

# ACOUSTIC SOURCE LOCALIZATION AND SPEED ESTIMATION BASED ON TIME-DIFFERENCES-OF-ARRIVAL UNDER TEMPERATURE VARIATIONS

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## ABSTRACT

In the context of acoustic source localization, knowing the actual propagation speed is crucial especially in uncontrolled environments where the temperature is subject to significant changes that influence the sound speed. In a recent paper we showed the effects of the assumed propagation speed on the localization performance of two closed-form localization algorithms based on TDOA measurements. It has been shown that the so-called unconstrained least squares method is not significantly influenced by a wrongly assumed propagation speed, whereas the constrained method, the one statistically more attractive, is impaired even by small speed deviations. In this article we study in more depth the causes of this disparity and we propose a novel technique to estimate the propagation speed and improve the localization performance when the environment conditions, i.e. the air temperature, are not known exactly.

## 1. INTRODUCTION

For the problem of localizing an acoustic source from Time-Differences-of-Arrival (TDOAs) there are a number of different solution approaches. Of certain interest are the methods capable of providing closed-form solutions in a linear fashion. Because of their efficiency, they are well suited for real time implementation on embedded systems using cheap, small and lightweight acoustic instrumentation available today. A valuable review of these methods can be found in [7], it focuses on methods which provide closed-form solutions given the so-called *spherical* or *non redundant* TDOA set [8]. This means that given a set of  $N + 1$  microphones  $N$  TDOAs are calculated with respect to the same reference microphone.

Most of these methods assume known propagation speed, which is a reliable assumption only under laboratory and controlled indoor conditions where the air temperature, influencing the sound speed, can be monitored. In all other cases it is meaningful to understand the effects of a wrongly assumed propagation speed as surveyed in [1].

Starting from these results we propose in this paper a novel technique to robustly estimate the actual sound speed from TDOAs. Such an estimate can be used to enhance the localization by making it aware of the environment conditions, i.e. the actual air temperature. Environment awareness is the vision of the ongoing EU-funded project SCENIC<sup>1</sup>. In the next section a brief overview of the two-dimensional source localization problem is given and some previous work is reviewed.

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## 2. SOURCE LOCALIZATION METHODS

### 2.1 Acoustic Source Localization from TDOAs

We consider the Euclidean space of  $D = 2$  dimensions where

- the acoustic source is located in an unknown position  $\mathbf{x} = [x \ y]^T$ ,
- $N + 1$  acoustic sensors are distributed at the known positions  $\mathbf{a}_n = [x_n \ y_n]^T$  for  $n = 0, \dots, N$ ,
- the reference sensor  $\mathbf{a}_0$  is located in the origin  $[0 \ 0]^T$  and  $N$  microphone pairs  $(\mathbf{a}_0, \mathbf{a}_n)$  are considered,
- the TDOA values  $\tau_n$ ,  $n = 1, \dots, N$  are estimated for each pair by suitable correlation between the two microphone signals.

The localization problem consists of finding  $\mathbf{x}$  given the sensor positions  $\mathbf{a}_n$  and the TDOAs  $\tau_n$ . Each TDOA can be expressed in terms of the travelled range difference as

$$\tau_n = \frac{1}{c} d_n, \quad n = 1, \dots, N, \quad (1)$$

where  $d_n$  denotes the source's range difference between the sensor  $\mathbf{a}_n$  and the reference  $\mathbf{a}_0$ , as depicted in Fig. 1.

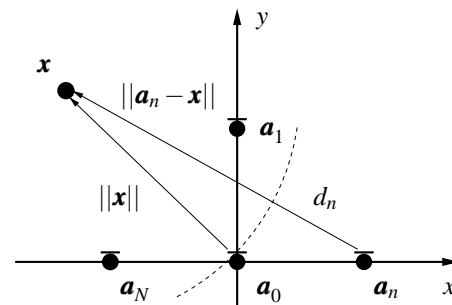


Figure 1: Geometry of the two-dimensional acoustic source localization problem using a microphone array. The extension of the array is sufficiently large to infer the source distance from the circular wave front.

In the literature the sound speed is typically assumed to be a known value  $c$ . However, as explained in Sec. 3, such a value might differ due to temperature variations from the actual sound speed  $c_s$ .

