DISTRIBUTED SPACE-TIME MSK TRELLIS CODES FOR AMPLIFY & FORWARD RELAYING

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ABSTRACT

In this paper, we consider using minimum shift keying, a special form of continuous phase modulation in relay systems. In this way, we aim at taking advantage of constant envelope, usage compatibility with low cost nonlinear amplifiers, and spectral efficiency of continuous phase modulation on relay/cooperative systems. We investigate optimum distributed space-time trellis codes for amplify & forward relaying method and propose novel codes which outperform their classical counterparts.

1. INTRODUCTION

Transmit antenna diversity is one of the main technique against fading effects on wireless channels and mostly deployed at base stations of cellular systems [1]. On the other hand, it might not be practical to place more than one antenna on the limited sized mobile terminals. Cooperative transmission methods should be considered to achieve transmit antenna diversity on those terminals [2]-[4]. The main idea of cooperative transmission is to send user information not only over the user's own antenna, but the appropriate mobile or stationary terminals' antennas around. Thereby, user's information reaches the destination terminal over different paths and transmit antenna diversity is constituted, even though the source terminal has a single antenna. In this way, error curbing performance can be increased. Cooperative diversity is based on the primary work done in [5] and [6] about using relays in communication channels. Relays can amplify the incoming user signal and transmit it to the destination terminal, which is called as amplify and forward (AF) method initially proposed in [2] and [7].

The performance can be further improved by making use of coding technique called as "coded cooperation" [8]. Different terminals may employ different codes or alternatively, different parts of a code may be operated on different terminals. Latter method is called as "distributed coding" and it is possible to obtain cooperative diversity by using classical space-time codes in a "distributed" fashion on relay networks [9], [10].

Continuous phase modulation (CPM) is a nonlinear multidimensional modulation method, which has rapidly decreasing side lobes in its power spectrum. This is due to the phase continuity between the transmitted consecutive signals [11]. Here, phase continuity is maintained by intrinsic memory elements, which yield a trellis code. Thereby, it is possible to decode the received signals using Viterbi algorithm, and benefit from power efficiency along with spectral efficiency. This is an important advantage over frequently encountered fading effects of wireless channels. In addition, CPM makes it possible to operate with low cost nonlinear power amplifiers and provides practically constant envelope for the transmitted signals.

In this paper we consider a special form of CPM, which is called minimum shift keying (MSK). We propose using MSK on relaying in order to achieve the mentioned advantages on cooperative systems. Distributed space-time MSK trellis codes are investigated within the scope of this paper. After an exhaustive search, new distributed space-time MSK trellis codes are designed and their error performances are simulated by computer programs.

2. SYSTEM MODEL

Continuous phase frequency shift keying (CPFSK) is a special case of CPM of which frequency pulse is a rectangular function [11]. Rimoldi has shown that a CPM modulator can be decomposed into a convolutional continuous phase encoder and a memoryless modulator, making use of the time invariant tilted phase definition [12]. A tilted phase CPFSK signal can be represented as

$$s(t,\boldsymbol{\beta})=\sqrt{\frac{2E}{T}}\cos\left(2\pi f_1 t+2\pi h\beta_n\frac{(t-nT)}{T}+\theta_n\right)\cdot \qquad (1)$$

Here, $f_1=f_c-h(U-1)/2T$ is the modified carrier frequency and $\boldsymbol{\beta}=\{\beta_n\}$is the modified data sequence where $\beta_n=(\alpha_n+U-1)/2$ is the modified data value for a U level CPFSK modulation. $\alpha_n \in \{\pm1, \pm3..., \pm(U-1)\}$ is the nth data symbol. Every data symbol takes values with equal probability of $1/U$. E is the symbol energy, and T is the symbol interval. $h=J/P$ is the modulation index, where J and P are relatively

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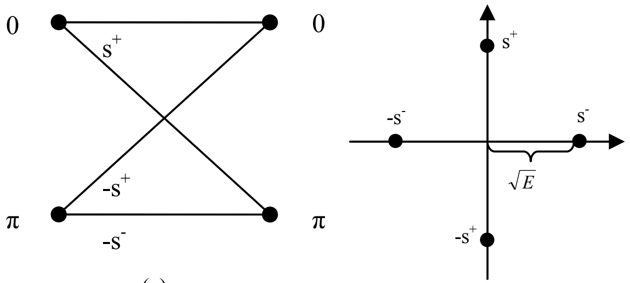
Table 1 – Protocol A, Protocol B and Protocol C

