MULTI-TARGET TRACKING WITH MCMC-BASED PARTICLE FILTERS

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ABSTRACT

In this article, we address the problem of multiple target tracking. Particle filter solutions must mainly cope with two problems: a high-dimensional problem and a data association problem. We propose to solve both problems simultaneously thanks to a particle filter based on a Gibbs sampler that simulates both state space and association variables. We present two possible implementations of this solution that differ in their inner structure: the first one samples the conditional densities with a Hastings-Metropolis algorithm while the second one uses importance sampling. These algorithms are shown to be as efficient as JPDA particle filters, with a dramatic reduction of the computational cost.

1. INTRODUCTION

Multi-target tracking is a well-known problem that consists of sequentially estimating the states of several targets from noisy data. It is encountered in many applications, for instance aircraft tracking from radar measurements [5] or football player tracking in a video sequence [9]. Solutions of this problem using particle filters have been proposed in the past ten years [5, 10, 6]. Two problems are generally faced:

- a dimensional problem: the state vector gathers all target states and its size increases with the number of targets. In this high-dimensional state space, particle filters tend to become inefficient.
- a data association problem: data consist of a set of measurements resulting from a thresholding procedure. Each measurement may correspond either to a target or to a false alarm. Conversely, a target may have been either detected or missed. How to determine which targets have been detected, and which measurement corresponds to each detected target?

Although this second problem may not be faced in some multitarget tracking applications, we will consider here situations where both dimensional and data association issues arise. The Joint Probabilistic Data Association (JPDA) filter is a classical approach to tackle the data association problem [2]. It considers all possible associations between targets and measurements, and solves the tracking problem by estimating the marginal posterior density of each target state. Interestingly, this approach also solves the dimensional problem since it effectively resorts to one filter per target [10]. Several JPDA particle filters have been proposed in the literature [3, 10, 1]. For a small number of targets and measurements, this approach is very efficient. However, the number of possible associations combinatorially grows with the number of targets and measurements. For target tracking at low SNR and a fixed detection probability, many false alarms occur and the JPDA particle filter becomes intractable.

Other alternative particle filters have been proposed to solve the multi-target tracking problem. A particle filter based on Probabilistic Multiple Hypothesis Tracker (PMHT) strategy has been proposed in [5]. It requires the estimation of false alarm and association probabilities. The problem is solved via a Gibbs sampler that estimates these densities, prior to the computation of the particle weights. However the PMHT approach which is based on an independence hypothesis generally leads to degraded performance compared to JPDA filters [11]. In [10], a new particle filter strategy has been proposed to solve the association problem at a tractable cost. In this solution, association variables are sampled as well as target states by means of a proposal distribution that permits to factorize the importance weights over the individual target associations and thus solve the dimensional problem. However this procedure involves additional resampling steps for the cumulative weights, and must be repeated several times to avoid depletion problems in the final Monte Carlo representation of the posterior density, thus leading again to important computational costs. Another solution for the dimensional problem, described in [6], consists in replacing the importance sampling approach in the classical particle filter by a Monte Carlo Markov Chain (MCMC) method [7]. However this interesting solution does not deal with the combinatorial problem.

We propose in this article two filters that can solve both dimensional and association problems at a computational cost linear in the number of targets and observations. These two solutions resort to a Gibbs sampler [4] to break the high-dimensional structure of the state space by sampling each target conditionally to the othersThe first solution estimates these conditional densities via a Hastings-Metropolis algorithm within the Gibbs sampler; it therefore presents a full MCMC structure. On the contrary, the second solution performs an importance sampling step within the Gibbs sampler in order to estimate the conditional densities. This hybrid structure permits to keep advantages of both MCMC and importance sampling algorithms.

This article is organised as follows: the multi-target model is presented in section 2. The two proposed algorithms are derived in section 3. Finally we present simulations and conclusions in section 4.

2. THE MULTI-TARGET MODEL

In this section, we describe the multi-target model. Note that we will assume throughout this article that the number of targets M_T is known and constant over time.

