LOW COMPLEXITY MEAN ENHANCED GREEDY ALGORITHMS FOR DYNAMIC SUBCARRIER ALLOCATION IN UPLINK LTE

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ABSTRACT

This paper proposes two lower complexity mean enhanced greedy (MEG) algorithms for dynamic subcarrier allocation (DSA) in uplink Long Term Evolution (LTE) systems. The computational savings are achieved by eliminating some of the iterative steps in the existing MEG algorithm. We also investigate the effect of different optimization criteria on DSA. Simulation results show that the proposed lower complexity algorithms, have near optimal uncoded bit error rate (BER) and capacity performances when the minimum BER optimization criterion is employed. This is in contrast to the poorer average BER performances achieved using maximum capacity or maximum channel gains as optimization criteria.

1. INTRODUCTION

Multicarrier wireless broadband systems have been shown to combat effectively, severe frequency selective channels. Although, orthogonal frequency division multiple access (OFDMA) is the most widely investigated multicarrier system, in this paper, we investigate DSA for single carrier - frequency division multiple access (SC-FDMA) [1]. The European third generation partnership project - long term evolution (3GPP-LTE) has chosen SC-FDMA as the uplink modulation technique, because it has a lower peak to average power ratio (PAPR) and a higher frequency diversity than the OFDMA system [2].

Dynamic subcarrier allocation (DSA) algorithms for OFDMA and SC-FDMA have been much explored in literature, [3]-[7] and the references therein. The real time nature of wireless communication systems restrict the use of complex optimal algorithms for DSA, therefore, most of the algorithms are based on sub-optimal greedy methods. Greedy algorithms have the ability to be fast with a low computational burden, however, the performance of these methods may be insufficient. Lots of work has been done on improving the performance of greedy algorithms, while keeping their computational complexity low. In [9], the mean enhanced greedy (MEG) algorithm was proposed, which has a near optimal BER and capacity performance with a much lower computational complexity than the optimal Hungarian algorithm [8]. It also significantly outperforms the 2-D greedy algorithm [6] [5] [3] with a comparable complexity. Another important factor affecting the performance of DSA algorithms is the optimization criterion used for finding the suitable allocations. The most commonly used criterion is the maximum capacity or spectral efficiency (SE) [3], where the aim is to find the allocation with the largest sum total capacity. In [4], [5] and [6], the maximum channel gain criterion is employed, where the allocation with the largest sum channel gain is found, with the aim of maximizing the sum capacity. To the best of our knowledge, is is only in [4], where a comparison of the maximum channel gains and maximum capacity criteria is performed, in terms of minimizing the total transmit power for an OFDMA system. The minimum BER criterion, which entails finding the allocation with the smallest sum BER across all the users is not explored in the literature.

In this paper, we propose two lower complexity mean greedy algorithms, which are achieved by eliminating some of the iterative steps in the MEG algorithm [9]. The first algorithm, referred to as the single mean enhanced greedy (SMEG) algorithm, eliminates the iterative calculation of the "mean stage", and replaces it with one mean calculation. The second algorithm, referred to as the random mean enhanced greedy (RMEG) algorithm, restricts the number of users allowed to search for the best available subcarrier block, and assigns subcarrier blocks randomly to those users not allowed to compete. These random allocations incur negligible computational cost, similar to the round robin system. The computation reductions have both BER and capacity performance penalties; however, this is greatly dependent on which of the optimization criteria is employed, which we show with simulations.

This paper is organized as follows; Section 2, describes the system model, while DSA and the optimiation criteria are discussed in Section 3. The proposed algorithms are described in Section 4. The complexity of the discussed algorithms and simulation results are presented in Sections 5 and 6, respectively. The paper is concluded in Section 7.