From geometrical reasoning follows that

$$d_n = \|\mathbf{a}_n - \mathbf{x}\| - \|\mathbf{x}\|, \quad n = 1, \dots, N, \quad (2)$$

where  $\|\cdot\|$  denotes the Euclidean vector norm, thus the candidate position  $\mathbf{x}$  must fulfill the above  $N$  equations.

## 2.2 Previous Works

It is well known, e.g. from [3], that by squaring Eq. (2) the following simpler set of equations can be obtained

$$d_n \|\mathbf{x}\| + \mathbf{a}_n^T \mathbf{x} = \frac{\|\mathbf{a}_n\|^2 - d_n^2}{2}, \quad n = 1, \dots, N.$$

The corresponding system of equations is described in matrix form as follows

$$\Phi \mathbf{y}(\mathbf{x}) = \mathbf{b}, \quad (3)$$

$$\mathbf{y}(\mathbf{x}) = \begin{bmatrix} \|\mathbf{x}\| \\ \mathbf{x} \end{bmatrix}, \quad \Phi = [\mathbf{d} | \mathbf{A}], \quad (4)$$

$$\mathbf{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_N \end{bmatrix}, \quad \mathbf{d} = \begin{bmatrix} d_1 \\ \vdots \\ d_N \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \mathbf{a}_1^T \\ \vdots \\ \mathbf{a}_N^T \end{bmatrix}, \quad (5)$$

$$b_n = \frac{1}{2} (\|\mathbf{a}_n\|^2 - d_n^2). \quad (6)$$

Solving the system in (3) with respect to  $\mathbf{x}$  is an estimation problem since the elements of  $\mathbf{A}$  and  $\mathbf{d}$  are subject to uncertainties and the system does not hold exactly, i.e.  $\Phi \mathbf{y}(\mathbf{x}) \approx \mathbf{b}$ . Typically the sensor positions  $\mathbf{a}_n$  of a well constructed sensor array are considered to be exactly known, whereas the values  $d_n$  are derived from (1) where the values  $\tau_n$  are correlation results, impaired by other acoustic sources and by room reverberation. The statistical analysis of the problem is far from the scope of this paper, therefore the reader is referred to [3, 5, 8], we restrict ourself to consider in Sec. 3 a deterministic error  $\Delta c = c_s - c$  in the propagation speed  $c$  assumed by the localization methods.

In practice the direct solution of such a nonlinear estimation problem is not attractive under real-time and low computational complexity constraints, therefore closed-form localization methods have been devised which provide approximate solutions in a linear fashion.

### 2.2.1 Unconstrained Least Squares Method

Several authors, e.g. [3, 5], showed that introducing a new scalar variable  $r$  independent of  $\mathbf{x}$  in place of the norm  $\|\mathbf{x}\|$  enables to address the problem as a linear least squares estimation of the unknown vector  $\mathbf{y} = [r \ \mathbf{x}]^T$ . Provided that  $N \geq D + 1$ , such a least squares estimate is given in terms of the pseudo-inverse  $\Phi^+$

$$\hat{\mathbf{y}} = \begin{bmatrix} \hat{r} \\ \hat{\mathbf{x}} \end{bmatrix} = \Phi^+ \mathbf{b} = (\Phi^T \Phi)^{-1} \Phi^T \mathbf{b}. \quad (7)$$

This estimate should be considered an approximate solution of (3) since in general  $\mathbf{y}(\hat{\mathbf{x}}) \neq \hat{\mathbf{y}}$  or quite simply  $\hat{r} \neq \|\hat{\mathbf{x}}\|$ , hence it is called unconstrained least squares method. In fact  $\hat{r}$  is usually considered a byproduct and only  $\hat{\mathbf{x}}$  is used as an estimate of the source position.