Let us denote by $\mathbf{x}_{k,n}$ the vector describing the state of the n^{th} target ($n \in \{1, M_T\}$) at time instant k. Similarly, at this time instant, the observations are provided by a set of M_k measurements $(\mathbf{z}_{k,j})_{j\in\{1,M_k\}}$. A variable $\theta_{k,n}$ can be introduced to associate target n to the observation it generated; this association variable is defined as

$$\theta_{k,n} = \begin{cases} j & \Rightarrow \begin{cases} \text{if the observation } \mathbf{z}_{k,j} \text{ is generated} \\ \text{by target } n \text{ with state vector } \mathbf{x}_{k,n}; \end{cases} \\ 0 & \Rightarrow \text{ if target } n \text{ has not been detected.} \end{cases}$$

State vectors $(\mathbf{x}_{k,n})_{n\in\{1,M_T\}}$, measurement vectors $(\mathbf{z}_{k,j})_{j\in\{1,M_k\}}$ and association variables $(\theta_{k,n})_{n\in\{1,M_T\}}$ can be concatenated into vectors $\mathbf{x}_k$, $\mathbf{z}_k$ and θ_k respectively.

With these notations, the n^{th} target state $\mathbf{x}_{k,n}$ can be modelled by the general state equation:

$$\mathbf{x}_{k,n} = f_k(\mathbf{x}_{k-1,n}, \mathbf{v}_{k-1,n}), \tag{1}$$

where $f_k(\cdot,\cdot)$ represents the target dynamics at time instant k, and $\mathbf{v}_{k,n}$ is a (possibly non gaussian) noise vector. Since measurements can be generated either by a real target or by a false alarm, the observation model is two-fold:

- If the measurement is generated by a target, it is modelled by a standard measurement equation;
- If the measurement corresponds to a false alarm, it is modelled by a uniform random variable [2, 10].

This can be summarized as follows:

$$\mathbf{z}_{k,j} = \begin{cases} h_k(\mathbf{x}_{k,n}, \mathbf{n}_{k,n}) & \text{if } \exists n \in \{1, M_T\} \text{ s.t. } \theta_{k,n} = j \\ \mathbf{u}_{k,j} & \text{otherwise,} \end{cases}$$

where $h_k(\cdot,\cdot)$ models the relationship between the target state and the observation, $\mathbf{n}_{k,n}$ is a (possibly non gaussian) noise vector, and $\mathbf{u}_{k,j}$ is a uniform variable of probability density $p(\mathbf{u}_{k,j}) = 1/\mathscr{V}$ over the observation window of volume $\mathscr{V}$.

3. GIBBS SAMPLER BASED PARTICLE FILTERS

The classical particle filter aims at estimating the objective density $p(\mathbf{x}_k|\mathbf{z}_{1:k})$ via a set of particles. In multitarget tracking, this leads to the two aforementioned issues of high dimension and association. The JPDA particle filter solves the high dimensional problem by sampling the marginal densities $p(\mathbf{x}_{k,n}|\mathbf{z}_{1:k})$ instead of the objective density $p(\mathbf{x}_k|\mathbf{z}_{1:k})$, and solves the association problem by considering all possible associations θ_k. However it faces a combinatorial growth of the number of possible associations with the number of targets and measurements. In this article, we consider instead a particle filter that estimates the posterior density $p(\mathbf{y}_k|\mathbf{z}_{1:k})$ of the completed variable $\mathbf{y}_k \triangleq [\mathbf{x}_k^T,\ \theta_k]^T$. For clarity in the calculations, we will decompose this variable in the form

$$\mathbf{y}_k = [\mathbf{y}_{k,1}^T, \mathbf{y}_{k,2}^T, \ldots, \mathbf{y}_{k,M_T}^T]^T \quad \text{with} \quad \mathbf{y}_{k,n} = [\mathbf{x}_{k,n}^T,\ \theta_{k,n}]^T.$$

Sampling the completed variable $\mathbf{y}_k$ with importance sampling as in standard particle filter leads to degraded performance compared to JPDA particle filter [10]. We propose here to use a Gibbs sampler [7] in order to draw samples according to the posterior distribution $p(\mathbf{y}_k|\mathbf{z}_{1:k})$. More precisely, our Gibbs sampler works by sampling the entries $\mathbf{y}_{k,n}$ according to the conditional densities $p(\mathbf{y}_{k,n}|\mathbf{y}_{k,-n}, \mathbf{z}_{1:k})$, where $\mathbf{y}_{k,-n} = [\mathbf{y}_{k,1}^T, \ldots, \mathbf{y}_{k,n-1}^T, \mathbf{y}_{k,n+1}^T, \ldots, \mathbf{y}_{k,M_T}^T]^T$. This one-target-at-a-time feature permits to break the high dimensionality of the problem. However, the conditional densities $p(\mathbf{y}_{k,n}|\mathbf{y}_{k,-n}, \mathbf{z}_{1:k})$ are themselves difficult to sample directly and we must resort to simulation techniques. We propose two solutions for the simulation of these conditional densities. The first one is based on a Hastings-Metropolis algorithm, and the second one on importance sampling.

