TAP AND TRANSMIT ANTENNA CORRELATION BASED PRECODING FOR MIMO-OFDM SYSTEMS

Y. Lebrun[1,2], *C. Hofbauer*[3], *V. Ramon*[2], *A. Bourdoux*[2], *M. Huemer*[3], *F. Horlin*[4] *and R. Lauwereins*[1,2]

[1]Katholieke Universiteit Leuven
Kasteelpark Arenberg 10
B-3001 Leuven, Belgium
web: www.kuleuven.ac.be

[2]IMEC
Kapeldreef 75
3001 Leuven, Belgium
Email: lebruny@imec.be
web: www.imec.be

[3]ES&S
Embedded Systems & Signal Processing Klagenfurt University
9020 Klagenfurt, Austria

[4]ULB
Université Libre de Bruxelles
Kapeldreef 75
3001 Leuven, Belgium
web: www.ulb.ac.be

ABSTRACT

Spatial Multiplexing (SM) is an effective means for enhancing the transmission data rate in Multiple-Input Multiple-Output (MIMO) systems, particularly when used in combination with precoding. Precoding based on the knowledge of the slow-varying statistics of the channel exploits this information at the transmitter to combat the effect of fading in SM systems. Moreover, this technique requires little feedback overhead and can therefore be used in fast fading environments. In this paper, we consider a MIMO-Orthogonal Frequency Division Multiplexing (OFDM) system. We exploit the knowledge of the tap correlation and the transmit antenna correlation present in the frequency-selective channel to propose a low-complexity precoding scheme that minimizes the sum of the symbol Mean Square Errors (MSEs) over all subcarriers. Simulations show that exploiting the knowledge of both the tap and the spatial correlation improves the performance of the MIMO-OFDM system.

1. INTRODUCTION

The presence of multiple antennas at both the transmitter and the receiver in communication systems can provide a considerable gain in terms of capacity, coverage and link reliability. Thanks to these benefits, Multiple-Input Multiple-Output (MIMO) techniques have become popular in emerging wireless standards (for example the 3GPP LTE [1]). Whereas any type of modulation scheme can be used with MIMO systems, the combination of multiple antennas techniques with the Orthogonal Frequency Division Multiplexing (OFDM) modulation scheme is seen as the preferred choice for the next generation of wireless standards. The OFDM scheme divides a frequency-selective channel into multiple narrow-band subchannels and enables low-complexity equalization.

While MIMO techniques already improve the performance when the receiver alone knows the channel, the achievable gain can be further enhanced when the transmitter has knowledge of the channel [2]. If only the statistical knowledge of the channel is available, precoding exploits these slow-varying properties (e.g. the channel covariance matrix) to perform signal shaping before transmission [3], [4].

Precoding is particularly attractive in the presence of correlation between the antennas (spatial correlation) or correlation between the taps of a frequency-selective channel. Indeed, it is then used to alleviate the performance reduction of a Spatial Multiplexing (SM) system due to these correlations.

Whereas the presence of spatial correlation is a well known phenomenon, tap correlation has been justified in the literature under some propagation scenarios [5], and is also known to be caused by the pulse shaping filters at the transmitter and the receiver [6], [7]. Following these assumptions, Yoon and al. [8] proposed a precoding scheme that exploits the knowledge of both the correlation between the transmit antennas and the correlation between the channel taps to improve the capacity of a MIMO-OFDM system.

In this paper, we develop a precoder that exploits the knowledge of these correlations to minimize the sum of the symbol Mean Square Errors (MSEs) over all subcarriers of a MIMO-OFDM system.

Following the assumptions made in [6], [7] and [8], we consider that the tap and the spatial correlations have independent effects and can therefore be separated. We then derive the form of our precoding matrix based on the Jensen's inequality and the derivations made in [3].

The outline of this paper is as follows. In section 2, we introduce the channel model (including both spatial and tap correlations) and the received signal model. In section 3, we provide the derivations of our precoding matrix. Simulations in section 4 present the performance of our precoder and section 5 concludes our work.

