ROBUST ADAPTIVE MODULATION WITH IMPERFECT CHANNEL INFORMATION

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ABSTRACT

Adaptive modulation is a promising technique to increase system throughput considerably. However, it relies on perfect channel state information (CSI), and is sensitive to errors in CSI. In this work, we maximize the system transmission rate based on a lower bound of average bit error rate (BER) while satisfying the transmit power and BER constraint. In order to further enhance the system throughput, adaptive modulation scheme is combined with a robust transmit beamformer to obtain extra diversity gain. Moreover, to pay the penalty for the lower bound of the average BER, we introduce a probabilistic constraint by keeping a low outage probability of signal-to-noise ratio (SNR). Simulation results show that the proposed scheme provides the maximum system throughput compared with several state-of-the-art robust adaptive schemes, and always guarantees the target BER.

1. INTRODUCTION

Adaptive modulation has the potential to increase the transmit rate by taking the advantage of favorable channel conditions [1] [2] [3] [4] [5] [6] [7]. Perfect channel state information is crucial to adaptive modulation, but is typically not available due to errors induced by the imperfect (quantized, erroneous, or outdated) feedback channel [8]. Thus, a robust adaptive modulation scheme is required based on imperfect CSI.

Although existing robust adaptive modulation schemes at transmitter [5] [6] [7] take errors in CSI into account, the system throughput does not achieve the maximum rate, due to the improperly-paid compensation on average BER. More specifically, the system throughput is determined by the target BER and the average BER that the system achieves. However, the latter is difficult to evaluate, and is usually replaced by its lower bound, which carries a performance penalty. To ensure that adaptive modulation still meets the BER target, the compensation can be employed in two ways. One approach is to artificially introduce a modifying factor which can only be empirically determined through extensive Monte Carlo simulations [5]. In another approach [6], the BER constraint is satisfied under the consideration of worst-case SNR scenario. Due to the excessive compensation, only a conservative throughput can be achieved. Therefore, it is necessary to investigate an efficient approach that employs appropriate compensation on the average BER.

Recently, transmit diversity has been well developed to enhance the performance of wireless communication when perfect CSI is not known [6] [9] [10]. In order to reduce its performance degradation caused by imperfect CSI, adaptive modulation scheme incorporates transmit beamforming technique and leads to further improvement of system throughput. For instance, in partial channel information scenarios, the transmit beamformers based Alamouti scheme provide extra two-dimensional diversity gain to adaptive modulation scheme, which increase the system throughput [3] [5]. By applying the transmit beamformer based on worst-case CSI scenario, the robust adaptive modulation scheme achieves the maximum transmission rate for any possible error in the uncertainty region [6]. In this work, the recently proposed transmit beamforming techniques [11] [12] are incorporated into adaptive modulation scheme.

We design robust adaptive modulation scheme for multiantenna transmissions with imperfect channel information. Under transmit power constraint, the transmitter here optimally adjusts the power allocation and the signal constellation to maximize the system throughput while maintaining a prescribed BER constraint. In order to obtain an extra diversity gain, the proposed adaptive modulation scheme is combined with the transmit beamformer. Thus, a necessary compensation is required. Here, we introduce a probabilistic constraint to efficiently pay for the penalty to keep the outage probability of SNR as low as possible. The proposed robust adaptive problem is transformed into maximization of SNR while satisfying a probabilistic constraint and transmit power constraint, which can be solved by standard mathematical tools. Simulation results show that the proposed adaptive scheme significantly increases the system throughput compared with other state-of-the-art robust adaptive modulation schemes, while guaranteeing the target BER.

This paper is organized as follows. The system model is described in Section 2. After a brief introduction of the standard adaptive modulation schemes in Section 3, the proposed robust adaptive modulation schemes is developed in Section 4. Simulation results are presented and discussed in Section 5. Concluding remarks are given in Section 6.

2. SYSTEM MODEL

Consider a single-user wireless communication system with N_t transmit antennas and N_r receive antennas ($N_t \geq N_r$). The channels are assumed as slow time-varying, and the transmitter can track the channel variations via feedback channel. However, perfect channel realization can not be accessed, leading the imperfection taken into account in real scenario. In this work, we assume that the transmitter can obtain the imperfect channel information and the error statistics over slow-fading channel.

Defining the perfect channel as $\mathbf{H} \in \mathbb{C}^{N_r \times N_t}$ and the estimate as $\hat{\mathbf{H}} \in \mathbb{C}^{N_r \times N_t}$, we have

$$\mathbf{H} := \hat{\mathbf{H}} + \mathbf{E} , \quad (1)$$

where the channel matrix is $\mathbf{H} = [\, \mathbf{h}_1, \ldots, \mathbf{h}_{N_t}]$, and the error matrix $\mathbf{E} \in \mathbb{C}^{N_r \times N_t}$ consists of i.i.d complex normally distributed entries with variance σ_e^2. The information-bearing symbol $\mathbf{s} \in \mathbb{C}^{P \times 1}$ is drawn from an appropriate signal constellation of size M with average energy E_s , spread by a precoding matrix $\mathbf{C} \in \mathbb{C}^{N_t \times P}$ and transmitted through multiple channels.