3.1 Hastings-Metropolis-within-Gibbs particle filter

The Hastings-Metropolis algorithm (HMA) [7] is based on a simple acceptation-reject principle. At each iteration, a proposed sample $\hat{\mathbf{y}}_{k,n}$ is drawn according to an instrumental law $q(\mathbf{y}_{k,n}|\mathbf{y}_{k,-n}, \mathbf{y}_{k-1}, \mathbf{z}_{1:k})$ and is accepted with a probability defined by the Hastings-Metropolis (HM) ratio, given by

$$\alpha = \frac{p(\hat{\mathbf{y}}_{k,n}|\mathbf{y}_{k,-n}, \mathbf{z}_{1:k})\, q(\mathbf{y}_{k,n}^{i-1}|\mathbf{y}_{k,-n}, \mathbf{y}_{k-1}, \mathbf{z}_{1:k})}{p(\mathbf{y}_{k,n}^{i-1}|\mathbf{y}_{k,-n}, \mathbf{z}_{1:k})\, q(\hat{\mathbf{y}}_{k,n}|\mathbf{y}_{k,-n}, \mathbf{y}_{k-1}, \mathbf{z}_{1:k})},$$

where $\mathbf{y}_{k,n}^{i-1}$ denotes the last accepted sample.

3.1.1 Computation of $p(\mathbf{y}_{k,n}|\mathbf{y}_{k,-n}, \mathbf{z}_{1:k})$

Using Bayes law, we can write

$$p(\mathbf{y}_{k,n}|\mathbf{y}_{k,-n}, \mathbf{z}_{1:k}) \propto p(\mathbf{z}_k|\mathbf{y}_k, \mathbf{z}_{1:k-1})\, p(\mathbf{y}_{k,n}|\mathbf{y}_{k,-n}, \mathbf{z}_{1:k-1}),$$

where the proportionality factor, equal to $p(\mathbf{z}_k|\mathbf{y}_{k,-n}, \mathbf{z}_{1:k-1})$, disappears in the HM ratio since it doesn't depend on target n. Conditionally to $\mathbf{y}_k$, the target contributions can be separated from false alarms in $p(\mathbf{z}_k|\mathbf{y}_k, \mathbf{z}_{1:k-1})$:

$$p(\mathbf{z}_k|\mathbf{y}_k, \mathbf{z}_{1:k-1}) = \frac{1}{\mathscr{V}^{\max(0, M_k - M_T)}} \prod_{q=1}^{M_T} p(\mathbf{z}_{k,\theta_{k,q}}|\mathbf{x}_{k,q}), \tag{2}$$

where we assumed that $p(\mathbf{z}_{k,\theta_{k,q}}|\mathbf{x}_{k,q}) = \mathscr{V}^{-1}$ if $\theta_{k,q} = 0$. The term $\mathscr{V}^{\max(0,M_k - M_T)}$ accounts for the case where there are more measurements than targets: these measurements in excess are false alarms. Of course additional false alarms arise for associations $\theta_{k,q} = 0$.

The density $p(\mathbf{y}_{k,n}|\mathbf{y}_{k,-n}, \mathbf{z}_{1:k-1})$ can be factorized as

$$p(\mathbf{y}_{k,n}|\mathbf{y}_{k,-n}, \mathbf{z}_{1:k-1}) = p(\mathbf{x}_{k,n}|\mathbf{z}_{1:k-1})\, p(\theta_{k,n}|\theta_{k,-n}). \tag{3}$$

Here the predictive density $p(\mathbf{x}_{k,n}|\mathbf{z}_{1:k-1})$ can be computed from the set of N_p particles $\mathbf{x}_{k-1}^i$ obtained at time $k-1$. Indeed, as particles $\mathbf{x}_{k-1,n}^i$ are distributed according to the posterior density $p(\mathbf{x}_{k-1,n}|\mathbf{z}_{1:k-1})$,

$$\begin{aligned} p(\mathbf{x}_{k,n}|\mathbf{z}_{1:k-1}) &= \int p(\mathbf{x}_{k,n}|\mathbf{x}_{k-1,n})\, p(\mathbf{x}_{k-1,n}|\mathbf{z}_{1:k-1})\, d\mathbf{x}_{k-1,n} \\ &\approx \frac{1}{N_p} \sum_i p(\mathbf{x}_{k,n}|\mathbf{x}_{k-1,n}^i). \end{aligned}$$

Finally, $p(\theta_{k,n}|\theta_{k,-n})$ in (3) must be assigned a prior density. We propose to use the following prior, inspired from [10]:

$$p(\theta_{k,n} = j|\theta_{k,-n}) = \begin{cases} 1 - P_D & \text{if } j = 0 \\ 0 & \text{if } \exists q \in \{1, M_T\} - n : \\ & \theta_{k,q} = j \neq 0 \\ \dfrac{P_D}{M_{u,-n}} & \text{otherwise} \end{cases} \tag{4}$$

with $M_{u,-n}$ the number of remaining measurements not assigned by variables $\theta_{k,-n}$, and P_D the detection probability.