The following notations are used in this paper. The vectors and matrices are in boldface letters, vectors are denoted by lower-case and matrices by capital letters. The superscripts $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian transpose operators, respectively. $E[\cdot]_H$ is the expectation operator over H, $\mathbf{I}_N$ is an identity matrix of size $(N \times N)$ and $[\mathbf{A}]_{ij}$ denotes the (i,j) element of the matrix $\mathbf{A}$. $\mathbb{C}^{N\times 1}$ denotes the set of complex vectors of size $(N\times 1)$, $\mathbb{C}^{N\times M}$ denotes the set of complex matrices of size $(N\times M)$ and $\mathbf{x} \sim CN(\mathbf{0},\mathbf{R})$ is the vector of zero-mean Gaussian distributed complex elements with covariance matrix $\mathbf{R}$.

2. SYSTEM MODEL

We consider a MIMO-OFDM communication system with K subcarriers, N_t transmit antennas and N_r receive antennas. We define N_s as the number of transmit streams, where $N_s \leq \min(N_r, N_t)$. In this section, we express the channel model in both the time domain (section 2.1) and the frequency domain (section 2.2) and give the per-subcarrier representation of the signal model (section 2.3).

2.1 Channel Model in the time domain

The L channel taps (frequency-selective) MIMO channel is described in a matrix form as

$$\mathbf{H}_{total} = [\mathbf{H}_0 \dots \mathbf{H}_{L-1}] \quad (1)$$

where $\mathbf{H}_l \in \mathbb{C}^{N_r \times N_t}$ represents the MIMO channel at the channel tap l and $\mathbf{H}_{total} \in \mathbb{C}^{N_r \times N_t L}$ denotes the whole channel matrix. We consider the presence of correlation between antennas at the transmitter whereas receive antennas are assumed to be decorrelated. Physically, this corresponds for example to a transmitter situated on a high location with waves coming from a small angular spread while the receiver is located in a rich scattering environment [9].

As introduced above, we assume the presence of correlation between the channel taps. We consider in this paper that the tap correlation and the spatial correlation have independent effects, as justified in [6], [7] and [8], and can therefore be separated. Following this assumption a particular representation of the channel model in the time domain can be expressed as mentionned in those papers:

$$\mathbf{H}_{total} = \mathbf{H}_w^E \left(\mathbf{R}_{mp}^{T/2} \otimes \mathbf{R}_{ta}^{1/2} \right) \tag{2}$$

where $\otimes$ is the Kronecker product, and

- $\mathbf{H}_w^E$ denotes an $N_r \times N_t L$ white Zero-Mean Circularly Symmetric Complex Gaussian (ZMCSCG) channel matrix with Independent and Identically Distributed (i.i.d.) elements of unit variance (Rayleigh fading).
- $\mathbf{R}_{ta}^{1/2}$ is an $N_t \times N_t$ matrix that represents the Cholesky matrix of the transmit antenna correlation matrix $\mathbf{R}_{ta}$. It is defined as

$$\mathbf{R}_{ta} = \left(\mathbf{R}_{ta}^{1/2} \right)^H \mathbf{R}_{ta}^{1/2}. \tag{3}$$

  The entries of $\mathbf{R}_{ta}$ are determined by the transmit antenna spacing and the angular spread. We model $\mathbf{R}_{ta}$ by an exponential correlation model $[\mathbf{R}_{ta}]_{ij} = \rho_{ta}^{|i-j|}$, where $\rho_{ta} = 0$ means no correlation and $\rho_{ta} = 1$ means full correlation. As we assume no receive antenna correlation, the columns $\mathbf{h}_1 \ldots \mathbf{h}_{N_r}$ of $\mathbf{H}_l^H$ can be assumed to be i.i.d. $\mathbf{h}_j \sim \mathbb{C}\mathcal{N}(\mathbf{0}, \mathbf{R}_{ta})$.
- $\mathbf{R}_{mp}$ denotes the $L \times L$ taps correlation matrix and is defined as

$$[\mathbf{R}_{mp}]_{l_1,l_2} = \sqrt{\gamma_{l_1}} \cdot \sqrt{\gamma_{l_2}} \cdot \rho_{l_1,l_2}^{mp} \tag{4}$$

  where γ_{l_i} is the average link power of the l_i^{th} tap, under the normalization constraint $\sum_{l_i=0}^{L-1} \gamma_{l_i} = 1$ and ρ_{l_1,l_2}^{mp} denotes the correlation factor between the taps l_1 and l_2, with its magnitude satisfying $0 \leq |\rho_{l_1,l_2}^{mp}| \leq 1$.