	1st Time Slot	2nd Time Slot	3rd Time Slot	...	(M-1)th Time Slot	M th Time Slot
Protocol A	S($\boldsymbol{s}_1$) → R$_1$ S($\boldsymbol{s}_1$) → D	S($\boldsymbol{s}_2$) → R$_2$ S($\boldsymbol{s}_2$) → D	S($\boldsymbol{s}_3$) → R$_3$ S($\boldsymbol{s}_3$) → D	...	S($\boldsymbol{s}_{M-1}$) → R$_{M-1}$ S($\boldsymbol{s}_{M-1}$) → D	R$_1$($\boldsymbol{s}_1$) → D R$_2$($\boldsymbol{s}_2$) → D R$_3$($\boldsymbol{s}_3$) → D ... R$_{M-1}$($\boldsymbol{s}_{M-1}$) → D S($\boldsymbol{s}_M$) → D
Protocol B	S($\boldsymbol{s}_1$) → R$_1$ S($\boldsymbol{s}_1$) → D	S($\boldsymbol{s}_2$) → R$_2$ R$_1$($\boldsymbol{s}_1$) → D S($\boldsymbol{s}_2$) → D	S($\boldsymbol{s}_3$) → R$_3$ R$_1$($\boldsymbol{s}_1$) → D R$_2$($\boldsymbol{s}_2$) → D S($\boldsymbol{s}_3$) → D	...	S($\boldsymbol{s}_{M-1}$) → R$_{M-1}$ R$_1$($\boldsymbol{s}_1$) → D R$_2$($\boldsymbol{s}_2$) → D R$_3$($\boldsymbol{s}_3$) → D ... R$_{M-2}$($\boldsymbol{s}_{M-2}$) → D S($\boldsymbol{s}_{M-1}$) → D	R$_1$($\boldsymbol{s}_1$) → D R$_2$($\boldsymbol{s}_2$) → D R$_3$($\boldsymbol{s}_3$) → D ... R$_{M-1}$($\boldsymbol{s}_{M-1}$) → D S($\boldsymbol{s}_M$) → D
Protocol C	S($\boldsymbol{s}_1$) → R$_1$	S($\boldsymbol{s}_2$) → R$_2$	S($\boldsymbol{s}_3$) → R$_3$	...	S($\boldsymbol{s}_{M-1}$) → R$_{M-1}$	R$_1$($\boldsymbol{s}_1$) → D R$_2$($\boldsymbol{s}_2$) → D R$_3$($\boldsymbol{s}_3$) → D ... R$_{M-1}$($\boldsymbol{s}_{M-1}$) → D S($\boldsymbol{s}_M$) → D

prime positive integers. It is possible to show that

$$\theta_{n+1} = \left[\theta_n + 2\pi h \beta_n\right] \bmod 2\pi \tag{2}$$

where θ_{n+1} and θ_n takes P different values from the set of {0, $(2\pi h)_{\text{mod}2\pi}$, $(4\pi h)_{\text{mod}2\pi}$, .. }. (2) corresponds to a P state trellis which have the values of θ_n as its states. MSK is a special case of CPFSK where U=2 and h=1/2. Here, possible values for β_n are {0,1} and for θ_n are {0,π} [13]. Figure 1a represents the trellis diagram of MSK. MSK symbols on the branches corresponds to $s^- \rightarrow \sqrt{(2E/T)}\cos 2\pi f_1 t$, $s^+ \rightarrow \sqrt{(2E/T)}\cos 2\pi (f_1+1/2T)t$, $-s^- \rightarrow -\sqrt{(2E/T)}\cos 2\pi f_1 t$, $-s^+ \rightarrow -\sqrt{(2E/T)}\cos 2\pi (f_1+1/2T)t$ within $[0,T]$ interval. MSK constellation has two dimensions and its corresponding vectors are as $s^- \rightarrow (\sqrt{E}, 0)$, $s^+ \rightarrow (0,\sqrt{E})$, $-s^- \rightarrow (-\sqrt{E}, 0)$, and $s^+ \rightarrow (0,-\sqrt{E})$ (Figure 1b).



(a) (b)

Figure 1 - (a) Trellis diagram for MSK, (b) MSK constellation.