According to the error model (1), the SNR is a function of the channel estimate $\hat{\mathbf{H}}$ and the random error $\mathbf{E}$,

$$\gamma = \frac{E_s}{N_0} \operatorname{tr}\left\{ \mathbf{C}^H (\hat{\mathbf{H}} + \mathbf{E})^H (\hat{\mathbf{H}} + \mathbf{E}) \mathbf{C} \right\} , \quad (2)$$

where N_0 is the energy of the additive white Gaussian noise (AWGN) with zero mean and variance $N_0/2$ per real and imaginary dimension.

3. STANDARD ADAPTIVE MODULATION

The goal of adaptive modulation is to maximize the system transmission rate, subject to BER constraint and power constraints. To simplify the design, we rely on the approximation of the instantaneous BER, which is a function of received SNR γ and constellation size 2^k [2]

$$\mathrm{BER}(k, \gamma) \approx 0.2 \exp\left(-\frac{1.6\gamma}{2^k - 1} \right) , \quad (3)$$

where k is the transmission rate. The average BER can be calculated by taking the expectation of the instantaneous BER with respect to γ, as follows

$$\overline{\mathrm{BER}}(k) = \int_0^\infty \mathrm{BER}(k, \gamma)\; p(\gamma) d\gamma . \quad (4)$$

Here, we define the BER constraint as

$$\overline{\mathrm{BER}}(k) \le \mathrm{BER}_0 , \quad (5)$$

where BER_0 is pre-specified value, usually defined as 10^{-3}.

According to (3), (4) and (5), the optimization problem can be formulated as

$$\max \;\; k , \quad (6)$$
$$\text{subject to} \quad \overline{\mathrm{BER}}(k) \le \mathrm{BER}_0 , \quad (7)$$

where the transmission rate k is parameterized by the average BER and BER constraint.

4. ROBUST DESIGN WITH IMPERFECT CHANNEL INFORMATION

In practice, the CSI can not be perfectly known, leading a significant degradation performance of system throughput. Thus, for robust adaptive modulation scheme, it is crucial to take the errors in CSI into account. In this section, we combine robust adaptive modulation scheme with recently developed robust transmit beamforming technique [11] [12], which can significantly enhance the system throughput.

Since the integral in (4) can not be calculated in closed form, a common method is to take the lower bound of average BER [5],

$$\overline{\mathrm{BER}}_L(k) = 0.2 \exp\left(-\frac{1.6\, \overline{\gamma}}{2^k - 1} \right) , \quad (8)$$

where $\overline{\gamma}$ is the average SNR. By considering BER constraint (5), a suboptimal transmission rate can be expressed as

$$k' = \log_2 \left(1 - \frac{1.6\, \overline{\gamma}}{\ln(5\, \mathrm{BER}_0)} \right) , \quad (9)$$

where k' denotes as the suboptimal transmission rate. Given a pre-specified BER constraint, the maximum achievable transmission rate increases with the average SNR [5].

However, according to Jensen's inequality, the average BER (4) may be larger than the target BER, leading the constraint (5) violated [5]. Two approaches are used to prevent this. One introduces a modifying factor to set a smaller BER target [5], which only can be empirically determined by extensive Monte Carlo simulation. Another approach [6] considers the worst-case SNR, which leads conservative solution due to extreme rare worst operational condition. In order to efficiently maximize the system throughput, we propose a novel approach which can intelligently and efficiently prevent the constraint violation.

In order to avoid the suboptimal transmission rate violating the average BER constraint, we introduce a probabilistic constraint that keeps a low outage probability of SNR.

To illustrate the novelty in the proposed scheme, we investigate the relationship between average BER and its lower bound. According to [13], the instantaneous BER, $\mathrm{BER}(k, \gamma)$, can be approximated by a Taylor series about the mean SNR $\overline{\gamma}$ that is truncated after the quadratic term, such as

$$\begin{aligned} \mathrm{BER}(k, \gamma) \;&\approx\; \overline{\mathrm{BER}}_L(k) + (\gamma - \overline{\gamma})\, \overline{\mathrm{BER}}\,'_L(k) \\ &+\; \frac{(\gamma - \overline{\gamma})^2}{2}\, \overline{\mathrm{BER}}\,''_L(k) + o(\gamma) , \end{aligned}$$