3.1.2 Choice of the instrumental law

We choose here an instrumental law that factorizes between the state and the association variable:

$$\begin{aligned} q(\mathbf{y}_{k,n}|\mathbf{y}_{k,-n}, \mathbf{y}_{k-1}, \mathbf{z}_{1:k}) = {} & q(\mathbf{x}_{k,n}|\mathbf{y}_{k,-n}, \mathbf{y}_{k-1}, \mathbf{z}_{1:k}) \\ & \times q(\theta_{k,n}|\mathbf{x}_{k,n}, \mathbf{y}_{k,-n}, \mathbf{y}_{k-1}, \mathbf{z}_{1:k}). \end{aligned} \tag{5}$$

For the target state, we choose an instrumental distribution derived from the target dynamics, expressed as:

$$q(\mathbf{x}_{k,n}|\mathbf{y}_{k,-n},\mathbf{y}_{k-1},\mathbf{z}_{1:k}) = \frac{1}{N_p}\sum_q p(\mathbf{x}_{k,n}|\mathbf{x}^q_{k-1,n}).$$

This choice consists in choosing randomly one particle in the set $(\mathbf{x}^i_{k-1,n})_{i\in\{1,N_p\}}$ and propagating it according to the state equation. It is the equivalent for the HMA to the instrumental law used in standard particle filter when the instrumental law is provided by the state equation. Besides it presents the great advantage of simplifying with the density $p(\mathbf{x}_k|\mathbf{z}_{1:k-1})$ in the HM ratio, thus lightening the ratio calculation. Note finally that this choice leads to an independent HMA; this particular class of HMA has strong convergence properties [8].

As for the association variable, we choose an instrumental distribution that takes into account the current observation, so that most probable associations will be favored in the sampling. Using Bayes, we can write

$$\begin{aligned} q(\theta_{k,n}|\mathbf{x}_{k,n},\mathbf{y}_{k,-n},\mathbf{y}_{k-1},\mathbf{z}_{1:k}) &\propto p(\mathbf{z}_k|\mathbf{y}_k,\mathbf{z}_{1:k-1}) \\ &\quad\times p(\theta_{k,n}|\mathbf{x}_{k,n},\mathbf{y}_{k,-n},\mathbf{y}_{k-1},\mathbf{z}_{1:k-1}). \end{aligned}$$

In this expression, $p(\mathbf{z}_k|\mathbf{y}_k,\mathbf{z}_{1:k-1})$ is given by (2), whereas $p(\theta_{k,n}|\mathbf{x}_{k,n},\mathbf{y}_{k,-n},\mathbf{y}_{k-1},\mathbf{z}_{1:k-1}) = p(\theta_{k,n}|\theta_{k,-n})$ is provided by (4). The instrumental distribution for the association variable can then straightforwardly be computed as:

$$q(\theta_{k,n}=j|\mathbf{x}_{k,n},\mathbf{y}_{k,-n},\mathbf{y}_{k-1},\mathbf{z}_{1:k}) = \begin{cases} \dfrac{1}{\mathscr{V}}\dfrac{(1-P_D)}{Q_{k,n}(\mathbf{x}_k,\theta_{k,-n})} & \text{if } j=0 \\ 0 & \text{if } \exists q\in\{1,M_T\}-n: \\ & \theta_{k,q}=j\neq 0 \\ \dfrac{P_D}{M_{u,-n}}\dfrac{p(\mathbf{z}_{k,j}|\mathbf{x}_k)}{Q_{k,n}(\mathbf{x}_k,\theta_{k,-n})} & \text{otherwise,} \end{cases} \tag{6}$$

with the normalization coefficient $Q_{k,n}(\mathbf{x}_k,\theta_{k,-n})$ given by:

$$Q_{k,n}(\mathbf{x}_k,\theta_{k,-n}) = \frac{1-P_D}{\mathscr{V}} + \frac{P_D}{M_{u,-n}}\sum_{\substack{j=1\\ j\notin\theta_{k,-n}}}^{M_k} p(\mathbf{z}_{k,j}|\mathbf{x}_k). \tag{7}$$