2. SYSTEM MODEL

We consider an uncoded uplink SC-FDMA [1] system, with U users, as illustrated in Fig. 1 and K blocks of subcarriers ($K = U$), with each block made up of M subcarriers. The M data symbols for the uth user are transformed into the frequency domain by the fast Fourier transform (FFT). The uth user's frequency domain data vector $\mathbf{X}_u = [X_u(0) \cdots X_u(M-1)]^T$ is then mapped onto the whole subcarrier set of size N ($N = KM$) subcarriers, localized [1] subcarrier mapping is employed, where contiguous subcarriers are allocated to the same user. The mapped data are then transferred back into the time domain by N-point inverse FFT (IFFT) and each block of N time domain symbols is prepended with a cyclic prefix (CP) before transmission. The CP is discarded at the receiver to remove the inter-block interference and make the

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channel appear to be circular [10]. The received signals are transferred into the frequency domain by N-point FFT, which is followed by subcarrier de-mapping. Frequency domain equalization is performed for each user, and the equalized signals are transferred back into the time domain by M-point IFFT. The total received signal at the base station is given by

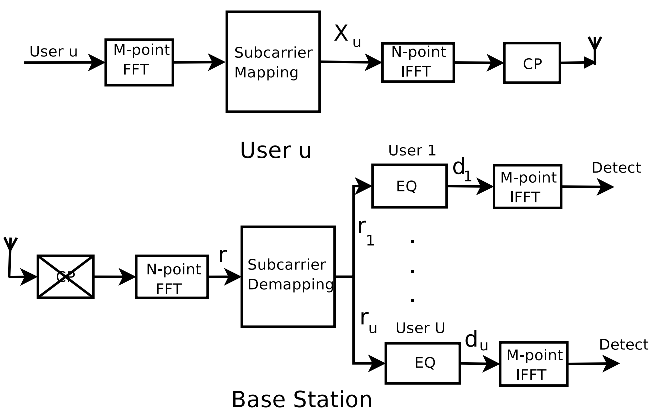

Figure 1: SC-FDMA Block Diagram

$$\mathbf{r} = \sum_{u=0}^{U-1} \mathbf{H}_u \mathbf{M}_u \mathbf{X}_u + \mathbf{n}_u \quad (1)$$

where $\mathbf{r}$ is the frequency domain received signal for all users. $\text{diag}\{\mathbf{H}_u\} = [H_u(0), \cdots, H_u(N-1)]^T$, $H_u(n) = \sum_{l=0}^{L} h_u(l) exp(-j2\pi nl/N)$, $(n = 0, \cdots N-1)$, is the discrete frequency response of the channel for the uth user, where $h_u(l)$ $(l = 0, 1, \cdots, L)$ is the lth path gain of channel impulse response (CIR) between the uth user and the base station, assumed to be an independent and identically distributed (i.i.d.) complex random variable with Rayleigh distributed amplitude and uniformly distributed phase. $\mathbf{n}_u$ is the complex additive white Gaussian noise (AWGN) with variance $N_0/2$ per dimension. The mapping matrix that determines the allocation of subcarriers to the uth user is given by $\mathbf{M}_u$ below

$$\mathbf{M}_u = \begin{bmatrix} \mathbf{0}_{(kM)\times M} \\ \mathbf{I}_M \\ \mathbf{0}_{(N-kM-M)\times M} \end{bmatrix} \quad (2)$$

where k $(0 \le k \le K)$ is the index of the block of contiguous subcarriers allocated to the uth user. Using the orthogonality principle [11], the optimal minimum mean square equalizer (MMSE) weight matrix $\mathbf{W}_u$ for each user is given by

$$\mathbf{W}_u = \bar{\mathbf{H}}_u^H (\bar{\mathbf{H}}_u \bar{\mathbf{H}}_u^H + N_0 \mathbf{I}_M)^{-1} \quad (3)$$

where superscript H is the Hermitian transpose operation, $\mathbf{I}_M$ the $M \times M$ identity matrix and $\bar{\mathbf{H}}_u = \mathbf{M}_u^T \mathbf{H}_u \mathbf{M}_u$ is the uth user's $M \times M$ effective channel matrix de-mapped from $\mathbf{H}_u$. Each user's data is equalized differently as shown in Figure 1. Therefore, the equalized data $\mathbf{d}_u$ for the uth user is $\mathbf{d}_u = \mathbf{W}_u \mathbf{r}_u$, where $\mathbf{r}_u = \mathbf{M}_u^T \mathbf{r}$, is the uth user's $M \times 1$ data vector de-mapped from $\mathbf{r}$. It can also be derived that the output SINR for the uth user is given by

$$\gamma_u = \frac{1}{\frac{1}{M}\sum_{m=0}^{M-1} \frac{1}{|H_u(kM+m)|^2/N_0+1}} - 1 \quad (4)$$

3. DYNAMIC SUBCARRIER ALLOCATION

In this section, we formulate the DSA problem as a discrete combinational assignment problem [8] and discuss suitable optimization criteria used in conjunction with algorithms to solve the described problem.