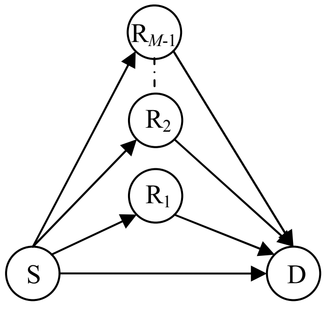


Figure 2 – Relay system.

We employ a relay system including M-1 relays which is shown in Figure 2. Source (S) sends information directly to destination (D) and/or making use of M-1 relays, using different time sharing protocols. Relays use AF method to transmit the faded information, which they received, to the destination. Canpolat, Uysal and Fareed [14] expanded the transmission protocols given by Nabar et al. [10] for multi-relay systems. The proposed protocols in [14] are named as Protocol A, Protocol B and Protocol C. Protocol A and Protocol C are the augmented versions of Protocol I and Protocol III, respectively [10]. In order to maintain phase continuity at the transmit antennas of the source and relay terminals, the protocols of [14] is slightly modified to transmit the information frame by frame instead of symbol by symbol. It is assumed that K MSK symbols are transmitted from the source and each relay, within a time slot. A frame length is $L=MK$ considering all time slots. $\boldsymbol{s}_m = \left[s_m^1 s_m^2 \cdots s_m^K\right]$ is the frame transmitted to the mth relay where $s_m^l \in \{s^-, s^+, -s^-, -s^+\}$ is the lth symbol of the frame (m = 1,2,..,M-1). The frame sent within last time slot is denoted by $\boldsymbol{s}_M = \left[s_M^1 s_M^2 \cdots s_M^K\right]$. The protocols used in this paper shown in Table 1 are also named as Protocol A, B, and C, respectively since there is no major difference between these and the ones in [14]. In this table, S($\boldsymbol{s}_m$)→R$_m$ and S($\boldsymbol{s}_m$)→D denote the transmission of $\boldsymbol{s}_m$ from source to the mth relay and to the destination, respectively. Similarly, R$_m$($\boldsymbol{s}_m$)→D denotes the transmission of faded and normalized signal related to $\boldsymbol{s}_m$ from mth relay to the destination.

Table 2 – Distributed space-time MSK trellis codes with 4 states

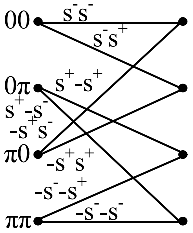
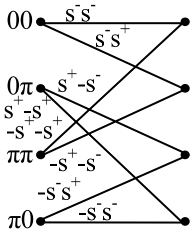
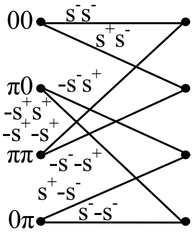
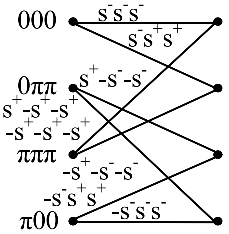
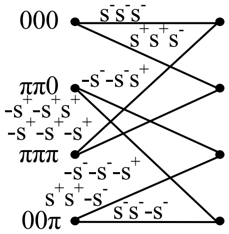
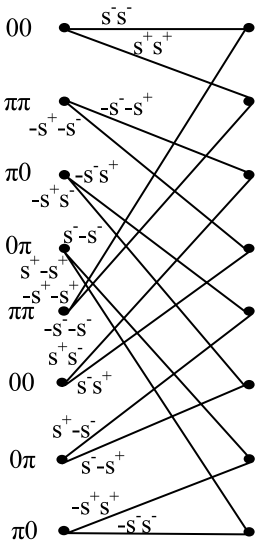
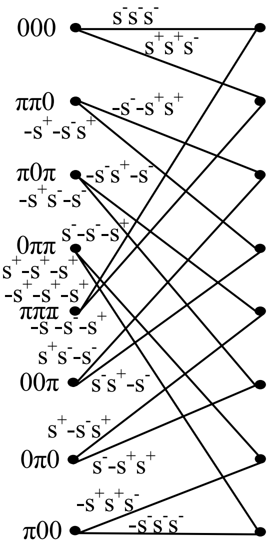
M=2			M=3	
STMSK1 [15],[16]	STMSK2 [15]	STMSK3	STMSK4 [15]	STMSK5
 $d^2{}_{cl}$=8 $d^2{}_{\mathrm{PrA}}$= $d^2{}_{\mathrm{PrB}}$=12	 $d^2{}_{cl}$=12 $d^2{}_{\mathrm{PrA}}$= $d^2{}_{\mathrm{PrB}}$=16	 $d^2{}_{cl}$=12 $d^2{}_{\mathrm{PrA}}$= $d^2{}_{\mathrm{PrB}}$=20	 $d^2{}_{cl}$=20 $d^2{}_{\mathrm{PrA}}$=32 $d^2{}_{\mathrm{PrB}}$=36	 $d^2{}_{cl}$=20 $d^2{}_{\mathrm{PrA}}$=36 $d^2{}_{\mathrm{PrB}}$=44

