]^T$, and \mathbf{H}_t is the equivalent channel response for active layers, and $\overline{\mathbf{H}}_t$ is the equivalent channel response for dropped layers. They are respectively defined by their elements as

$$[\mathbf{H}_t]_{i,k} = \varepsilon_{i,k} \tilde{H}_{t-i+1}^{(k)}, \tag{7}$$

$$[\overline{\mathbf{H}}_t]_{i,k} = \overline{\varepsilon}_{i,k} \tilde{H}_{t-i+1}^{(k)}, \tag{8}$$

for $1 \le i \le F_{\max} + 1$ and $1 \le k \le K$, where $\varepsilon_{i,k}$ takes the value of '1' if the k-th layer is not correctly decoded in the (t-i+1)-th transmission but it is still active, and '0' otherwise. while $\overline{\varepsilon}_{i,k}$ takes the value of '1' if the k-th layer is not correctly decoded in the (t-i+1)-th transmission but it has been dropped, and '0' otherwise. Note that in (6), the dropped layers form an additional source of noise for active layers. Therefore, a care must be taken in the design of the HARQ retransmission protocol in order to limit this effect.

The advantages of the equivalent MIMO model can be resumed by the following points:

- The packet combining method can be easily generalized to MIMO channels.
- It gives some insights in designing the system parameters {ρ_k} and {θ_k} in order to maximize the transmission diversity in the system based on previously known results for MIMO channels.
- The possible use of space-time coding techniques as layer-time coding schemes in order to reduce the interlayer interference.

These points are currently under investigations for future works. Now, having determined the equivalent MIMO channel model for multiple HARQ transmission, we can apply classical methods for MIMO channels in order to separate layers as in [10]. Since the equivalent channel has a particular form with correlated entries, this can simplify the detection problem as shown in Section 4.

4. RECEIVER STRUCTURE

The receiver structure is shown in Fig. 3. For each received blocks, the cyclic-prefix is first removed and a DFT is applied in order to facilitate equalization. The obtained signal is stored in the receiving buffer. Successive interference cancellation is used in order to separate different layers. In the successive interference cancellation receiver, the detection of different layers is performed in the descending order of the received powers. The total received power for the *k*-th layer is the sum of squared amplitudes of the elements of the *k*-th column of the equivalent MIMO channel, and it is given by

$$P_t^{(k)} = \sum_{i=0}^{F_{t,k}} \rho_k^2 \gamma_{t-i},$$

where $\gamma_{t-i} = ||\mathbf{h}_{t-i}||^2$. Let $\{k_1, k_2, \dots, k_K\}$ be the indices of layers in the descending order of $P_t^{(k)}$. The SIC receiver performs the detection starting from the k_1 -th layer up to the k_K -th layer. In the following, we describe the different constituent blocks.

4.1 Interference canceller

Prior to the detection of each layer, a soft interference cancellation is performed using the soft estimates of the coded

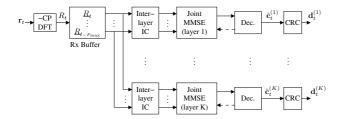


Figure 3: Receiver structure for packet combining of multilayer HARQ transmissions using successive interference cancellation and joint equalization.

bits $\hat{\mathbf{e}}^{(k)}$ from other decoded layers. The soft estimates are interleaved and soft mapped into a sequence of estimated symbols to $\bar{\mathbf{s}}_t^{(k)}$. After converting this sequence to the frequency-domain, the soft interference cancellation is performed by

$$Z_{t-i} = \underline{R}_{t-i} - a^{(k)} H_t \bar{S}_t^{(k)}, i = 0, \dots, F_{t,k}.$$

The obtained signals are then processed by a joint equalizer in order to estimate the current layer.

4.2 Joint equalizer

In this section we present the joint equalizer with *a priori* for possible use of iterative processing in the purpose of achieving the desired performance-complexity trade-off as it will be seen in simulation results.

For the k-th layer, the joint equalizer estimates the transmitted symbols $S^{(k)}$ form the $F_{t,k}+1$ signals Z_{t-i} containing that layer after the soft interference cancellation. The joint equalizer is implemented in the frequency-domain based on the MMSE criterion. To simplify our notations, we denote $F=F_{t,k}+1$ and we refer to the input signal by Z_f for $f=1,\cdots,F$ and the layer index k is omitted. The joint MMSE equalizer includes multiple forward linear filters A_f and a backward filter B. According to this structure, the linear estimate \hat{S} of S is given by

$$\hat{S} = \sum_{f=1}^{F} A_f Z_f - B\bar{S},$$

Following the same analysis as in [11, 12], the derivation of the MMSE filters assuming perfect cancellation of other layers leads to the following solution

$$A_f = rac{(ilde{H}_f)^*}{\sigma_w^2 + v \sum_{f=1}^F | ilde{H}_f|^2}, \ B = \sum_{f=1}^F A_f ilde{H}_f - \mu,$$