3.1 Problem Formulation

The objective is to allocate one block of subcarriers to one user with the aim of minimizing or maximizing the sum BER or capacity, respectively. Therefore, to find the optimal allocation, the following cost function must be optimized

$$J = \sum_{u=0}^{U-1} \sum_{k=0}^{K-1} b_{uk} s_{uk} \quad (5)$$

subject to:

$$s_{uk} \in \{0, 1\} \quad (6)$$

$$\sum_{u=0}^{U-1} s_{uk} = 1 \quad (7)$$

$$\sum_{k=1}^{K-1} s_{uk} = 1 \quad (8)$$

where b_{uk} may be some suitable wireless communication optimization criteria of the kth block of M subcarriers for the uth user. When the kth block of subcarriers is allocated to the uth user $s_{uk} = 1$, otherwise $s_{uk} = 0$. Constraints (6) - (8) ensure that only one block of subcarriers is allocated to a user (subcarriers cannot be shared).

3.2 Optimization Criteria

Here, we describe different optimization criteria used to solve (5) - (8). The optimization criterion is represented by b_{uk} in (5). The discussed criteria are the maximum channel gain, maximum capacity and minimum BER criteria.

Maximum Channel Gain: This criterion is evaluated by averaging the M subcarrier gains in the K blocks of subcarriers for the uth user, given by

$$b_{uk} = \frac{1}{M} \sum_{n=kM}^{kM+M-1} |H_u(n)| \quad (9)$$

It is employed in [5] and [6], for the clustered OFDMA and SC-FDMA systems, respectively.

Maximum capacity: This criterion is evaluated by estimating the capacity for all the available blocks of subcarriers. The capacity is related to the output SINR by the equation, $C_u = \log_2(1+\gamma_u)$. Therefore, the capacity for each of the K blocks of subcarriers is given by

$$b_{uk} = BW\, log_2 \left(\frac{1}{\frac{1}{M}\sum_{n=kM}^{kM+M-1} (|H_u(n)|^2/N_0 + 1)^{-1}} \right) \quad (10)$$

where BW is the total bandwidth for all the subcarriers in

Algorithm 1 SMEG Algorithm

1: **for all** u **do**
2: $\quad T_u = \frac{1}{K}\sum_{k=1}^{K} b_{uk}$
3: **end for**
4: $U_{order} = sort\{T_u\}$ {rank to get user order}
5: **while** $elements\_in\_(U_{order}) > 1$ **do**
6: $\quad \tilde{k} = max(b_{\tilde{u}k})$ {highest gain for that user}
7: $\quad A \leftarrow b_{\tilde{u}\tilde{k}}$
8: $\quad b_{\tilde{u}k} = \emptyset$ {remove user gains}
9: $\quad b_{u\tilde{k}} = \emptyset$ {remove allocated subcarriers}
10: **end while**

Algorithm 2 RMEG Algorithm

1: **for all** u **do**
2: $\quad T_u = \frac{1}{K}\sum_{k=1}^{K} b_{uk}$
3: **end for**
4: $U_{order} = sort\{T_u\}$ {rank to get user order}
5: $U_1 \subset U$ {select the orders needed}
6: **while** $element\_in\_(U_1) > 1$ **do**
7: $\quad \tilde{k} = max(b_{\tilde{u}k})$ {highest gain for that user}
8: $\quad A \leftarrow b_{\tilde{u}\tilde{k}}$
9: $\quad b_{\tilde{u}k} = \emptyset$ {remove user gains}
10: $\quad b_{u\tilde{k}} = \emptyset$ {remove allocated subcarriers}
11: **end while**
12: Randomly assign the remaining subcarrier blocks to the remaining users not un set U_1

a block. This criterion is used in many OFDMA subcarrier algorithms, and in [3] for SC-FDMA.

Minimum BER: The BER of each block of subcarriers is related to the output SINR by the equation, $\text{BER}_u = Q\sqrt{\gamma_u}$, where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-\frac{t^2}{2}}\,dt$ is the Q function. Therefore, the BER for each of the K blocks of subcarriers is given by

$$b_{uk} = Q\left(\sqrt{\frac{1}{\frac{1}{M}\sum_{n=kM}^{kM+M-1}(|H_u(n)|^2/N_0+1)^{-1}} - 1}\right) \quad (11)$$

This is the least investigated DSA optimization criterion in literature.