Table 3 – Distributed space-time MSK trellis codes with 8 states

M=2	M=3
STMSK6 [15]	STMSK7 [15]
 $d^2{}_{cl}$=20 $d^2{}_{\mathrm{PrA}}$= $d^2{}_{\mathrm{PrB}}$=32	 $d^2{}_{cl}$=28 $d^2{}_{\mathrm{PrA}}$=48 $d^2{}_{\mathrm{PrB}}$=60

3. DESIGN CRITERIA AND CODES

Nabar et al. [10] showed that the rank-determinant criteria, which is valid for classical space-time codes in quasi-static Rayleigh fading channels, is also valid for distributed space-time codes using AF method under the assumption of appropriate power control. It is shown in [14] that rank-determinant criteria is not valid as relay number increases, and new criteria – similar to the Euclidean distance criterion - should be used for Protocol A, B, and C. Analyses in [14] are done assuming that S→R and S→D links are Rayleigh faded and R→D links are Rician faded. In order to simplify the complicated results of mentioned analyses, it is suggested to use the criteria where R→D links are static [14]. Under the perfect power control assumption, mentioned criteria for Protocol A, B, and C are

$$d_{\mathrm{Pr\,A}}^2 = \Lambda_K^M + 2\sum_{m=1}^{M-1} \Lambda_K^m \,, \tag{3}$$

$$d_{\mathrm{Pr\,B}}^2 = \sum_{m=1}^{M} \Lambda_K^M + \sum_{m=1}^{M-1} \left(M-m\right)\Lambda_K^m \,, \tag{4}$$

$$d_{\mathrm{PrC}}^2 = \sum_{m=1}^{M} \Lambda_K^m \,, \tag{5}$$

respectively, where $\Lambda_K^m = \sum_{l=1}^{K} \left| s_m^l - \hat{s}_m^l \right|^2$. It can be shown that classical Euclidean distance is $d_{cl}^2 = \sum_{m=1}^{M} \Lambda_K^m$. In (3), (4), and (5) d^2 values have a close relationship with classical Euclidean distance, where $d^2{}_{\mathrm{PrC}}$= $d^2{}_{cl}$ for Protocol C. It can also be seen that Protocol A is equivalent to Protocol B, while M=2. Novel distributed space-time QPSK trellis codes are designed in [14] and it is shown that they perform better than classical space-time trellis codes on single and multi relay systems. The code search criterion is to maximize $d^2{}_{cl}$ first, and then maximize the corresponding d^2 for each mentioned protocol.

Since MSK can be viewed as a quadrature linear modulation [13], [17] and, since we assume quasi-static fading over the frame length of L, the criteria given above can be directly used. Following this approach in our paper, 4 or 8 state distributed space-time MSK trellis codes are investigated for one or two relay AF systems. All possible trellises are considered by exhaustive search. Two novel codes are found and named as STMSK3 and STMSK5. All proposed and benchmark codes are shown in Table 2 and Table 3 with their d^2 values for each protocol. Note that STMSK1, STMSK2, STMSK4, STMSK6, and STMSK7 were given in the literature for classical space-time structure. We found that STMSK6 and STMSK7 are optimum codes for the relay case as well.

4. ERROR PERFORMANCES

Error performance curves for the space-time MSK trellis codes are derived using a computer program. Frame error rate (FER) is calculated for different values of signal to noise ratios (SNRs). 1, 2 or 4 receive antennas (Rx) are assumed at the destination terminal. All links are modelled as quasi-static Rayleigh fading. A subframe is constituted using K=130 MSK symbols. The signal to noise ratio defined as E_{ij}/N_0 where $ij \in$ {SR, RD, SD}. E_{ij} is the average energy available at terminal j. We assume that all channels have additive Gaussian noise with zero mean and $N_0/2$ variance per dimension after proper normalization as in [14]. It is also assumed that $E_{SD}/N_0=E_{RD}/N_0$. Viterbi algorithm is used at the destination.

