MODIFIED MODULUS TRANSFORMATION FOR HIGH RESOLUTION DIRECTION FINDING

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ABSTRACT

The presence of impulsive noise can severely degrade the accuracy performance of conventional direction of arrival estimation algorithms. In this paper, we propose a non-linear transformation function to suppress the impulsive noise. The proposed function is called Modified Modulus Transformation and it is used to preprocess the impulsive noise contaminated signal prior to covariance estimation. Simulation results are presented to illustrate the efficacy of the proposed approach for high resolution direction finding in highly-impulsive environments. This observation is also corroborated with real data.

1. INTRODUCTION

Direction of arrival (DOA) estimation is an important problem in sensor array processing research and is relevant to many applications such as radar and wireless communications. Over the past decades, a number of high resolution direction finding (DF) algorithms have been proposed [1–4].

The noise in practical radio environments, especially in the HF and VHF bands, is non-Gaussian and impulsive in nature [5]. In addition to natural phenomenon, e.g. lightning, impulsive noise sources are increasingly attributed to human activities [5]. However, most of the conventional DOA estimation methods are not robust against impulsive noise and break down rapidly in their presence.

Maximum-likelihood (ML) estimation and robust covariance matrix estimation, have been proposed to overcome this limitation. The ML approach requires prior knowledge of the noise distribution [6–8]. While optimal, these estimators are computationally intensive and require knowledge of the noise parameters as well as user-defined threshold values and weighting functions.

Robust covariance estimation methods based on normalized sample covariance matrix have been proposed [9–11] and have enabled conventional DOA estimation to achieve good performance in the presence of impulsive noise. In [12], the authors showed that the performance of conventional DOA estimator can be improved by preprocessing the amplitude of the received signal with Gaussian-tailed zero-memory nonlinearity (GZMNL) before covariance estimation.

In this paper, we propose an alternative non-linear function, called Modified Modulus Transformation (MMT), to suppress the impulsive noise in the received signals prior to DOA estimation with conventional methods. It is worthwhile to point out that the MMT does not require prior information about the statistics (temporal distribution) of the impulsive noise nor parameters tuning, though it is dependent on the amplitude distribution. As shown later in the results analysis, MMT can offer an improved DOA estimation performance through better impulsive noise suppression, as compared to other robust covariance estimation methods proposed in [9–12].

The organization of the paper is as follows: In Section 2, the signal model of the antenna array system is discussed. Section 3 presents the proposed modified modulus transformation function. Simulation and real-data results are shown in Section 4 and a short conclusion is drawn in Section 5. In the sequel, the superscripts $(\cdot)^T$ and $(\cdot)^H$ denote the transpose and Hermitian operations respectively.

2. DATA MODEL

Consider an array of N sensors, which receives L narrowband signals emitting from far-field sources of DOAs $\{\theta_l\}_{l=1}^{L}$. The DF problem is to estimate the $(L < N)$ signal DOAs

$$\mathbf{\Theta} = [\theta_1, \theta_2, \ldots, \theta_L]^T \tag{1}$$

from the independent array snapshots $\{\mathbf{x}(t)\}_{t=1}^{M}$, modeled as:

$$\mathbf{x}(t) = \mathbf{A}(\mathbf{\Theta})\mathbf{s}(t) + \mathbf{i}(t) + \mathbf{w}(t), \quad t = 1, 2, \ldots, M \tag{2}$$

where

$$\mathbf{A}(\mathbf{\Theta}) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_L)] \tag{3}$$

is the $(N \times L)$ direction matrix and $\mathbf{a}(\theta_l)$ is the $(N \times 1)$ array steering vector. The signal-waveforms vector is denoted by $\mathbf{s}(t)$. The vector $\mathbf{w}(t)$ models identically and independent distributed (i.i.d.) sensor noise. The vector $\mathbf{i}(t)$ is due to impulsive noise.