4. LOW COMPLEXITY MEAN ENHANCED GREEDY SUBCARRIER ALLOCATION ALGORITHMS

The motivation behind the proposal of the two algorithms discussed in this section, is the need for a reduction in the computational complexity of DSA, because a reduction in complexity is necessary for the efficient realization of DSA in real time wireless communication systems. The proposed algorithms are based on the MEG algorithm [9]; however, they are of much lower complexity. The MEG algorithm was originally developed to be used with the maximum channel gain criterion (9), but its complexity is relatively high due to the iterative calculation of the mean of users' optimization criterion.

4.1 Single Mean Enhanced Greedy (SMEG) Algorithm

The order with which the users are allowed to greedily compete for subcarriers is determined by a *single* calculation of the means for all the users b_{uk}'s and ranking them. This is unlike the original MEG algorithm that dynamically determines this order at every iteration by re-calculating the mean of the available b_{uk}'s after every user allocation. So for a system with large number of users, the complexity is greatly reduced by cutting out the computational time used for the mean re-calculations. This algorithm is described in Algorithm 1.

4.2 Random Mean Enhanced Greedy (RMEG) Algorithm

This algorithm extends the computational complexity savings obtained in SMEG algorithm. The mean of the users b_{uk}'s are calculated once, however, greedy search is limited to a fixed number of users. After these users have iteratively searched for subcarrier blocks, the remaining users are allocated subcarrier blocks *randomly*. The users with average poor performing subcarriers are allowed to compete (made active) for subcarriers based on the quality of their criterion, while those users that have on average, good subcarriers are not given the opportunity to compete (left idle), and are allocated the remaining available subcarriers. For example, in a $U = 100$ user system, $U_1 = 50$ poorly ranked users are allowed to search for subcarrier blocks based on the performance of their b_{uk}'s, while the remaining users are allocated subcarriers blocks randomly without any consideration of their b_{uk} performance. The complexity of the system is reduced because only U_1 greedy search iterations are carried out, while the random allocation incurs negligible computational costs (similar to the fixed allocation/round-robin system). The percentage of randomness is given by $R\% = (1 - \frac{U_1}{U})100$, so for $U_1 = 75$ the amount of randomness is $R = 25\%$. This algorithm is described in Algorithm 2.

Note that in both algorithms, when the criterion is the minimum BER, the mean of the b_{uk}'s would be sorted in descending order, while line 6 in SMEG and line 7 in RMEG would be minimized instead of maximized.

5. COMPLEXITY ANALYSIS

The SMEG algorithm has two main parts that affect the computational complexity. Firstly, the calculation of the mean of the b_{uk}'s for all the users and secondly, the comparisons made by each user to find the best subcarrier block out of the available blocks. The number of computations needed for calculating the mean in a U user system is given by U^2, while the total number of computations for the iterative greedy search for all the users is given by $1/2(U^2 - U)$ [9]. Therefore, the total number of computations needed for a full allocation is $3/2U^2 - 1/2U = 1/2(U^2 - U) + U^2$.

The complexity for the RMEG algorithm is similar. It requires the same number of computations to calculate the mean of the b_{uk}s for all the users. However, only a certain number of users U_1, are allowed to carry out the iterative greedy search, with the number of computations given by $1/2(U_1^2 - U_1)$. Therefore, the total number of computations is given by $1/2(U_1^2 - U_1) + U^2$, where U_1 is a fraction of U.

Table 1 shows the computational complexity for all the discussed algorithms. The Hungarian, MEG and 2-D greedy

Table 1: Number of operations for the algorithms

Algorithm	# of operations
Hungarian [8]	$(11U^3+12U^2+31U)/6$
MEG [9]	$\frac{1}{3}U^3+U^2-\frac{1}{3}U$
2-D Greedy [6][5]	$\frac{1}{3}U^3+\frac{1}{2}U^2-\frac{5}{6}U$
SMEG	$3/2U^2-1/2U$
RMEG	$1/2(U_1^2-U_1)+U^2$

Table 2: Normalized complexity of the algorithms

Algorithm	Normalized Complexity
Hungarian	124
MEG	22.96
2-D Greedy	22.63
SMEG	1
RMEG (25% randomness)	0.85
RMEG (50% randomness)	0.75