3.1.3 HM-within-Gibbs algorithm (HMWG)

We can now derive the HM ratio. The previous choices make the computation very easy since quantities independent of target n get simplified, as well as terms induced by the target dynamics. The final expression of the ratio is simply:

$$\alpha = \frac{Q_{k,n}(\hat{\mathbf{x}}_k,\hat{\theta}_{k,-n})}{Q_{k,n}(\mathbf{x}^{i-1}_k,\theta^{i-1}_{k,-n})}, \tag{8}$$

where $\hat{\mathbf{y}}_k = [\hat{\mathbf{x}}_k,\hat{\theta}_k]$ is the candidate sample. The first method we propose is summarized by Algorithm 1 and called HMWG. Note that in this algorithm, targets are considered in a random order λ over the target set $\{1,M_T\}$. This common strategy insures certain convergence properties.

The HMWG algorithm presents some interesting features: the inner HM structure requires no importance weights and therefore no intermediate resampling steps, unlike the multi-target particle filter presented in [10]. The outer Gibbs sampler structure solves the association problem by MCMC sampling instead of considering all possible associations. The computation cost becomes linear in the number of targets (targets are sampled individually) and in the number of measurements (from the computation of $Q_{k,n}$). On the dark side, it looses the variance reduction feature of the importance sampling, and requires a burn-in period before the generated Markov chain converges to its stationary distribution.

Algorithm 1 Metropolis-within-Gibbs algorithm (HMWG)

for $i=1$ to N_p **do**
  draw a random permutation λ of the target set $\{1,M_T\}$
  for $n=1$ to M_T **do**
    Propagation: draw a candidate particle $\hat{\mathbf{y}}_{k,\lambda_n} \sim \frac{1}{N_p}\sum_q p_n(\hat{\mathbf{y}}^i_{\lambda_n}|\mathbf{x}^q_{k-1,\lambda_n})$;
    Draw a candidate association θ_{k,λ_n} according to the distribution $q(\theta_{k,\lambda_n}|\theta_{k,-\lambda_n},\mathbf{x}_k)$;
    Compute $Q_{k,\lambda_n}(\mathbf{y}^i_k,\theta_{k,\lambda_{-n}})$ from (7) ;
    Compute the HM ratio α as expressed in (8);
    Draw a uniform variable on $[0,1]$: $u\sim\mathscr{U}_{[0,1]}$;
    if $u <= \alpha$ **then**
      Accept: $\mathbf{x}^i_{k,\lambda_n} = \hat{\mathbf{y}}^i_{\lambda_n}$;
    else
      Reject: $\mathbf{x}^i_{k,\lambda_n} = \mathbf{x}^{i-1}_{k,\lambda_n}$;
    end if
  end for
end for

3.2 Hybrid Importance-Sampling-Gibbs particle filter

We now present a hybrid algorithm that combines advantages of both MCMC and importance sampling (IS). Recall that the HMA was previously used to draw particles according to the conditional densities $p(\mathbf{y}_{k,n}|\mathbf{y}_{k,-n},\mathbf{z}_{1:k})$. This HMA step can be replaced by an IS step in order to draw weighted particles representing the conditional densities $p(\mathbf{y}_{k,n}|\mathbf{y}_{k,-n},\mathbf{z}_{1:k})$.

IS can be used in two ways. We can use independent sets of particles at each time step to estimate the objective density; then particles are generated using the same instrumental distribution as the HMA, and weights are not propagated through time. Or we can use sequential IS as in standard particle filter: particles are propagated through time, and their weights computed recursively. Then the instrumental law must be chosen accordingly, and a resampling step must be added to deal with the possible degeneracy of the particle set. Note that if systematic resampling is used, these two strategies are equivalent. We will detail now the second strategy.