2.2 Channel model in the frequency domain

Here, we give an equivalent per-subcarrier representation of the channel model. Based on the time domain channel model (2), we express the frequency representation of the channel for the k^{th} subcarrier ($0 \leq k \leq K-1$), as introduced in [8]

$$\mathbf{H}_k = \mathbf{H}_w^E \left(\mathbf{R}_{mp}^{T/2} \mathbf{w}_k \otimes \mathbf{R}_{ta}^{1/2} \right) \tag{5}$$

where $\mathbf{w}_k$ denotes the discrete Fourier transform vector for the k^{th} subcarrier, given as $\mathbf{w}_k = [e^{-j\frac{2\pi}{K}k\times 0} \ldots e^{-j\frac{2\pi}{K}k\times(L-1)}]^T$.

2.3 Signal Model

We consider a MIMO-OFDM system as shown in the block diagram in Fig. 1. At the channel output, the received vector for the k^{th} subcarrier is denoted by $\mathbf{y}_k \in \mathbb{C}^{N_r \times 1}$ and can be expressed as

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{F}_k \mathbf{x}_k + \mathbf{n}_k \tag{6}$$

where $\mathbf{x}_k \in \mathbb{C}^{N_s \times 1}$ denotes the transmit symbol vector, $\mathbf{F}_k \in \mathbb{C}^{N_t \times N_s}$ is the precoding matrix and $\mathbf{n}_k \in \mathbb{C}^{N_r \times 1}$ is the zero-mean circularly symmetric complex additive white Gaussian noise with unilateral power spectral density noise N_0. At the receiver, after suppression of the cyclic prefix, serial-to-parallel conversion and Fast Fourier Transform (FFT), the samples are processed by a Minimum Mean Square Error (MMSE) equalizer.

3. DERIVATION OF THE PRECODING MATRIX

In this section, we derive a per-subcarrier based precoder that minimizes the sum of the symbol MSEs over all subcarriers for a MIMO-OFDM system. It is based on the knowledge of both the tap and the spatial correlation matrices. We first express the optimization problem for the MSE criterion (section 3.1). Then, we simplify the objective function based on our channel model (section 3.2) and reformulate it using the Jensen's inequality in section 3.3. Finally, we give the form of the precoding matrix (section 3.4).

3.1 Definition of the Optimization Problem

The problem consists in finding the set of matrices $\mathbf{F}_k$ that minimizes the sum of the symbol MSEs over all subcarriers, i.e.

$$\text{MSE} = \min_{\hat{\mathbf{x}}_k} \sum_{k=0}^{K-1} E_H \left[tr\left((\hat{\mathbf{x}}_k - \mathbf{x}_k)(\hat{\mathbf{x}}_k - \mathbf{x}_k)^H \right) \right] \tag{7}$$

$$= \min_{\mathbf{F}_k} \sum_{k=0}^{K-1} E_H \left[tr\left((\mathbf{I}_{N_s} + \xi_0 \mathbf{F}_k^H \mathbf{H}_k^H \mathbf{H}_k \mathbf{F}_k)^{-1} \right) \right] \tag{8}$$

and subject to the power constraint

$$\sum_{k=0}^{K-1} tr(\mathbf{F}_k \mathbf{F}_k^H) \leq P_{total}. \tag{9}$$

We denote by $\hat{\mathbf{x}}_k$ the output of the linear MMSE equalizer and assume $E\left[\mathbf{x}_k \mathbf{x}_k^H\right] = \mathbf{I}_{N_s}$ and $\xi_0 \triangleq 1/N_0$.

3.2 Simplification of the Objective Function

Using the Singular Value Decomposition (SVD) of the vector $\mathbf{R}_{mp}^{T/2} \mathbf{w}_k$ and the matrix $\mathbf{R}_{ta}^{1/2}$, we obtain

$$\mathbf{R}_{mp}^{T/2} \mathbf{w}_k = \mathbf{U}_k \mathbf{\Lambda}_k \mathbf{v}_k^H \tag{10}$$

$$\mathbf{R}_{ta}^{1/2} = \mathbf{U}_{ta} \mathbf{\Lambda}_{ta}^{1/2} \mathbf{V}_{ta}^H. \tag{11}$$