3. COVARIANCE MATRIX ESTIMATION IN THE PRESENCE OF IMPULSIVE NOISE BASED ON A MODIFIED MODULUS TRANSFORM

Conventionally, the array covariance matrix is estimated:

$$\hat{\mathbf{R}} = \frac{1}{M}\left[\mathbf{X}\mathbf{X}^H\right] \tag{4}$$

where

$$\mathbf{X} = [\mathbf{x}(1),\ \mathbf{x}(2),\ \ldots,\ \mathbf{x}(M)]. \tag{5}$$

In the presence of impulsive noise, DF methods using the array covariance matrix estimated as in (4) will result in poor DOA estimation accuracy performance.

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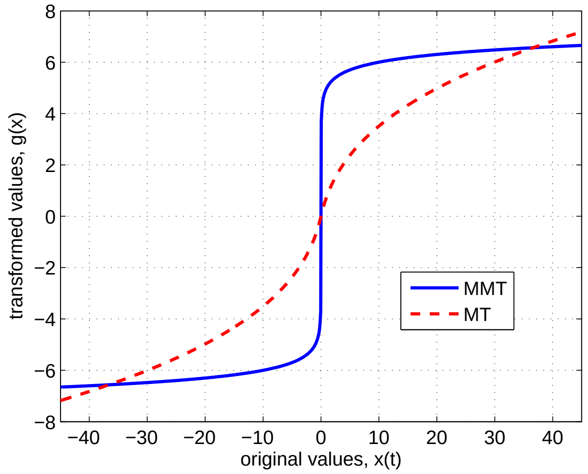

Figure 1: Transfer function of MMT.

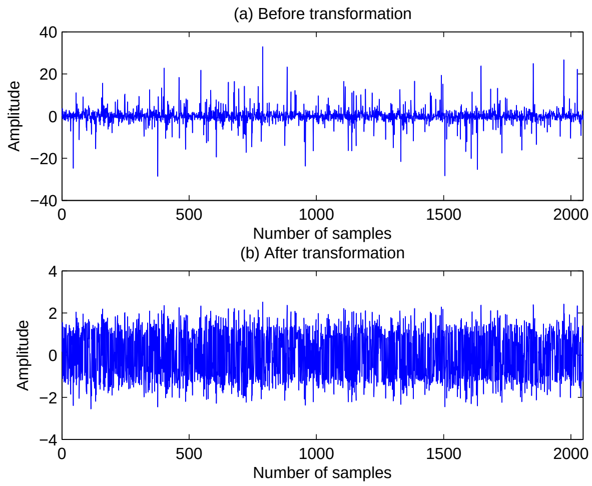

Figure 2: Transformation of a Gaussian signal contaminated with impulsive noise by applying MMT.

In this paper, we propose to use a non-linear transformation function known as Modified Modulus Transform (MMT) to suppress impulsive noise prior to covariance estimation. It is a modification from [13, 14] and is given by:

$$g(x) = \begin{cases} \operatorname{sign}(x_r)\left[\log_{10}\left(|x_r|\right)+1\right]+\ldots & \\ \quad j\,\operatorname{sign}(x_i)\left[\log_{10}\left(|x_i|\right)+1\right], & x \neq 0 \\ \quad 0, & x = 0 \end{cases} \tag{6}$$

where x_r and x_i denotes the real and imaginary parts respectively of x and sign(.) denotes the sign operation. It suffices to point out that, unlike the modulus transformation proposed in [13, 14], the proposed MMT does not require any tuning parameter. Furthermore it does not make any assumption about the temporal distribution of the impulsive noise. As with other existing non-linear transformations, the MMT in (6) is performed on the real and imaginary parts of $\mathbf{x}(t)$ independently.