3.2.1 Weight propagation

Similarly to the classical sequential IS particle filter, weights for particles $\mathbf{y}^i_{k,n}$ can be recursively computed as:

$$w^i_{k,n} \propto w^i_{k-1,n}\frac{p(\mathbf{z}_k|\mathbf{y}^i_k,\mathbf{z}_{1:k-1})\,p(\mathbf{y}^i_{k,n}|\mathbf{y}^i_{k,-n},\mathbf{y}^i_{k-1},\mathbf{z}_{1:k-1})}{q(\mathbf{y}^i_{k,n}|\mathbf{y}^i_{k,-n},\mathbf{y}^i_{k-1},\mathbf{z}_{1:k})}.$$

In this expression, the density $p(\mathbf{z}_k|\mathbf{y}^i_k,\mathbf{z}_{1:k-1})$ has already been provided by (2). Besides we have:

$$\begin{aligned} &p(\mathbf{y}^i_{k,n}|\mathbf{y}^i_{k,-n},\mathbf{y}^i_{k-1},\mathbf{z}_{1:k-1}) \\ &= p(\mathbf{x}^i_{k,n}|\theta^i_{k,n},\mathbf{y}^i_{k,-n},\mathbf{y}^i_{k-1},\mathbf{z}_{1:k-1})\,p(\theta^i_{k,n}|\mathbf{y}_{k,-n},\mathbf{y}^i_{k-1},\mathbf{z}_{1:k-1}) \\ &= p(\mathbf{x}^i_{k,n}|\mathbf{x}^i_{k-1,n})\,p(\theta^i_{k,n}|\theta^i_{k,-n}). \end{aligned}$$