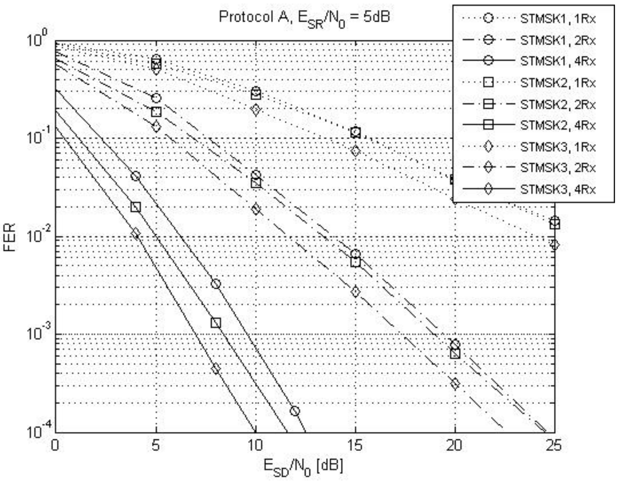


Figure 3 – FER performance of 4 state STMSK codes, E_{SR}/N_0=5dB, Protocol A, M=2.

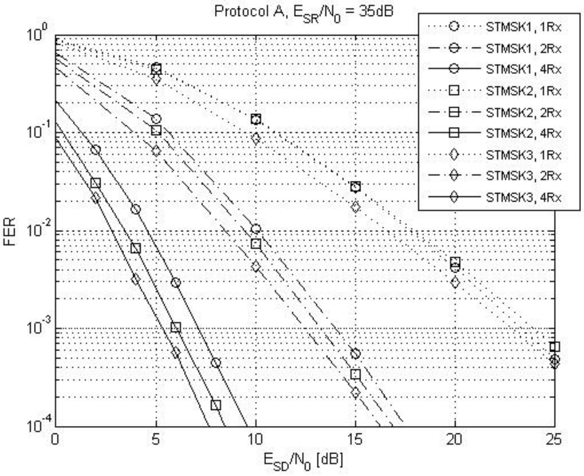


Figure 4 – FER performance of 4 state STMSK codes, E_{SR}/N_0=35dB, Protocol A, M=2.

The FER performances of STMSK1 [15], [16], STMSK2 [15], and STMSK3 using Protocol A are shown in Figure 3 and Figure 4 for E_{SR}/N_0=5dB and E_{SR}/N_0=35dB, respectively. It is seen that the novel code STMSK3 outperforms STMSK1 and STMSK2 for all considered SNRs and Rx numbers. This is due to the larger d^2_{PrA} value of the STMSK3 code. It is also seen that error performance is getting better as receive antenna number increases at the destination terminal and FER performance improves when

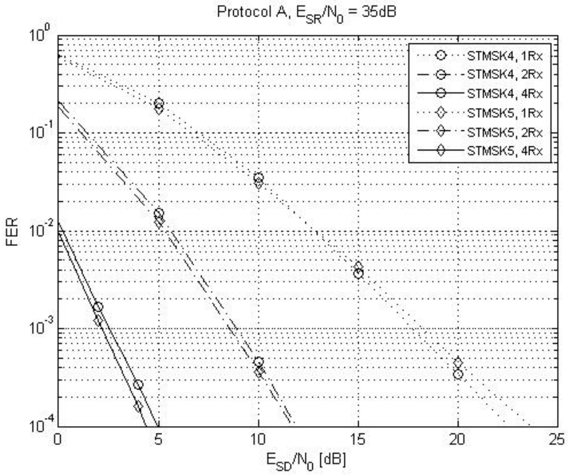


Figure 5 – FER performance of 4 state STMSK codes, E_{SR}/N_0=35dB, Protocol A, M=3.

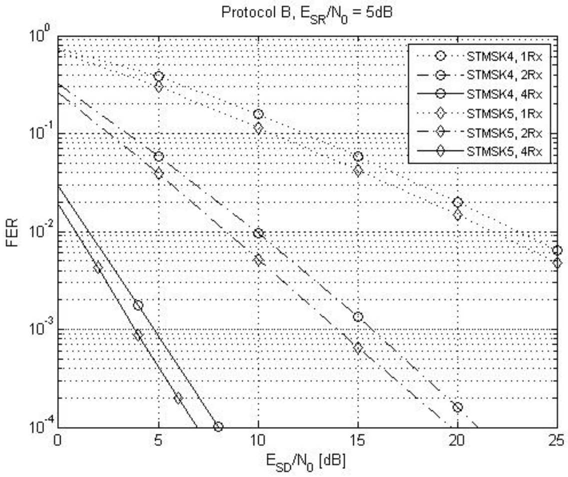


Figure 6 – FER performance of 4 state STMSK codes, E_{SR}/N_0=5dB, Protocol B, M=3.