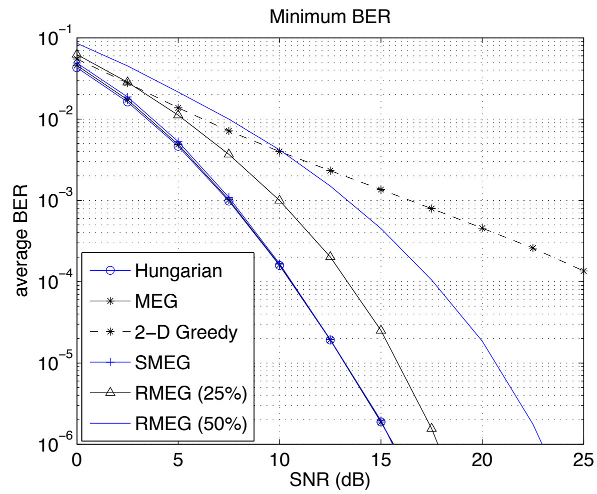

Figure 2: Average BER performance of the algorithms using the min. BER as criterion, $U = 100$ users

algorithms all have cubic expressions for the total number of computations needed for a complete allocation based on the number of users U in the system, while the proposed algorithms have square expressions, with the complexity of the RMEG algorithm based mostly on the number of users in the chosen set U_1. Furthermore, Table 2, provides a normalized numerical complexity weight based on a system with 100 users. Each of the values are normalized with the numerical value for the proposed SMEG greedy algorithm. The proposed algorithms Hungarian, MEG and 2-D greedy algorithms are 124, 22.96 and 22.63 as complex as the proposed SMEG algorithm, respectively. The RMEG algorithm has a fraction of the complexity of the SMEG algorithm which depends on the amount of randomness allowed.

6. SIMULATION RESULTS

The uplink SC-FDMA model for 3GPP-LTE is employed with a bandwidth of 18 MHz. This bandwidth is divided into 1200 subcarriers which are grouped into 100 blocks with 12 subcarriers in each block. There is a subcarrier spacing of 15 KHz, therefore, each block of subcarriers is 180 KHz wide. The channel model used for all simulations is the standard typical urban (TU) area channel with 6 taps, with an approximate RMS delay spread of 1 μs [2] and perfect channel state information (CSI) is assumed. All data are QPSK modulated and normalized to unit energy and the SNR is defined as the average ratio of the received signal power to noise power. RMEG algorithm has 25% and 50% randomness settings, which corresponds to $U_1 = 75$ and $U_1 = 50$, respectively. Monte Carlo simulation is used to determine the required performances.

There are two major observations to note that aid the results interpretation. i) the algorithms are general and can be used with any optimization criteria (b_{uk}). ii) Performances measures are average BER and total capacity. Therefore, the notion of an optimal performance not only depends on the algorithm, but also on the criteria used. For example, Hungarian (max. capacity) means the Hungarian algorithm used with the maximum capacity criterion. This is optimal when the total capacity performance is investigated, but will not be optimal for the average BER performance, the optimal combination of algorithm and criterion for the average BER performance is the Hungarian with minimum BER criterion, which is denoted by the Hungarian (min. BER) in the figures.

Fig. 2 shows the average BER performance for the algorithms with the minimum BER as criterion. The MEG and the proposed SMEG achieve near optimal performance and are almost indistinguishable from the optimal performance given by the Hungarian algorithm. Furthermore, the RMEG algorithm, with 25% randomness has about a 2 dB performance penalty compared to the optimal case, while it has approximately 11 dB performance gain over the 2-D greedy algorithm at a BER $= 10^{-4}$. In addition, the RMEG algorithm with 50% randomness also outperforms the existing 2-D greedy algorithm by at least 7.5 dB at BER $= 10^{-4}$; however, it has a performance loss of about 7 dB when compared to the MEG algorithm. Nevertheless, this algorithm provides a suitable trade-off between performance and complexity which is achieved by choosing a suitable number of users in U_1. Lower U_1 means higher randomness, lower complexity and poorer performance.