In [1] an alternative expression for the unconstrained least squares estimate has been derived. It shows 1) the dependency on the assumed speed  $c$  and 2) how this dependency influences the estimated range  $\hat{r}$  and the estimated position  $\hat{\mathbf{x}}$ , respectively,

$$\hat{r}(c) = \frac{1}{c} \Theta \mathbf{b}(c), \quad \hat{\mathbf{x}}(c) = \Gamma \mathbf{b}(c), \quad (8)$$

where

$$\Theta = (\mathbf{P}_A^\perp \boldsymbol{\tau})^+, \quad \Gamma = (\mathbf{P}_\tau^\perp \mathbf{A})^+, \quad (9)$$

with the orthogonal projection matrices as shown in [1],

$$\mathbf{P}_A^\perp = \mathbf{I} - \mathbf{A}(\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T, \quad \mathbf{P}_\tau^\perp = \mathbf{I} - \frac{1}{\|\boldsymbol{\tau}\|^2} \boldsymbol{\tau} \boldsymbol{\tau}^T, \quad (10)$$

$$\mathbf{b}(c) = \begin{bmatrix} b_1(c) \\ \vdots \\ b_N(c) \end{bmatrix}, \quad \boldsymbol{\tau} = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_N \end{bmatrix}, \quad (11)$$

$$b_n(c) = \frac{1}{2} (\|\mathbf{a}_n\|^2 - c^2 \tau_n^2). \quad (12)$$

It shall be noted that in ideal conditions Eqns. (8) provide exact and compatible values of range and position if and only if the sound speed  $c$  is correctly assumed, i.e.  $c = c_s$  and  $\Delta c = 0$ .

### 2.2.2 Constrained Least Squares Method

Constrained methods aim at a more robust localization by finding an estimate  $\hat{\mathbf{y}} = [\tilde{r} \ \tilde{\mathbf{x}}]^T$  which obeys the constraint between range and position, i.e.  $\tilde{r} = \|\tilde{\mathbf{x}}\|$ . A constrained least squares solution of (3) may be obtained employing the Lagrange multipliers technique and an iterative procedure as shown in [4, 5]. More attractive is its linear approximation which benefits from the closed-form estimate given in [7]. A short derivation is given below, since some of its elements are needed in Sec 3.2.

The residual function corresponding to (3) may be written in terms of  $\hat{\mathbf{y}}$  as

$$\boldsymbol{\varepsilon}(\mathbf{x}) = \Phi(\mathbf{y}(\mathbf{x}) - \hat{\mathbf{y}}), \quad (13)$$

and then linearized at the estimate  $\hat{\mathbf{x}}$  from (7)

$$\boldsymbol{\varepsilon}(\mathbf{x}) \approx \boldsymbol{\varepsilon}(\hat{\mathbf{x}}) + \boldsymbol{\varepsilon}'(\hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}}). \quad (14)$$

The term  $\boldsymbol{\varepsilon}(\hat{\mathbf{x}})$  and the Jacobian matrix  $\mathbf{J} = \boldsymbol{\varepsilon}'(\hat{\mathbf{x}})$  follow with (4) as

$$\boldsymbol{\varepsilon}(\hat{\mathbf{x}}) = \Phi \boldsymbol{\delta}, \quad \mathbf{J} = \boldsymbol{\varepsilon}'(\hat{\mathbf{x}}) = \Phi \mathbf{G}, \quad (15)$$

with

$$\boldsymbol{\delta} = \begin{bmatrix} \|\hat{\mathbf{x}}\| - \hat{r} \\ \mathbf{0} \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \frac{\hat{\mathbf{x}}^T}{\|\hat{\mathbf{x}}\|} \\ \mathbf{I} \end{bmatrix}. \quad (16)$$

Now an improved estimate  $\tilde{\mathbf{x}} = \hat{\mathbf{x}} + \Delta \mathbf{x}$  can be found for which the residual  $\boldsymbol{\varepsilon}(\tilde{\mathbf{x}})$  becomes a minimum in the least squares sense, i.e.

$$\boldsymbol{\varepsilon}(\tilde{\mathbf{x}}) \approx \boldsymbol{\varepsilon}(\hat{\mathbf{x}}) + \boldsymbol{\varepsilon}'(\hat{\mathbf{x}}) \Delta \mathbf{x} = \Phi \boldsymbol{\delta} + \mathbf{J} \Delta \mathbf{x}, \quad (17)$$

with

$$\tilde{\mathbf{x}} = \hat{\mathbf{x}} - \mathbf{J}^+ \Phi \boldsymbol{\delta}. \quad (18)$$

Here  $\boldsymbol{\varepsilon}(\hat{\mathbf{x}}) = \Phi \boldsymbol{\delta}$  and the pseudo-inverse  $\mathbf{J}^+$  may be interpreted as the intensity and the direction, respectively, of one iteration of the Gauss-Newton algorithm initialized at  $\hat{\mathbf{x}}$  [7].