Since the product of $\mathbf{R}_{mp}^{T/2} \mathbf{w}_k$ results in a vector, we have $\mathbf{v}_k^H = 1$ (unitary matrix of dimension 1×1). We can thus express $\mathbf{H}_k$ as

$$\mathbf{H}_k = \mathbf{H}_w^E \left(\mathbf{U}_k \mathbf{\Lambda}_k \otimes \mathbf{U}_{ta} \mathbf{\Lambda}_{ta} \mathbf{V}_{ta}^H \right). \tag{12}$$

Then, using the following properties of the Kronecker product

$$\mathbf{a} \otimes \mathbf{BC} = (\mathbf{a} \otimes \mathbf{B}) \mathbf{C} \tag{13}$$

$$\mathbf{AB} \otimes \mathbf{CD} = (\mathbf{A} \otimes \mathbf{C})(\mathbf{B} \otimes \mathbf{D}) \tag{14}$$

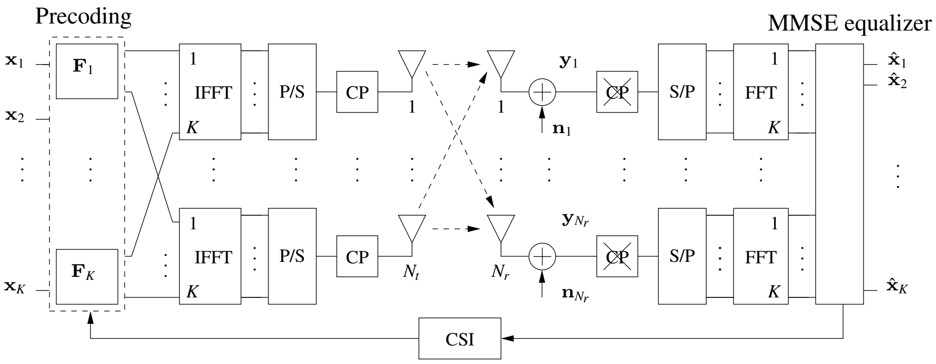

Figure 1: MIMO-OFDM with precoding

equation (12) becomes

$$\mathbf{H}_k = \mathbf{H}_w^E(\mathbf{U}_k \otimes \mathbf{U}_{ta})(\mathbf{\Lambda}_k \otimes \mathbf{\Lambda}_{ta})\mathbf{V}_{ta}^H. \tag{15}$$

The Kronecker product of two unitary matrices leads to a unitary matrix and the properties of a ZMCSCG matrix ($\mathbf{H}_w^E$) do not change when multiplied by a unitary matrix. Thus, we have

$$\mathbf{H}_k \sim \mathbf{H}_w^E\left(\mathbf{\Lambda}_k \otimes \mathbf{\Lambda}_{ta}\right)\mathbf{V}_{ta}^H \tag{16}$$

where $\mathbf{A} \sim \mathbf{B}$ indicates that $\mathbf{B}$ has the same distribution as $\mathbf{A}$. Also, since the product $\mathbf{R}_{mp}^{T/2}\mathbf{w}_k$ is a vector, it has a single non-zero eigenvalue denoted by $[\mathbf{\Lambda}_k]_{1,1}$. We can then write

$$\begin{aligned}\mathbf{H}_w^E(\mathbf{\Lambda}_k \otimes \mathbf{\Lambda}_{ta}) =& [\mathbf{\Lambda}_k]_{1,1} \cdot \mathbf{H}_w \cdot \mathbf{\Lambda}_{ta} \\ =& \sqrt{g_k} \cdot \mathbf{H}_w \cdot \mathbf{\Lambda}_{ta}\end{aligned} \tag{17}$$

with $\sqrt{g_k} \triangleq [\mathbf{\Lambda}_k]_{1,1}$ and where $\mathbf{H}_w$ is an $N_r \times N_t$ matrix containing the leftmost N_t columns of $\mathbf{H}_w^E$. Substituting (17) into (16) we obtain

$$\begin{aligned}\mathbf{H}_k &\sim \sqrt{g_k} \cdot \mathbf{H}_w \cdot \mathbf{\Lambda}_{ta} \cdot \mathbf{V}_{ta}^H & (18)\\ &\sim \sqrt{g_k} \cdot \mathbf{H}_w \cdot \mathbf{R}_{ta}^{1/2}. & (19)\end{aligned}$$