where $v=\frac{1}{N}\sum_{n=0}^{N-1}\mathrm{var}(\bar{s}(n)), \ \mu=\frac{1}{N}\sum_{n=0}^{N-1}\frac{H_F^2(n)}{\sigma_w^2+\nu H_F^2(n)}, \ \mathrm{and}\ H_F$ is the compound channel defined by its squared value $H_F^2=\sum_{f=1}^F|\tilde{H}_f|^2$ and v is the reliability of the decoder feedback, where v=0 indicates a perfect feedback, and v=1 for no a priori. The output of the MMSE estimator can be written in the time domain after the IDFT using the Gaussian model for the estimated symbols as

$$\hat{s}(n) = \mu s(n) + \eta(n),$$

where η is a complex Gaussian noise with zero mean and variance $\sigma_{\eta}^2 = \mu(1 - \nu \mu)$. This allows computing the soft estimates of the encoded bits as in [13]. For separate detection and decoding, the equalizer's soft input can be put to zero $(\nu = 1)$. For an iterative processing, the decoder's output is interleaved and returned to the equalizer to be used in the next iteration.

After channel decoding and in the case of a correct decoding of the k-the layer, the buffered signals are updated by removing the contribution of this layer from $\{\underline{R}_{t-i}\}_{i=0}^{F_{t,k}}$. To this end, a replica signal $S_t^{(k)}$ is generated from the decoded information packet \mathbf{d}^k and then the buffered signals are updated using (3). In the case of a decoding failure, the soft estimates of the packet's bits are used by the IC blocks of other layers.

The complexity of the joint MMSE equalizer in the frequency-domain is almost the same as for an MMSE equalizer with a single input. To show that, we note that the numerator of each forward filter is the matched filter to the channel that does not change with turbo-iterations. Hence, it is performed once per transmission. The denominator is common between all forward filters and the division can be performed after summation of the matched filters' outputs. Consequently, for each new reception, the accumulated sum of the matched filters is updated and so the squared compound channel. Other operations are the same as for the equalizer with a single input.

5. SIMULATION RESULTS

We present some computer simulations showing the performance of the proposed combining technique. We consider a multi-layer transmission with two modulation layers K=2, and a maximum of allowable retransmissions $F_{\rm max}=2$. The channel code is rate-1/2 convolutional code with the generator polynomial (133,171) in octal notations. We consider a static power allocation scheme with $\rho_1^2=0.8$ and $\rho_2^2=0.2$. The relative power ratio between the two layers is about 6 dB. The phase-shift are both zeros $\theta_1=\theta_2=0$. This corresponds to a partitioned 16-QAM modulation with two levels. Each packet contains 2N=1024 bits leading to N=512 QPSK symbols. We consider a frequency-selective channel of length L=5 with uniform power profile and the channel changes independently from one transmission to the next.

First we consider a non-iterative receiver where the detection and the decoding operation is performed only once for each layer at each block reception. We evaluate the frame error rate performance (FER) versus the SNR for the proposed combining scheme and we compare the obtained performance with the performance of the LLR combining. In LLR combining scheme, the SIC receiver has a single input which is the current received signal. For successive decoding, it uses the stored LLRs values of the non detected layers from previous receptions. The output of the decoder for one layer is accumulated with previous outputs for the same layer and this is the sum which is used by the IC for other layers. In Fig. 4, shows the obtained performance for both combining schemes. For comparison purpose, the performance of the bit-interleaved coded modulation scheme (BICM) using a Gray mapped 16-QAM modulation with joint equalization are shown on the same figure. First, we can see clearly the advantage of the joint equalization combining scheme for

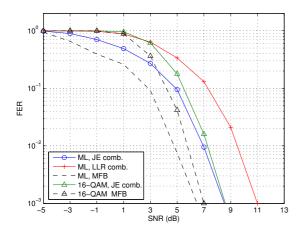


Figure 4: FER performance of the proposed packet combining scheme.

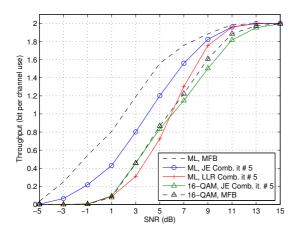


Figure 5: Throughput performance of the proposed packet combining scheme.

multi-layer retransmission over the LLR combining method. About 2.5 dB of gain is obtained at FER=10⁻². In addition, we can see the superiority of the multi-layer HARQ transmission scheme in comparison with BICM scheme, especially for low SNR values.

Now we consider an iterative receiver where the SIC is iterated many times in order to enhance the detection performance at the expense of a higher complexity. We have performed five turbo-iterations for both combining schemes. Fig. 5 shows the corresponding throughput performance. We note that the BICM scheme achieve its matched filter bound after five iterations with similar performance to the LLR combining scheme, but the performance of the multi-layer scheme are still better with an iterative receiver.

6. CONCLUSIONS

We presented in this paper a packet combining scheme for multi-layer transmissions by performing joint equalization for each layer within a successive interference canceller. The equivalent MIMO channel model for a multi-layer HARQ retransmission scheme is derived. The structure of the successive interference cancellation with joint equalization detection scheme is presented. Simulation results show that a significant SNR gain is obtained using the proposed combining scheme compared to the LLR combining method for both a linear and an iterative receiver.

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