### 2.2.3 Speed Estimation Methods

Some authors [6,9] form from Eq. (2) a linear system in  $D+2$  unknowns, from which a propagation speed estimate along with the position estimate can be obtained from an unconstrained least squares estimation of  $D+2$  unknowns. These methods avoid making initial assumptions on the sound speed  $c$ , but unfortunately the corresponding system matrix might be easily ill-conditioned and moreover the so-obtained speed estimate is not robust to noise. Thus they are not further investigated here.

## 3. ACOUSTIC SOURCE LOCALIZATION UNDER TEMPERATURE VARIATIONS

### 3.1 Speed of Sound under Temperature Variations

It is well known that the actual speed of sound depends on the air temperature through the following relation [2]

$$c_s(T) = \sqrt{\gamma RT}, \quad (19)$$

where  $R$  is the gas constant,  $\gamma = 1.4$  and  $T$  the absolute temperature. Linearizing yields the customary expression  $c_s(\theta) \approx 331 \frac{\text{m}}{\text{s}} + 0,6 \frac{\text{m}}{\text{s}^\circ\text{C}} \theta$ , where  $\theta$  is the air temperature in degrees Celsius. A popular value results from the air temperature  $\theta_0 = 20^\circ\text{C}$  as

$$c_0 = c_s(\theta_0) = 343 \frac{\text{m}}{\text{s}}. \quad (20)$$

The actual sound speed for other temperatures can be written as

$$c_s = c_0 + \Delta c. \quad (21)$$

In practice, temperature variations are not recorded and neither the speed deviation  $\Delta c$  nor the actual sound speed  $c_s$  are known.

The contribution of this paper is to present a novel technique to estimate the actual sound speed avoiding the problems mentioned in Sec. 2.2.3. This estimate is further applied to enhance the localization performance of the unconstrained and constrained methods that are differently impaired by a wrongly assumed sound speed as shown in [1]. The following sections investigate the cause of this disparity and show how to exploit it to achieve a robust sound speed estimate and to ensure reliable localization also in case of temperature variations.

### 3.2 Sound Speed Estimation

In ideal conditions and knowing the actual sound speed, the vector  $\boldsymbol{\varepsilon}(\hat{\mathbf{x}}) = \boldsymbol{\Phi}\boldsymbol{\delta}$  from (15) vanishes since  $\hat{r} = \|\hat{\mathbf{x}}\|$ . On the other hand when  $c \neq c_s$  such a vector engenders the disparity between the unconstrained solution and the constrained solution. Using Eqns. (8) it may be expressed as function of the assumed speed  $c$ , i.e.

$$\boldsymbol{\varepsilon}(c) = \boldsymbol{\Phi}(c) \begin{bmatrix} \delta(c) \\ \mathbf{0} \end{bmatrix} = c\delta(c)\boldsymbol{\tau}, \quad (22)$$

with

$$\delta(c) = \|\hat{\mathbf{x}}(c)\| - \hat{r}(c) = \|\boldsymbol{\Gamma}\mathbf{b}(c)\| - \frac{1}{c}\boldsymbol{\Theta}\mathbf{b}(c). \quad (23)$$

Then the value  $\hat{c}$  which annihilates the vector in (22) has to be the actual sound speed  $c_s$  since Eqns. (8) provide compatible estimates  $\hat{r}$  and  $\hat{\mathbf{x}}$ . Indeed the searched speed value has to be a zero of the scalar function  $\delta(c)$  (obviously the trivial solution  $\hat{c} = 0$  is not of interest). In the following an efficient way for finding such a sound speed value is given.

#### 3.2.1 Linear Approximation

The function  $\delta(c)$  involving the Euclidean norm of  $\hat{\mathbf{x}}$  is non-linear, nonetheless near the actual sound speed  $c_s$  it can be shown that it has a fairly linear behavior, see Fig 2.