Following the derivations above, we then include the statistical behavior of the channel given in (19) into the MSE expression (8). We obtain

$$\begin{aligned}\text{MSE} =& \min_{\mathbf{F}_k} \sum_{k=0}^{K-1} E_H\left[tr\left(\left(\mathbf{I}_{N_s}\right.\right.\right.\\ &\left.\left.\left. + g_k\xi_0\mathbf{F}_k^H\mathbf{R}_{ta}^{H/2}\mathbf{H}_w^H\mathbf{H}_w\mathbf{R}_{ta}^{1/2}\mathbf{F}_k\right)^{-1}\right)\right].\end{aligned} \tag{20}$$

Next, we replace $\mathbf{R}_{ta}^{1/2}$ by its SVD (11) into (20) and get

$$\begin{aligned}\text{MSE} =& \min_{\mathbf{F}_k} \sum_{k=0}^{K-1} E_H\left[tr\left(\left(\mathbf{I}_{N_s} + g_k\xi_0\right.\right.\right.\\ &\left.\left.\left. \times \mathbf{F}_k^H\mathbf{V}_{ta}\mathbf{\Lambda}_{ta}^{1/2}\mathbf{U}_{ta}^H\mathbf{H}_w^H\mathbf{H}_w\mathbf{U}_{ta}\mathbf{\Lambda}_{ta}^{1/2}\mathbf{V}_{ta}^H\mathbf{F}_k\right)^{-1}\right)\right].\end{aligned} \tag{21}$$

3.3 Reformulation of the Objective Function

Since the properties of a ZMCSCG matrix do not change when multiplied by a unitary matrix, we have $\mathbf{H}_w\mathbf{U}_{ta} \sim \mathbf{H}_w$. Then, we use the Jensen's inequality to move the expectation operator inside the trace operator of (21). Since the function $f(\mathbf{X}) = tr(\mathbf{X}^{-1})$ is a convex function, we get an upper bound on the expression of the MSE:

$$\begin{aligned}\text{MSE} \geq& \min_{\mathbf{F}_k} \sum_{k=0}^{K-1} tr\left(\left(\mathbf{I}_{N_s} + g_k\xi_0\right.\right.\\ &\left.\left. \times \mathbf{F}_k^H\mathbf{V}_{ta}\mathbf{\Lambda}_{ta}^{1/2}\underbrace{E_H\left[\mathbf{H}_w^H\mathbf{H}_w\right]}_{\mathbf{I}_{N_t}}\mathbf{\Lambda}_{ta}^{1/2}\mathbf{V}_{ta}^H\mathbf{F}_k\right)^{-1}\right).\end{aligned} \tag{22}$$

Using the following property of the trace

$$tr\left((\mathbf{I}_N + \mathbf{AB})^{-1}\right) = tr\left((\mathbf{I}_M + \mathbf{BA})^{-1}\right) + C \tag{23}$$

where $\mathbf{A}$ is of size $(N \times M)$, $\mathbf{B}$ is of size $(M \times N)$ and $C = M - N$. In our case, C can be omitted because it does not depend on $\mathbf{F}_k$ and thus, does not affect the optimization problem. We can then write the expression of the trace in (22) as

$$tr\left(\left(\mathbf{I}_{N_t} + g_k\xi_0\mathbf{\Lambda}_{ta}^{1/2}\mathbf{V}_{ta}^H\mathbf{F}_k\mathbf{F}_k^H\mathbf{V}_{ta}\mathbf{\Lambda}_{ta}^{1/2}\right)^{-1}\right). \tag{24}$$

Defining

$$\mathbf{B}_k = \mathbf{\Lambda}_{ta}^{1/2}\mathbf{V}_{ta}^H\mathbf{F}_k\mathbf{F}_k^H\mathbf{V}_{ta}\mathbf{\Lambda}_{ta}^{1/2} \tag{25}$$

we obtain

$$\text{MSE} \geq \min_{\mathbf{B}_k} \sum_{k=0}^{K-1} tr\left((\mathbf{I}_{N_t} + g_k\xi_0\mathbf{B}_k)^{-1}\right). \tag{26}$$