Algorithm 2 Hybrid algorithm

```
for i = 1 to N_p do
    draw a random permutation λ of the set of targets {1, M_T}
    for n = 1 to M_T do
        Prediction: draw particle x^i_{k,n} according to p_n(x^i_{k,n} | x^i_{k-1,n});
        Draw association θ_{k,λ_n} according to the distribution q(θ_{k,λ_n} | θ_{k,-λ_n}, x_k);
        Compute Q_{k,λ_n}(y^i_k, θ_{k,λ_{-n}}) from (7) ;
        Update weights w^i_{k,n} according to (9);
    end for
end for
Normalize weights: w̃^i_{k,n} = w^i_{k,n} / Σ_{j=1}^q w^j_{k,n} ;
Resample if necessary.
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Here $p(\mathbf{x}^i_{k,n}|\mathbf{x}^i_{k-1,n})$ is easily obtained from the state equation, while $p(\theta^i_{k,n}|\theta^i_{k,-n})$ can be expressed by prior (4).

3.2.2 *Choice of the instrumental law*

As previously, we choose an instrumental law that verifies the factorization property (5). For the state of target n, the instrumental distribution is provided by the state equation:

$$q(\mathbf{x}^i_{k,n}|\mathbf{y}^i_{k,-n},\mathbf{y}^i_{k-1},\mathbf{z}_{1:k}) = p(\mathbf{x}^i_{k,n}|\mathbf{x}^i_{k-1,n}).$$

This choice implies that each particle is propagated according to the corresponding target dynamics. It is formally equivalent to the distribution used in the HMA. For the association variable, we choose the law provided by (6) which permits to draw sequentially samples of associations $\theta_{k,n}$ taking into account the current observation as well as the associations $\theta_{k,-n}$.

3.2.3 *The hybrid algorithm*

With these instrumental laws, the update equation for the weights becomes:

$$w^i_{k,n} \propto Q_{k,n}(\mathbf{x}_k,\theta_{k,-n})w^i_{k-1,n}, \quad (9)$$

where $Q_{k,n}(\mathbf{x}_k,\theta_{k,-n})$ is given by (7). As the weights are defined up to a proportionality constant, they must be normalized in order to get an estimate of the objective density. Moreover, as the particle weights are propagated from time k-1 to time k, the particle cloud may degenerate, and a resample step must be added if necessary as in classical particle filter, which is not the case for the HMA.

This second method is summarized by Algorithm 2. It gathers advantages of both MCMC and IS approaches. The Gibbs sampler structure solves both high-dimensional and association problems at a cost linear in the number of targets and measurements. The variance of the state estimate is reduced thanks to the IS step used to sample the target states.

4. SIMULATIONS, RESULTS AND CONCLUSIONS

For the simulations, we consider a typical ground radar scenario: several targets move in the x-y plane at unknown constant velocity. Each target state is determined by a vector $\mathbf{x}_{k,n} = [x_{k,n}, y_{k,n}, v^x_{k,n}, v^y_{k,n}]^T$ representing the position and velocity in x and y directions. The state equation is given by

$$\mathbf{x}_{k,n} = \begin{bmatrix}1 & T_s\\ 0 & 1\end{bmatrix} \otimes \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \mathbf{x}_{k-1,n} + \mathbf{v}_{k-1,n},$$

where $\otimes$ denotes a Kronecker product, $\mathbf{v}_{k-1,n}$ is an additive gaussian noise and T_s is the interval between two consecutive samples. Measurements are provided by a predetection step performed with a quadratic detector. The detection probability is set to $P_D = 0.95$ and the false alarm probability is computed at a given SNR assuming a non fluctuant model (Swerling 0). Each detection provides an observation in the form of a distance and an angle between the radar and the target: $\mathbf{z}_{k,j} = [r_{k,j}, \varphi_{k,j}]^T$. If measurement j has been produced by target n, the observation equation can then be written as

$$\mathbf{z}_{k,j} = \left[\sqrt{x^2_{k,n}+y^2_{k,n}},\ \tan^{-1}\left(\frac{y_{k,n}}{x_{k,n}}\right)\right]^T + \mathbf{n}_{k,j} = h(\mathbf{x}_{k,n}) + \mathbf{n}_{k,j},$$

where $h(\mathbf{x}_{k,n})$ is a non linear function of the target state and $\mathbf{n}_{k,j}$ is an additive gaussian noise.

We consider scenarios where targets are very close from one another and are therefore located in the same cluster. Benchmark performance is provided by the JPDA particle filter. A typical scenario with five close targets is presented in figure 1. Simulations are run for various SNR values, 1000 Monte Carlo simulations per SNR value and number of targets. For the sake of comparison, the three algorithms are run with the same number of particles, here 500 particles per target. The HMWG particle filter uses part of this 500 particles as a burn-in period, and the remain for estimation.