where $\overline{\mathrm{BER}}\,'_L(k)$ and $\overline{\mathrm{BER}}\,''_L(k)$ are defined as first derivative and second derivative of $\overline{\mathrm{BER}}_L(k)$ with respect to k. Ignoring higher order terms and taking the expectation of BER, we have

$$\begin{aligned} \overline{\mathrm{BER}}(k) \;&\approx\; \int_0^\infty \Big[\, \overline{\mathrm{BER}}_L(k) + (\gamma - \overline{\gamma}) \overline{\mathrm{BER}}\,'_L(k) \\ &+\; \frac{(\gamma - \overline{\gamma})^2}{2} \overline{\mathrm{BER}}\,''_L(k) \,\Big]\, p(\gamma) d\gamma \\ &=\; \overline{\mathrm{BER}}_L(k) + 0.2 \frac{\mathrm{var}\{\gamma\}}{2} \overline{\mathrm{BER}}\,''_L(k) \\ &=\; \overline{\mathrm{BER}}_L(k) \left(1 + \frac{\mathrm{var}\{\gamma\}}{10} \left(\frac{-1.6}{2^k - 1} \right)^2 \right) . \quad (10) \end{aligned}$$

Note that the first-order term vanishes as a result of the expectation operation, and the approximation will be accurate if the instantaneous SNR is well concentrated about its mean, namely the variance of SNR $\mathrm{var}\{\gamma\}$ is small. It also clearly indicates that the penalty of lower bound BER comes from the ignored higher order terms.

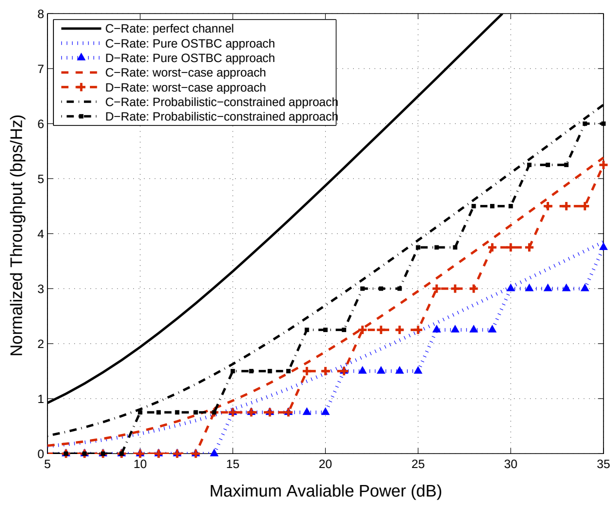

Figure 1: Average normalized throughput comparison, $\gamma_{th} = 0.95$ and $p_{out} = 10\%$

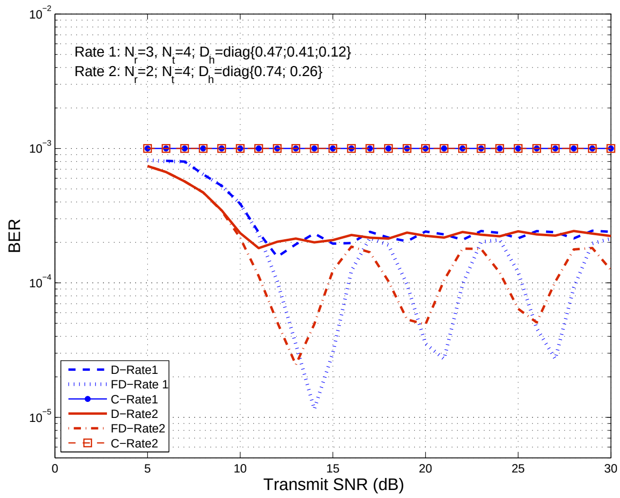

Figure 2: BER for the proposed robust adaptive modulation scheme, $\gamma_{th} = 0.95$ and $p_{out} = 10\%$

In recently proposed probabilistic-constrained transmit beamforming techniques [11] [12], we find that the high-order terms in (10) can be reasonably taken into account with a properly defined outage probability constraint. We define the probabilistic constraint that the SNR γ falls below a threshold,

$$Pr\{\gamma \leq \gamma_{th}\} \leq p_{out} \ , \tag{11}$$

where the SNR threshold is defined as γ_{th}, and p_{out} is a pre-specified probability value that satisfies QoS requirements, and $Pr\{A\}$ stands for the probability of event A. Note that by setting the threshold equal to or larger than the average SNR, $\overline{\gamma}$, and the outage probability at a low level, leading well-concentrated random variables γ, correspondingly, the difference between average BER and its lower bound is reduced without any extra compensation.