We then perform the eigenvalue decomposition of $\mathbf{B}_k$

$$\mathbf{B}_k = \mathbf{U}_{B_k}\mathbf{\Lambda}_{B_k}\mathbf{U}_{B_k}^H \tag{27}$$

where $\mathbf{U}_{B_k}$ and $\mathbf{\Lambda}_{B_k}$ are square matrices and have same dimensions. Replacing it into (26), we get

$$\begin{aligned}\text{MSE} &\geq \min_{\mathbf{\Lambda}_{B_k}} \sum_{k=0}^{K-1} tr\left((\mathbf{I}_{N_t} + g_k\xi_0\mathbf{U}_{B_k}\mathbf{\Lambda}_{B_k}\mathbf{U}_{B_k}^H)^{-1}\right) & (28)\\ &\geq \min_{\mathbf{\Lambda}_{B_k}} \sum_{k=0}^{K-1} tr\left((\mathbf{I}_{N_t} + g_k\xi_0\mathbf{\Lambda}_{B_k})^{-1}\right). & (29)\end{aligned}$$

Equation (29) means that looking for a matrix $\mathbf{F}_k$ which leads to a matrix $\mathbf{B}_k$ that is diagonal, does not make the optimization problem suboptimal. Nevertheless, we still have to examine now, how the choice of a diagonal matrix impacts the transmit power constraint. Using the following cyclic property of the trace operator: $tr(\mathbf{AB}) = tr(\mathbf{BA})$, where the product of the matrices $\mathbf{A}$ and $\mathbf{B}$ is a square matrix, we can write the transmit power constraint as

$$\sum_{k=0}^{K-1} tr(\mathbf{F}_k\mathbf{F}_k^H) = \sum_{k=0}^{K-1} tr(\underbrace{\mathbf{V}_{ta}\mathbf{V}_{ta}^H}_{\mathbf{I}_{N_t}}\mathbf{F}_k\mathbf{F}_k^H) \quad (30)$$

$$= \sum_{k=0}^{K-1} tr(\mathbf{V}_{ta}^H\mathbf{F}_k\mathbf{F}_k^H\mathbf{V}_{ta}). \quad (31)$$

We can then equivalently write (31) as

$$\sum_{k=0}^{K-1} tr(\mathbf{\Lambda}_{ta}^{-1/2}\underbrace{\mathbf{\Lambda}_{ta}^{1/2}\mathbf{V}_{ta}^H\mathbf{F}_k\mathbf{F}_k^H\mathbf{V}_{ta}\mathbf{\Lambda}_{ta}^{1/2}}_{\mathbf{B}_k}\mathbf{\Lambda}_{ta}^{-1/2}) \quad (32)$$

$$= \sum_{k=0}^{K-1} tr(\mathbf{\Lambda}_{ta}^{-1/2}\mathbf{B}_k\mathbf{\Lambda}_{ta}^{-1/2}) = \sum_{k=0}^{K-1} tr(\mathbf{\Lambda}_{ta}^{-1}\mathbf{B}_k) \leq P_{total}. \quad (33)$$

Because it can be proved using the majorization theory [3] that

$$\sum_{k=0}^{K-1} tr(\mathbf{\Lambda}_{ta}^{-1}\mathbf{\Lambda}_{B_k}) \leq \sum_{k=0}^{K-1} tr(\mathbf{\Lambda}_{ta}^{-1}\mathbf{B}_k), \quad (34)$$

we also have

$$\sum_{k=0}^{K-1} tr(\mathbf{\Lambda}_{ta}^{-1}\mathbf{\Lambda}_{B_k}) \leq P_{total} \quad (35)$$

which means that a diagonal matrix Λ_{B_k} does not relax the power constraint with respect to a generic matrix $\mathbf{B}_k$. The optimization problem is now expressed by (29) together with the transmit power constraint in (35) and where the matrix $\mathbf{\Lambda}_{B_k}$ has been defined in (25) and (27).

3.4 Solution of the Optimization Problem

We have shown above that a precoding matrix $\mathbf{F}_k$ that diagonalizes the MSE expression is a solution to the optimization problem. Actually, the matrix $\mathbf{F}_k$ must have the form:

$$\mathbf{F}_k = \mathbf{V}_{ta}\mathbf{\Lambda}_{F_k} \quad (36)$$

where $\mathbf{V}_{ta}$ denotes the eigenvectors of the transmit antennas correlation matrix, these eigenvectors indicate the statistically preferred directions for transmission, and $\mathbf{\Lambda}_{F_k}$ is a diagonal matrix that allocates power across the transmit streams. Such a result has been obtained for flat fading channels in [4]. It can be easily extended to the OFDM case, with no dependence of matrix $\mathbf{V}_{ta}$ with respect to k, because the correlation between the transmit antennas, $\mathbf{R}_{ta} = \mathbf{V}_{ta}\mathbf{\Lambda}_{ta}\mathbf{V}_{ta}^H$ (combine (3) and (11)), is seen as constant over all subcarriers in our channel model. A similar result with OFDM has been obtained in [8] for the capacity criterion. Looking at definition (25) of $\mathbf{B}_k$, it can be verified that $\mathbf{F}_k = \mathbf{V}_{ta}\mathbf{\Lambda}_{F_k}$