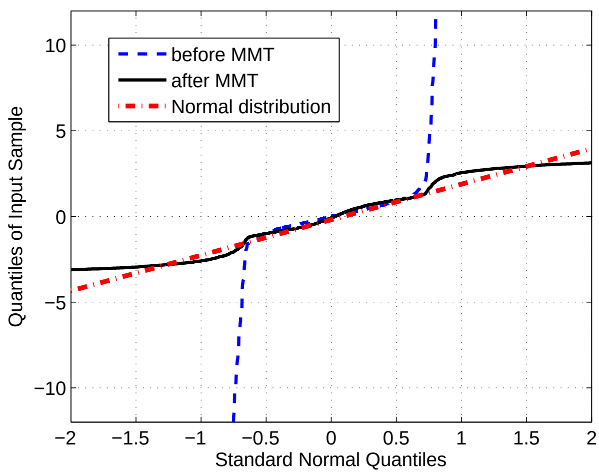

Figure 3: QQ-plot of the contaminated data in (2), before and after applying MMT.

The transfer function of MMT (6), as depicted in Fig. 1, is a companding function where the small valued signals are scaled almost linearly while the larger ones are compressed. Within this region, the MMT does not cause any phase perturbations. The transfer function of the modulus transformation is also included as a comparison. For extremely high values of the impulsive noise, the MMT shows a better suppression performance.

Fig. 2(a) and 2(b) plot a Gaussian signal contaminated by impulsive noise[1] and its MMT transformed counterpart. The QQ-plot in Fig. 3 shows that the MMT transformed signal is no longer heavy-tailed and is approximately Gaussian distributed.

It follows that with the application of MMT on the array data, the transformed data will comprise instances of $\mathbf{x}(t)$ that are almost linearly scaled and those that are compressed due to the presence of large impulses. As a result, the estimated array covariance matrix using MMT transformed data can provide a reasonably good approximation of the true array covariance matrix, due to the suppression of the large valued impulses, to allow DOA estimation with conventional methods. For the same impulsive noise model, the MMT performs better than the modulus transformation (MT) in [13, 14]. This will be illustrated in the next Section.

4. RESULTS ANALYSIS

In this section, we compare the DF performance of MMT with other existing robust covariance estimation methods, for both simulated and real data. For the simulations, we consider two far-field sources impinging on an uniform linear array of $N = 8$ omnidirectional sensors from the DOAs $\boldsymbol{\Theta} = [\theta_1 = -5^\circ \quad \theta_2 = 5^\circ]^T$ and $M = 200$ snapshots are used for DOA estimation. The RMSE performance metric is averaged over 10,000 Monte Carlo runs.

For the purpose of simulation and performance analysis, each entry in the vector $\mathbf{i}(t) + \mathbf{w}(t)$ from (2) is given as $r\,\exp(j\phi)$, where the random phase, ϕ, is drawn from a

[1] The impulsive noise is generated using the model described in (7).

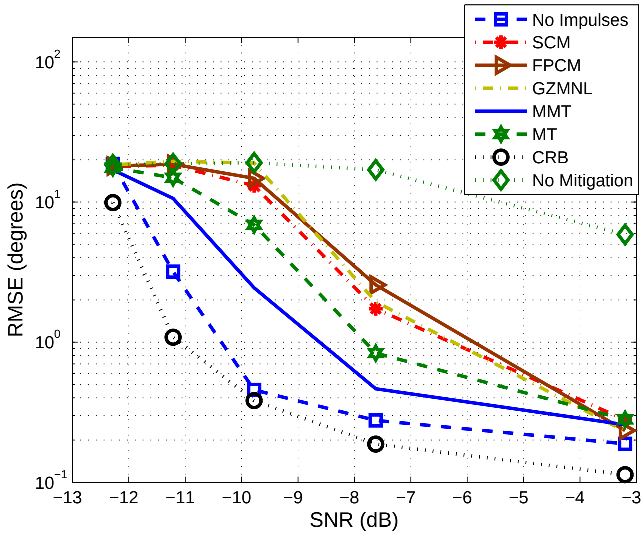

Figure 4: DF estimation accuracy of first DOA, $\theta_1 = -5°$: RMSE versus SNR by varying ε, with $\kappa = 75$ and $\gamma^2 = 1$. The corresponding values of ε are $[0.1,\ 0.3,\ 0.5,\ 0.7,\ 0.9]$.