By taking the lower bound of average BER (8) and introducing the probabilistic constraint (11), our adaptive modulation scheme (6)-(7) can be formulated as follows

$$\max \quad \log_2\left(1 - \frac{1.6\,\overline{\gamma}}{\ln(5\overline{\mathrm{BER}}_L(k))}\right) , \tag{12}$$

subject to

$$\overline{\mathrm{BER}}_L(k) \leq \mathrm{BER}_0 \ , \tag{13}$$

$$Pr\{\gamma \leq \gamma_{th}\} \leq p_{out} \ . \tag{14}$$

It indicates that the system throughput is determined by the achievable average SNR and outage probability of varied SNR. Applying the robust transmit beamforming [11] [12], the average SNR is maximized while the robustness is achieved by taking errors in CSI proportionally. Compared to other popular robust designs [10] [6], simulation results show that the probabilistic constraint approach has the best performance. Consequently, it leads to the highest transmit rate in the proposed adaptive modulation scheme.

5. SIMULATION RESULTS

In our simulation, we consider a single-user MIMO system with multiantenna at both transmitter and receiver sides ($N_t \geq N_r$). 10^5 Monte-Carlo runs are used to obtain each point. The proposed adaptive modulation scheme is compared with other adaptive schemes based on different approaches, such as the worst-case approach [6] and the orthogonal space-time block code (OSTBC) approach [14]. Without any loss of generality, we assume the following:

- Channel parameters : The channel between pth and qth transmit antennas can be presented as [15]

$$[\mathbf{H}^H\mathbf{H}]_{p,q} \approx \frac{1}{2\pi}\int_0^{\pi} \exp\left[-j2\pi(p-q)\Delta\frac{d_t}{\lambda}\sin\theta\right] d\theta,$$

  where angle of spread Δ is related to the channel state information, λ is the wavelength of a narrow-band signal, and d_t the antenna spacing and Δ the angle of spread. We set $d_t = 0.5\lambda$ and $\Delta = 30°$.

- Error in CSI : We assume that the error is Gaussian distributed with zero mean and covariance matrix $\sigma_e^2\mathbf{I}$, i.e.

$$\mathbf{E}_{N_r \times N_t} \sim \mathscr{CN}(0, \sigma_e^2\mathbf{I}).$$

  In our simulation, the variance of the error is set as 0.6.

- Other parameters : We set the target BER as 10^{-3} . The SNR threshold is $\gamma_{th} = 0.95$, and $E_s/N_0 = 1$. The outage probability is $p_{out} = 10\%$.

In Fig. 1 , the average throughput [6] has been normalized with respect to the code rate, so that the gains provided by the robust technique itself for different number of transmit antennas can be compared directly. In this case, 4 transmit antennas and 3 receive antennas are considered. The eigenvalues of $\hat{\mathbf{H}}^H\hat{\mathbf{H}}$ are $(0.4676, 0.4104, 0.1220, 0)$. With the same channel condition, the proposed scheme requires less transmit power among other schemes to fulfill the BER constraint, thus larger constellation size is allowed to modulate the transmit symbols, consequently, leading to the maximum normalized system throughput.

In Fig. 2, it shows that the average BER performance has been well controlled below 10^{-3} under the proposed scheme. Here, we consider the BER performance with three different constellation rates

$$M_i = 2^k \begin{cases} k \geq 2,\ k \in \mathscr{R}^+ : & \text{Continuous Rate (C-Rate)}, \\ k \in \{2,3,4,\ldots\}: & \text{Discrete Rate (D-Rate)}, \\ k \in \{2,4,6,8\} : & \text{Finite Discrete Rate (FD-Rate)}, \end{cases}$$

and two different numbers of receive antennas : $N_r = 2$ and $N_r = 3$. It indicates that no matter the number of receive antennas, the BER achieves the target by using the continuous rate. Note that the BER bound of 10^{-3} breaks down at low SNR, since (4) is not applicable to BPSK. Furthermore, because the BER increases monotonically with decreasing constellation size, the exact average BER is much lower than 10^{-3} with both discrete rates, such as D-Rate and FD-Rate.

6. CONCLUSION

We propose a novel robust adaptive modulation scheme that significantly improves the system throughput while satisfying the BER constraint. In contrary to the conventional schemes, the proposed scheme introduces the probabilistic constraint to control the varied SNR, which efficiently minimizes the penalty for the lower bound on average BER by keeping the probability that the SNR falls below a threshold low. Under imperfect channel conditions, the robust adaptive modulation scheme gains an extra transmit diversity gain by combining with transmit beamforming, and a high average SNR. With the robustness provided by probabilistic constraint, the resulting system throughput achieves the maximum rate. Simulation results demonstrate the proposed robust adaptive scheme provides the most significant improvement of the normalized system throughput among the state-of-art robust adaptive schemes, and guarantees the target BER in different scenarios, such as different numbers of receive antennas and different constellation rates.

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