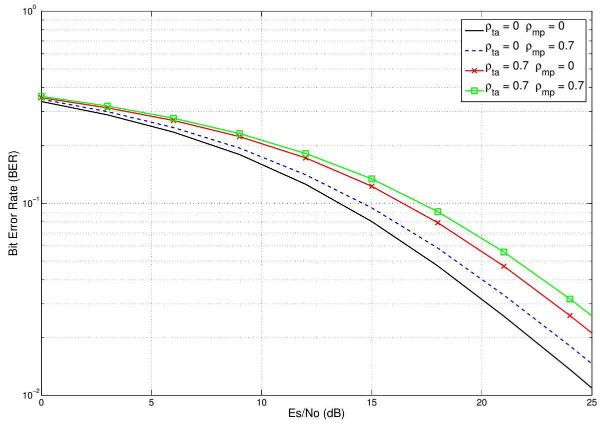

Figure 2: BER for a non-precoded MIMO-OFDM system ($K = 64$, $L = 4$, $N_s = 2$ and $N_r = N_t = 2$) for different values of correlation between the channel taps and between the transmit antennas. At 20 dB, a tap correlation factor of 0.7 introduces a 1.2 dB loss. An antenna correlation of 0.7 brings a loss of 3 dB and the presence of both tap and transmit antenna correlations ($\rho_{ta} = \rho_{l_i,l_j}^{mp} = 0.7$) bring a loss of 4 dB.

makes $\mathbf{B}_k$ diagonal. Based on this result, the problem becomes

$$\text{MSE} \geq \min_{\mathbf{\Lambda}_{F_k}} \sum_{k=0}^{K-1} tr[(\mathbf{I}_{N_t} + g_k\xi_0 \times \mathbf{\Lambda}_{ta}^{1/2}\mathbf{V}_{ta}^H\mathbf{V}_{ta}\mathbf{\Lambda}_{F_k}\mathbf{\Lambda}_{F_k}\mathbf{V}_{ta}^H\mathbf{V}_{ta}\mathbf{\Lambda}_{ta}^{1/2})^{-1}] \quad (37)$$

$$\geq \min_{\mathbf{\Lambda}_{F_k}} \sum_{k=0}^{K-1} tr\left[\left(\mathbf{I}_{N_t} + g_k\xi_0\mathbf{\Lambda}_{ta}^{1/2}\mathbf{\Lambda}_{F_k}\mathbf{\Lambda}_{F_k}\mathbf{\Lambda}_{ta}^{1/2}\right)^{-1}\right] \quad (38)$$

where the diagonal elements of the matrix $\mathbf{\Lambda}_{F_k}$ are computed by a waterfilling algorithm based on the statistical knowledge of the channel. Its inputs are the number of transmit streams (N_s), the maximum power (P_{total}) and the eigenvalues of the matrices

$$\mathbf{M}_k = g_k\xi_0\mathbf{\Lambda}_{ta} \quad \text{for } 0 \leq k \leq K-1 \quad (39)$$

This power allocation algorithm is derived from the algorithms presented in [10] (pages 127-135).

4. RESULTS

We consider a MIMO-OFDM system with 64 subcarriers ($K = 64$) and a frequency-selective channel with 4 equally spaced channel taps ($L = 4$). All the simulations are for a 16 QAM modulation scheme and without channel coding. We consider no correlation between the receive antennas and model the transmit antenna correlation matrix as $[\mathbf{R}_{ta}]_{i,j} = \rho_{ta}^{|i-j|}$. The elements of the tap correlation matrix $[\mathbf{R}_{mp}]_{l_i,l_j}$ are modeled as introduced in (4) with $\gamma_l/\gamma_{l+1} = 3$dB.