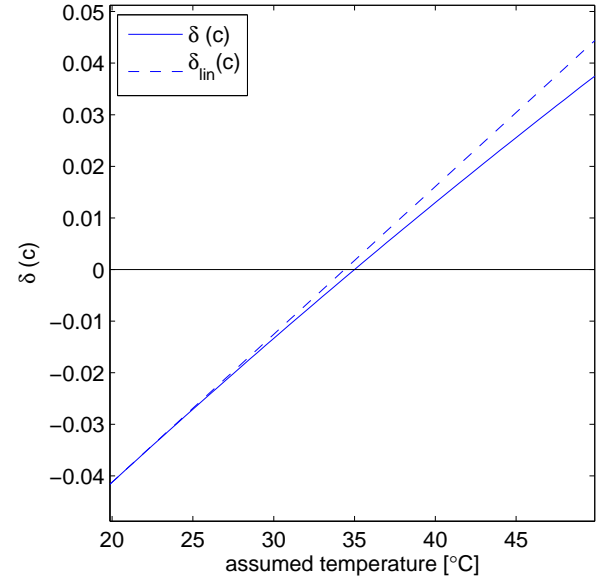


Figure 2: Behavior of the function  $\delta(c)$  and its linearized version  $\delta_{\text{lin}}(c)$  assuming an initial temperature guess of  $\theta_0 = 20^\circ\text{C}$ . The actual air temperature of  $\theta = 35^\circ\text{C}$  can be inferred from the zero-crossing of both functions.

This means that given a reliable initial guess  $c_0$ , e.g. the nominal value from (20), the following first order Taylor expansion  $\delta_{\text{lin}}(c)$  is a useful approximation of  $\delta(c)$

$$\delta_{\text{lin}}(c) = \delta_0 + \delta'_0(c - c_0), \quad (24)$$

with

$$\delta_0 = \delta(c_0) \quad \text{and} \quad \delta'_0 = \left. \frac{d\delta(c)}{dc} \right|_{c=c_0}. \quad (25)$$

Thus the desired speed value corresponding to the zero-crossing  $\delta_{\text{lin}}(\hat{c}) = 0$  is given by

$$\hat{c} = \frac{\delta_0 - \delta'_0 c_0}{\delta'_0}. \quad (26)$$

The value of the first order derivative at  $c_0$  can be calculated with simple derivation rules from (23)

$$\delta'_0 = \frac{\hat{\mathbf{x}}_0^T}{\|\hat{\mathbf{x}}_0\|} \boldsymbol{\Gamma}\mathbf{b}'_0 + \frac{\hat{r}_0}{c_0} - \boldsymbol{\Theta}\mathbf{b}'_0, \quad (27)$$

where  $\hat{r}_0$  and  $\hat{\mathbf{x}}_0$  are the unconstrained estimates of range and position calculated with the initial guess  $c_0$  while  $\mathbf{b}'_0$  is a vector containing the derivatives of (12) evaluated at  $c = c_0$ , i.e.

$$\mathbf{b}'_0 = -c_0 \boldsymbol{\tau}^2. \quad (28)$$

### 3.2.2 Further Enhancement

We suggest a further enhancement which makes the speed-temperature estimation more robust in noisy conditions. So far the employed TDOA set  $\boldsymbol{\tau}$  has been computed merely regarding the reference microphone  $\mathbf{a}_0$  positioned at the origin. Actually apart from a translation of the coordinate system the localization can be carried out regarding arbitrarily positioned references. This means that given an array of  $M = N + 1$  microphones,  $M$  different spherical TDOA sets  $\boldsymbol{\tau}_j$ ,  $j = 0, \dots, N$  may be calculated from the recorded signals. If there is no speed deviation unconstrained and constrained solutions provide in ideal conditions the same location regardless which microphone is chosen as reference, thus normally only one spherical TDOA set is used and the others are considered redundant.

However if a speed deviation  $\Delta c$  occurs, it can be shown that both localization methods give different solutions with respect to different reference microphones, this effect is prominent for the constrained method (see the range distortion map in Fig. 3).