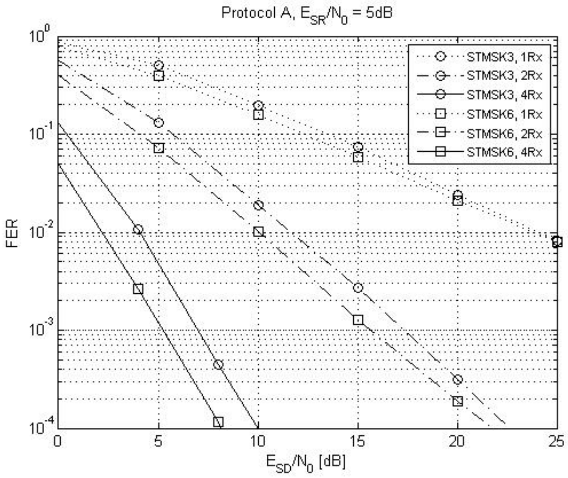


Figure 7 – FER performance of STMSK3 and STMSK6 codes, E_{SR}/N_0=5dB, Protocol A.

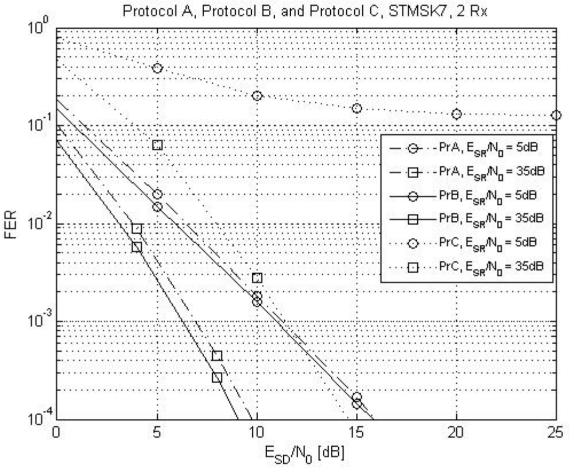


Figure 8 – FER performance of STMSK7 code, 2 Rx, Protocol A, Protocol B, and Protocol C.

E_{SR}/N_0 is increased, as expected. Figure 5 shows the FER performance of two relay system using STMSK4 [15] and STMSK5 for E_{SR}/N_0=35dB for each relay. Again, since the new code STMSK5 has a larger d^2_{PrA} value than that of STMSK4, it provides slightly better performance. If we use Protocol B, the performance improvement is more apparent because STMSK5 has even larger d^2_{PrB} value (Figure 6).

STMSK3 [15] and STMSK6 [15] are compared using Protocol A when E_{SR}/N_0=5dB in Figure 7. Since STMSK6 is an 8-state code and therefore has a larger d^2_{PrA} value, it provides significantly better performance especially when the number of Rx is increased at the destination. The FER performance of STMSK7 [15] is shown in Figure 8 for E_{SR}/N_0=5dB and E_{SR}/N_0=35dB using Protocol A, Protocol B and Protocol C. As seen, Protocol A and Protocol B lead to better FER performance with respect to Protocol C since they have S→D links. When the channel between S and R has low SNR, Protocol C suffers from error floor. This is not the case for other protocols.

5. CONCLUSION

In this paper, 4 or 8-state distributed space-time MSK trellis codes have been investigated for one or two relay AF systems. Two novel 4-state distributed space-time MSK trellis codes were proposed and it was shown that they outperform the corresponding classical space-time MSK codes on AF relaying systems, which use Protocol A or Protocol B. Those codes are suitable for use in AF relaying systems in order to exploit advantages of CPM in cooperative transmission. It was also shown that performance of Protocol B is slightly better than that of Protocol A, and both of these protocols significantly outperform Protocol C. We showed that the 8-state space-time MSK trellis codes proposed for classical space-time systems are also optimum for use in one or two relay AF systems.

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