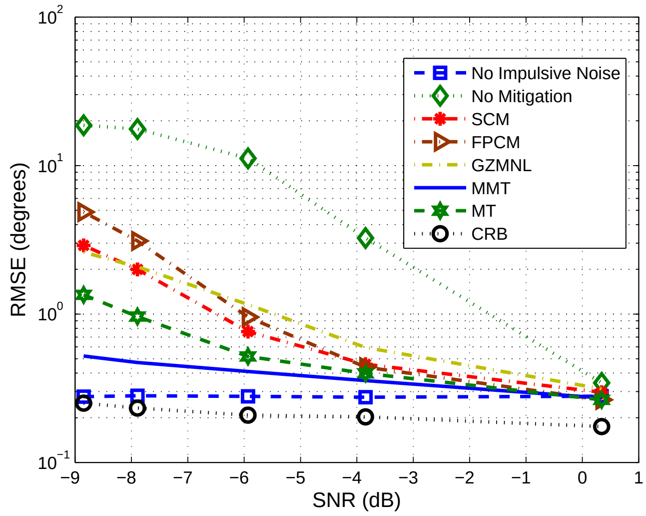

Figure 5: DF estimation accuracy of first DOA, $\theta_1 = -5°$: RMSE versus SNR by varying κ, with $\varepsilon = 0.3$, and $\gamma^2 = 1$.The corresponding range of κ is $[10,\ 30,\ 50,\ 80,\ 100]$.

uniform distribution between $[-\pi,\ \pi)$ while r is the magnitude of a random variable drawn from the ε-contaminated Gaussian mixture model (ε-cGMM) [15]:

$$f_\varepsilon(v) = (1-\varepsilon)\, f_G\left(v;\gamma^2\right) + \varepsilon f_G\left(v;\kappa\gamma^2\right), \qquad (7)$$

and where $0 \le \varepsilon \le 1$ represents the amount of contamination. The term $f_G\left(v;\gamma^2\right)$ is a Gaussian distribution with variance γ^2 and $\kappa > 1$ represents the relative strength of the impulsive component.

In this simulation, we used the MUSIC algorithm [1] for DOA estimation with array covariance matrix estimated after applying MMT. We also compared the DF accuracy performance against the MT method (with $\lambda = 0$); SCM method [11], fixed point covariance matrix (FPCM) estimate [9, 10], GZMNL method [12] and the no impulsive noise case, where perfect knowledge of the locations of the contaminated samples is available. The CRB on the accuracy of estimating the DOA vector Θ in (1) is also included as a comparison. The derivation of the CRB is given by equations (4)-(9) from [8].

Figs. 4-5 show the DF estimation accuracy of the first DOA, $\theta_1 = -5°$, versus the SNR, which is obtained by varying ε and κ respectively. In both Figures, throughout the range of SNR values, we note that our proposed method performs better than the existing techniques.

Fig. 6 shows an example of the high-resolution DF capability of our proposed methods, when three sources $\Theta = [-15°, 30°, 50°]^T$ are present. The results show that some existing methods fail to resolve the two closely-separated sources at 30° and 50°. In comparison, our proposed method performs very well in this scenario.

We also show the small-sample performance of our proposed method in Fig. 7. Even when the number of available array snapshots is reduced, we can see that the DF estimation accuracy of MMT still outperforms the other methods. It has the closest performance to the CRB among the methods considered here.

Besides the simulated data, the proposed method is also verified using real data. This data is obtained by collecting high frequency (HF) radio wave using the following measurement setup: Four Active HF antennas are connected to the data acquisition system developed by the Interactive Circuits and Systems (ICS) Limited and placed on the roof top of Block S2 School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. Fig. 8 shows the experimental setup.