Fig. 3 illustrates the average BER performance for the algorithms with the maximum channel gain as criterion. Under this criterion, the proposed low complexity algorithms have almost linear improvements in their BER performance SNR values above 10 dB, compared to the exponential BER improvement under the minimum BER performance in Fig. 2. Also note that the Hungarian algorithm under this criterion has approximately 2 dB performance loss at BER $= 10^{-5}$ compared to the optimal case (Hungarian algorithm with minimum BER as criterion). Furthermore, comparing Fig. 2 and Fig. 3, we see that the MEG algorithm is insensitive to the criterion used, it provides almost identical performances under both criteria.

Fig. 4 compares the total capacity performance of the proposed algorithms with the minimum BER criterion and to the optimal case (Hungarian algorithm with the maximum capacity as criterion). The SMEG algorithm has a near-optimal performance, with approximately 4 Mbps (about 40

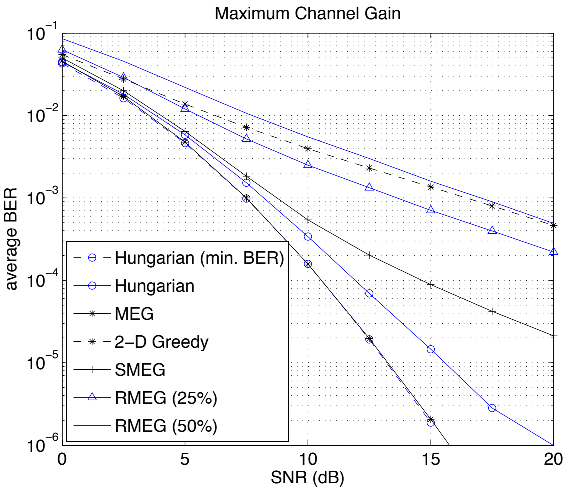

Figure 3: Average BER performance of the algorithms using the max. channel gains as criterion, $U = 100$ users

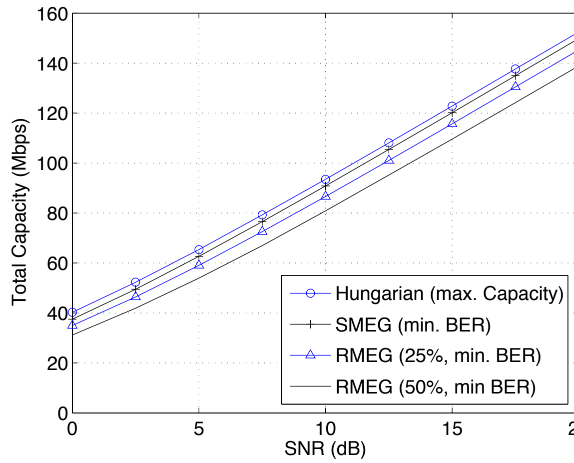

Figure 4: Average capacity performance of the algorithms with the Hungarian algorithm using the max. capacity as criterion and the other algorithms using min. BER as criterion, $U = 100$ users

Kbps for each of the 100 users) penalty at all SNR levels. The 25% and 50% versions of the RMEG algorithm have slightly worse performance penalties, with about 10 Mbps (100 Kbps per user) and 15 Mbps (150 Kbps per user) loss, respectively. The capacity performances of the other algorithms coupled with the other criteria have been omitted for clarity's sake; however, their performances, will lie in-between the optimal and the SMEG performance, which is rather a negligible performance gap. It can be observed in Fig.4 that there are smaller capacity performance gaps between the algorithms, compared to the BER performance gaps in Fig. 2. This implies that, both the BER and capacity performances should be evaluated to determine the complete performance of an algorithm. Note that, the poor BER performance of an algorithm, (e.g., the 2-D greedy algorithm) would require a lot of re-transmissions of the data detected in error, which would result in a severe degradation of the system throughput, even though fig. 4 shows that its theoretical capacity performance is almost optimal.

7. CONCLUSION

In this paper, we have proposed two subcarrier allocation algorithms for the uplink LTE system, which have a much lower computational complexity than the MEG algorithm [9]. The SMEG algorithm provides large computational complexity reduction, with a negligible performance penalty, while the RMEG algorithm offers a complexity - performance trade-off. We also show that these performance degradations can be kept to a minimum if the minimum BER criterion is coupled with these algorithms for optimization. Furthermore, we have also shown that the original MEG algorithm is very insensitive to the optimization criterion, and it provides a good performance, irrespective of the optimization criterion used, while the other algorithms BER performances are very sensitive to the optimization criteria used.

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