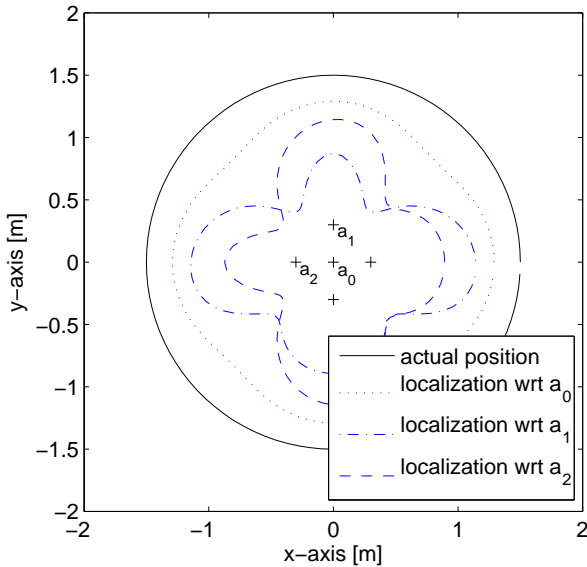


Figure 3: Simulation results which show how the constrained localization depends on the chosen reference microphone given a speed deviation  $\Delta c$  corresponding to a temperature variation of  $\Delta T = 25$  K. It turns out that the central microphone  $\mathbf{a}_0$  of the array produces the smallest range distortion.

In brief, the idea is to exploit this dependency in order to enhance the obtained sound speed value, which has to be the one minimizing the difference between solutions corresponding to different reference microphones. This is possible by minimizing the function in (23) in the least squares sense regarding all reference microphones, i.e. the corresponding

least squares criterion is

$$\sum_{j=0}^N \delta_j(c)^2 = \sum_{j=0}^N \left( \|\hat{\mathbf{x}}_j(c)\| - \hat{r}_j(c) \right)^2, \quad (29)$$

where  $\hat{\mathbf{x}}_j$  and  $\hat{r}_j$  are the unconstrained estimates for position and range obtained with the TDOA set  $\boldsymbol{\tau}_j$ .

An efficient way for solving the above estimation problem is to use the linear approximation in (24), which leads to the following least squares estimate for the sound speed

$$\hat{c} = (\boldsymbol{\delta}'_0)^+ (\boldsymbol{\delta}_0 - \boldsymbol{\delta}'_0 c_0), \quad (30)$$

where

$$\boldsymbol{\delta}_0 = \begin{bmatrix} \delta_{01} \\ \vdots \\ \delta_{0N} \end{bmatrix}, \quad \boldsymbol{\delta}'_0 = \begin{bmatrix} \delta'_{01} \\ \vdots \\ \delta'_{0N} \end{bmatrix}. \quad (31)$$

As long as a reliable value  $\hat{c}$  of the sound speed is available, it can be used instead of the initial guess  $c_0$  and much better localization results are expected from both estimators, especially from the constrained one. The following section is devoted to experimental results which confirm the theoretical reasoning described so far.

## 4. EXPERIMENTAL RESULTS

We used the same cross-array ( $N = 4$ ) from [1] and we performed the localization of 48 different source positions (loudspeakers emitting white noise) distributed on a circle of 1,5 m radius in a laboratory environment with  $T_{60} \approx 0,25$  s. The TDOA values  $\boldsymbol{\tau}$  are obtained with GCC-PHAT [4] processing signal windows of 1024 samples acquired at 48 kHz.

We calculated initially unconstrained and constrained estimates from Eqns. (7) and (18), respectively, assuming a wrong propagation speed  $c_0$  corresponding to a variation  $\Delta T = 25$  K from the actual air temperature of  $24,1$  °C (measured with an electronic thermometer). The localization results are depicted in Fig. 4, the range distortion produced by the temperature variation is clearly visible especially for the constrained method.

For each position the corresponding TDOAs and the initial guess  $c_0$  are also used to calculate the speed estimate in (30) which is then applied to perform an enhanced localization. The corresponding results are shown in Fig. 5. It is clear that the range distortion is compensated through this enhancement, the constrained method benefits the most from this and becomes reliable again.

Tab. 1 shows the mean value and standard deviation of the range error. The enhanced localization with the estimated speed value  $\hat{c}$  according to Eqns. (26) and (30) yields a significant improvement in terms of bias reduction, both for the unconstrained and the constrained localization. The results using least squares estimation of the sound speed according to Eq. (30) are slightly superior to the simpler estimate from Eq. (26).