The receiving ICS system has a 4-channel 14-bit analog-to-digital converter module in PCI Mezzanine Card (PMC) format, with sample rates of up to 100 MHz/channel and a digital down converter (DDC) option. In this experiment, the receiver frequency is 30MHz and set the effective sampling rate is 12.5MHz for each channel. Due to the congested nature of the HF band, we use short-time Fourier transform (STFT) to reduce the wide receiver bandwidth to a much narrower band so that we can observe the signals-of-interest at each channel.

The results of applying the MMT on the real parts of the data, from one channel, is shown in Fig 9. Similar results are observed for the other channels and are not produced here. It can be seen that the MMT gives good impulsive noise suppression performance. Though the magnitude of the data is changed (this happens for all 4 channels), the MMT still allows for high resolution DF. Fig 10 shows this capability. The MMT is able to pick up the 2 signal sources and with a better resolution, while the other existing methods do not perform so well. The results show the robustness of the proposed method. In both Figs. 6 and 9, the higher MUSIC spectral peak, after applying MMT, indicates that a more accurate covariance was likely to have been estimated.

5. CONCLUSION

We have proposed a modified modulus transformation to mitigate the impact of impulsive noise for high-resolution di-

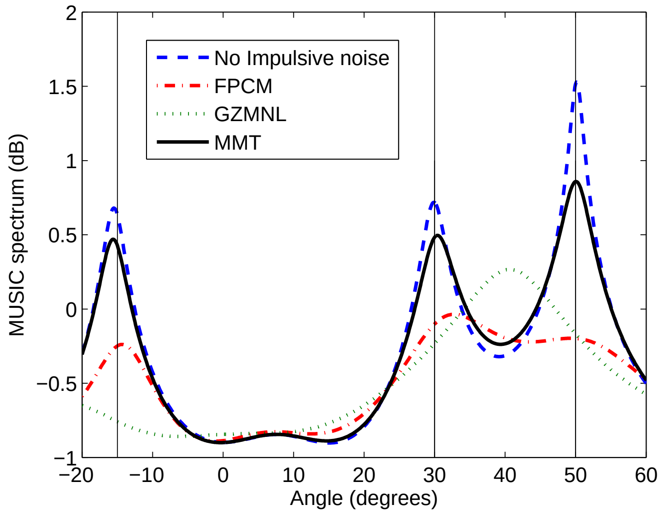

Figure 6: DF in the presence of three sources: MUSIC spectrum with $\varepsilon = 0.5$, $\kappa = 100$ and $\gamma^2 = 1$.

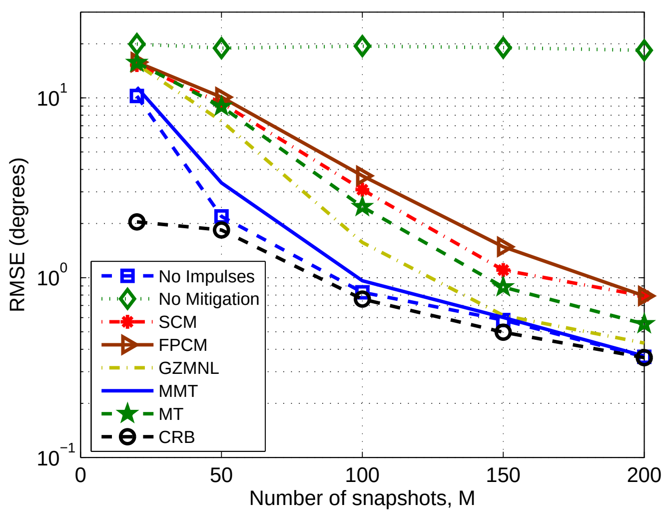

Figure 7: DF estimation accuracy of first DOA, $\theta_1 = -5°$: RMSE versus number of array snapshots M, with $\varepsilon = 0.5$, $\kappa = 100$ and $\gamma^2 = 1$.

rection finding. This transformation is simple to implement and works well against heavy-tailed data. This is illustrated by simulation and real data results, which show better accuracy performance of direction of estimation algorithms for the proposed method over existing approaches.