Fig. 2 shows the impact of the correlation on the performance of a MIMO-OFDM system with $N_r = N_t = 2$ and $N_s = 2$. The Bit Error Rate (BER) curves are drawn for a non-precoded scheme. We consider a correlation between

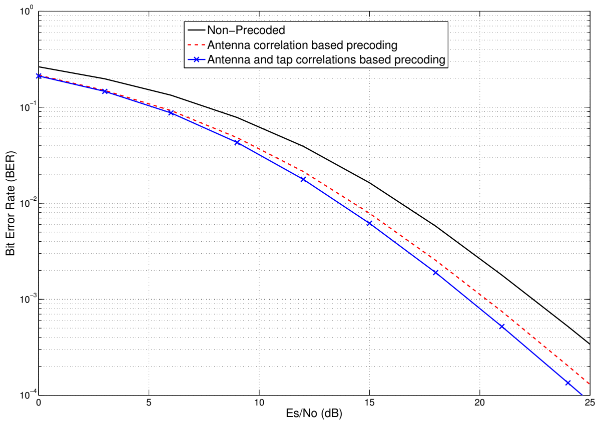

Figure 3: Performance gain of exploiting the knowledge of the correlation between the channel taps (with $K=64, L=4$, $\rho^{mp}_{l_i,l_j}=\rho_{ta}=0.7, N_s=1$ and $N_r=N_t=2$). In this configuration, exploiting the knowledge of the correlation between the channel taps brings an extra SNR gain of 0.6 dB.

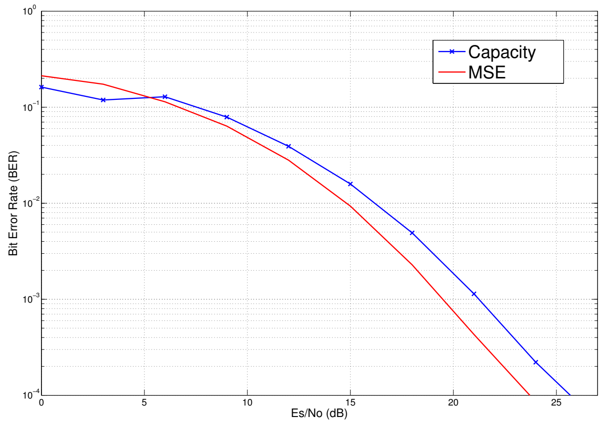

Figure 4: BER for the capacity and the MSE criteria (with $K=64, L=4, \rho^{mp}_{l_i,l_j}=\rho_{ta}=0.7, N_s=2$ and $N_r=N_t=4$). We can observe that above a 5 dB SNR the MSE curve reaches a lower BER than the capacity-based precoder. At 20 dB we have a gain of 1.8 dB compared to its capacity counterpart.

the transmit antennas (ρ_{ta}) of 0 and 0.7 and a tap correlation factor ($\rho^{mp}_{l_i,l_j}$) of 0 and 0.7 for all l_i and l_j. We observe that the performance of the system degrades with the presence of these correlations.

In Fig. 3, we show the performance obtained for a MIMO-OFDM system (with $N_r=N_t=2$, $N_s=1$ and $\rho^{mp}_{l_i,l_j}=\rho_{ta}=0.7$) for the non-precoded scheme and for two precoding schemes:

- one for which the precoder exploits only the knowledge of the correlation between the transmit antennas (antenna correlation based precoding)
- and the other one for which the precoder exploits the knowledge of both the correlation between the transmit antennas and the correlation between the channel taps (antenna and tap correlations based precoding).

Fig. 4 compares the BER curves obtained with our MSE precoder to that obtained with the capacity precoder of [3] (with $N_r=N_t=4$, $N_s=2$ and $\rho^{mp}_{l_i,l_j}=\rho_{ta}=0.7$). We can observe that the MSE precoder achieves a better BER than its capacity counterpart. Except at low SNR where the capacity-based precoder suppresses more data streams than the proposed MSE precoder. Simulated results (not reported in this paper) logically support that in terms of bit per second/ Hertz (bps/Hz), the capacity criterion performs better than our MSE-based precoder.

5. CONCLUSIONS

In this paper, we have proposed a precoding scheme for minimizing the sum of the symbol MSEs over all subcarriers for a MIMO-OFDM system. The precoder exploits the knowledge of the correlation between the channel taps and also that between the transmit antennas. Simulation results show that exploiting this knowledge brings extra performance gain compared to a non-precoded scheme. We can also observe that the proposed precoder achieves a lower BER than the capacity-based precoding.

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