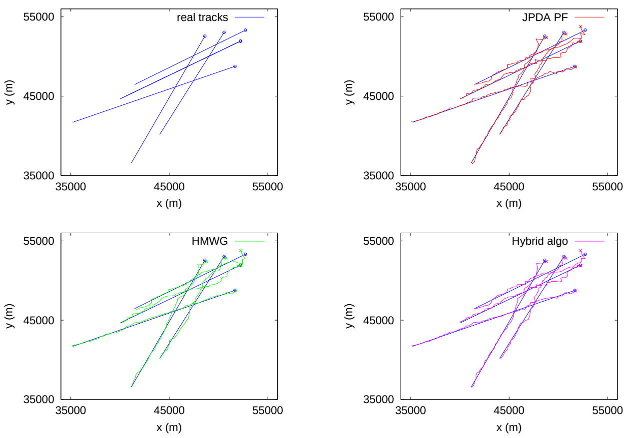

Figure 1: Example of a 5-target scenario. Upper left plot: real tracks. Other plots: tracking estimators.

Performances measured in terms of mean square error between estimated and real positions and velocities, are presented in figure 2. We see that performance of the HMWG method is slightly worse than the JPDA one. This is partly because less particles are used in HMWG to compute the state estimator (burn-in period), and partly because the IS algorithm used in the JPDA particle filter reduces the variance of the state estimator. However JPDA performance can be equalized or even outperformed by HMWG algorithm at a smaller computational cost by using more particles. Conversely, the proposed hybrid algorithm matches JPDA performance for the same number of particles.

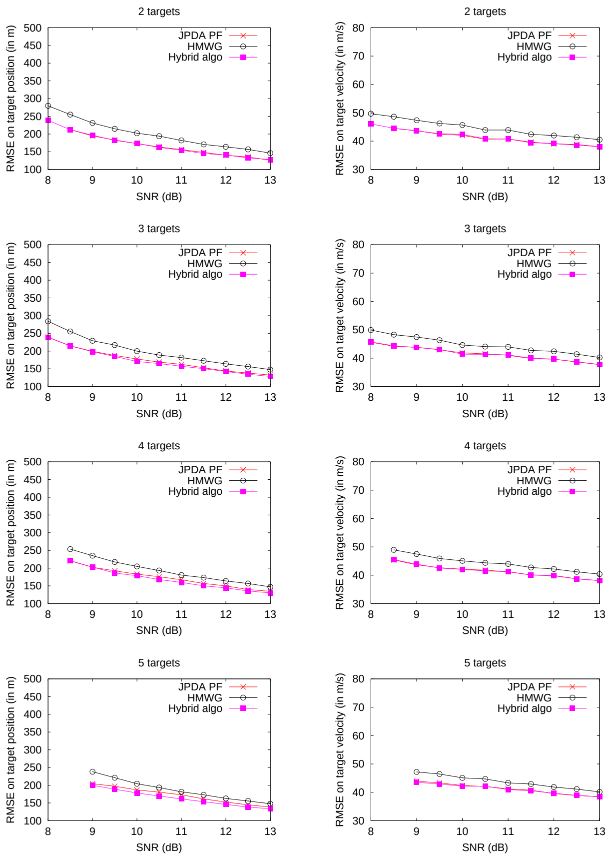

Figure 2: Root Mean Square Error for the position (left) and the velocity (right) for different numbers of targets.

A comparison of the three algorithms in terms of computational time versus SNR is presented in figure 3 for different numbers of targets. In this regard, it is clear that the Gibbs sampling dramatically improves the computational cost, particularly in two situations which both lead to a very large number of associations: at low SNR, the two proposed algorithms present only a small increase in their computational costs, due to the increase in the number of false alarms, while the JPDA particle filter computational cost grows very rapidly due to the combinatorial growth of the number of possible associations; similarly, for large numbers of targets, we notice only a slight increase in the computational cost of the two proposed algorithms, while the JPDA computational cost increases rapidly, once again because of the combinatorial growth in the number of possible associations. Note that the smallest SNR values have not been considered for the scenarios with more targets because the JPDA particle filter becomes too costly for those SNR values.

As a conclusion, we have proposed two MCMC-based particle filters that solve the association problem in multitarget tracking with a computational cost linear in the number of targets and measurements. In particular, performance of the proposed hybrid algorithm matches JPDA performance at a much smaller cost. Finally, note that we assumed in this article that the number of targets is known. In a forthcoming work, we will therefore focus on more complex scenarios where targets may enter or leave the observation area.

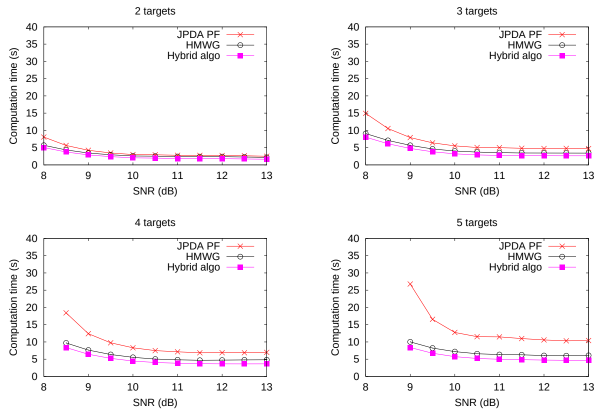

Figure 3: Run time vs SNR for different numbers of targets.

REFERENCES

[1] M. Ekman. Particle filters and data association for multi-target tracking. In *Proc. of 5th Int. Conf. on Information Fusion*, 2008.

[2] T. Fortmann, Y. Bar-Shalom, and M. Scheffe. Sonar Tracking of Multiple Targets Using Joint Probabilistic Data Association. *IEEE Journal of Oceanic Engineering*, 8(3):173–184, 1983.

[3] O. Frank, J. Nieto, J. Guivant, and S. Scheding. Multiple Target Tracking using Sequential Monte Carlo Methods and Statistical Data Association. In *Proc. of IEEE Int. Conf. on Intelligent Robots and Systems*, pages 2718–2723, 2003.

[4] S. Geman and D. Geman. Stochastic relaxation,Gibbs distribution and the Bayesian restoration of images. *IEEE Tr. Pattern Anal. Mach. Intell.*, 6:721–741, 1984.

[5] C. Hue, J. P. Le Cadre, and P. Pérez. Sequential Monte Carlo Methods for Multiple Target Tracking and Data Fusion. *IEEE Trans. on Signal Processing*, 50(2):309–325, 2002.

[6] Z. Khan, T. Balch, and F. Dellaert. MCMC-Based Particle Filtering for Tracking a Variable Number of Interacting Targets. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, 27(11):1805–1819, 2005.

[7] C. Robert and G. Casella. *Monte Carlo Statistical Methods*. Springer-Verlag, 2004.

[8] A. Sankanarayanan, A. Srivastava, and R. Chellappa. Algorithmic and Architectural Optimizations for Computationally Efficient Particle Filtering. *IEEE Trans. on Image Processing*, 17(5):737–748, 2008.

[9] J. Vermaak, A. Doucet, and P. Pérez. Maintaining Multi-Modality through Mixture Tracking. In *Proc. of the IEEE Int. Conf. on Computer Vision*, volume 2, pages 1110–1116, 2003.

[10] J. Vermaak, S. J. Godsill, and P. Pérez. Monte Carlo Filtering for Multi-Target Tracking and Data Association. *IEEE Trans. on Aerospace and Electronic Systems*, 41(1):309–332, 2005.

[11] P. Willett, Y. Ruan, and R. Streit. PMHT: Problems and Solutions. *IEEE Trans. on Aerospace and Electronic Systems*, 38(3):738–754, 2002.