We are developing theoretical analysis of the proposed transformation, in terms of its detection performance. Notwithstanding this, we have provided the motivation of the proposed transformation, through the transfer function and the results analysis.

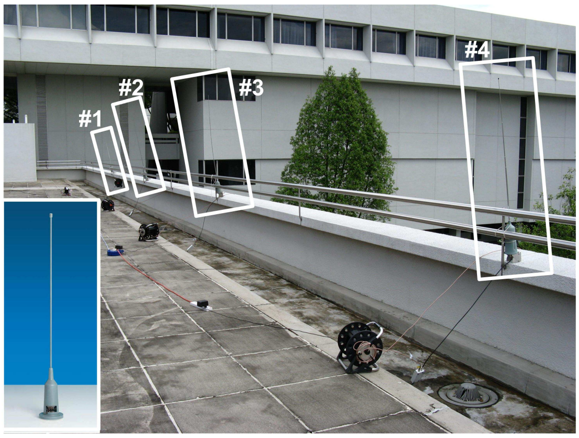

Figure 8: Experimental setup.

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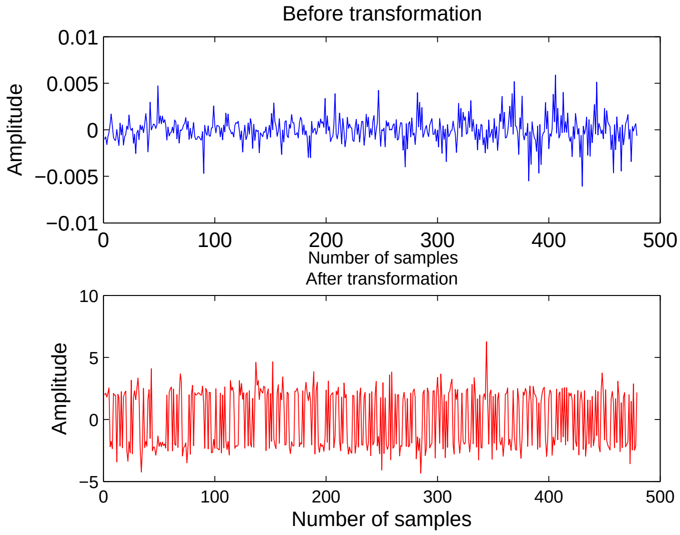

Figure 9: Transformation of the data (real parts), from one channel, by applying MMT.

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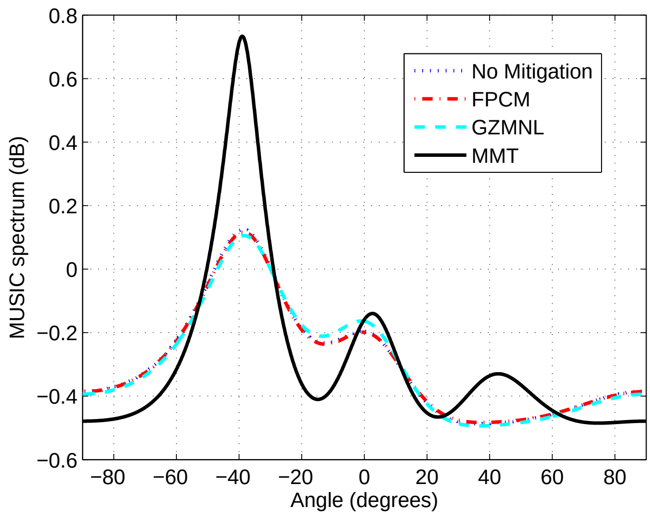

Figure 10: DF using real HF data, with no control over the transmission.