Since the localizations of each position have been carried out sequentially in a short period of time, the corresponding temperature estimates derived from Eqns. (26) and (30) have been averaged leading to a mean temperature of  $23,2$  °C and  $24,2$  °C respectively, where the latter is very close to the air temperature of  $24,1$  °C measured during the experiment.

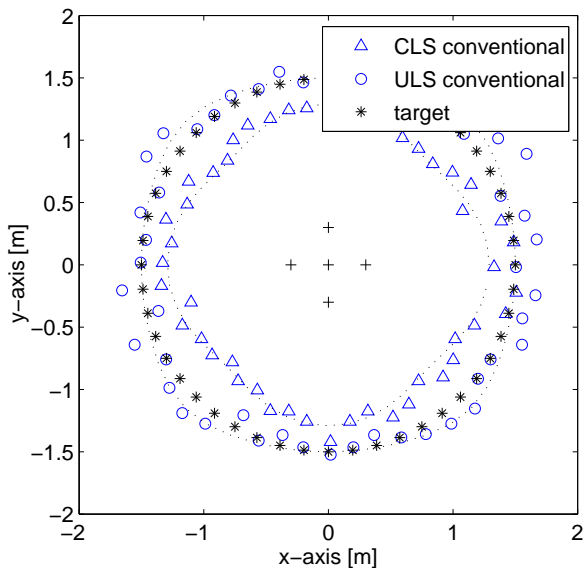


Figure 4: Experimental results with a speed deviation  $\Delta c$  corresponding to a temperature variation of  $\Delta T = 25$  K. The triangles are the positions estimated with the conventional constrained least squares method (CLS), the circles are the positions estimated with the conventional unconstrained least squares method (ULS).

|             | $c_0$  |      | $\hat{c}(26)$ |      | $\hat{c}(30)$ |      |
|-------------|--------|------|---------------|------|---------------|------|
|             | CLS    | ULS  | CLS           | ULS  | CLS           | ULS  |
| <b>mean</b> | -23.59 | 5.43 | -0.52         | 0.55 | -0.41         | 0.14 |
| <b>std</b>  | 9.47   | 9.66 | 8.66          | 9.09 | 8.40          | 8.71 |

Table 1: Mean value and standard deviation in cm of the range error for conventional and enhanced localization using the speed estimates from Eqns. (26) and (30).

## 5. CONCLUSION

In this paper we described a novel technique to obtain a robust sound speed estimate from TDOA measurements, i.e. in an unsynchronized source-sensors scenario, where the air temperature is unknown or subject to significant changes. It is based on the so called unconstrained and constrained least squares methods for source localization. The obtained sound speed estimate can be used to infer the actual air temperature and to enhance the performance of localization methods which require accurate knowledge about the propagation speed. Experimental results confirmed the effectiveness of the proposed technique.

## 6. ACKNOWLEDGMENT

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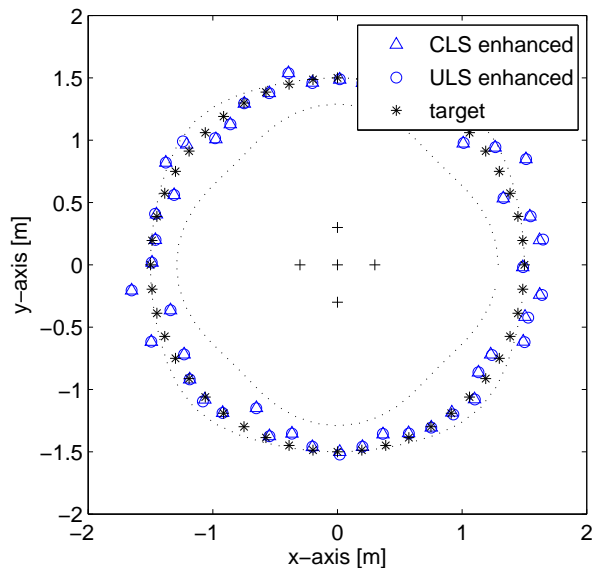


Figure 5: Experimental results using the speed estimation technique with a speed deviation  $\Delta c$  corresponding to a temperature variation of  $\Delta T = 25$  K. The triangles are the positions estimated with the enhanced constrained least squares method (CLS), the circles are the positions estimated with the enhanced unconstrained least squares method (ULS).

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