ADAPTIVE CIRCULAR BEAMFORMING USING MULTI-BEAM STRUCTURE

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ABSTRACT

In this paper, a broadband beamspace adaptive microphone array that constructs a multibeam network is proposed. A beampattern synthesis method that has its frequency invariant property focusing on the desired direction only was used to form the beams. The main idea is to replace the blocking matrix and the quiescent vector in the conventional General Sidelobe Canceler (GSC) with the frequency invariant beampattern synthesis method. Simulation results show that this proposed method has faster convergence rate than an existing algorithm.

1. INTRODUCTION

Adaptive beamforming has found numerous applications in radar, sonar, seismology and microphone array speech processing. In speech processing, the sound waves are considered a broadband signal as it covers over 4 octaves in frequency. Conventionally, for broadband signal, adaptive beamformer uses a bank of linear transversal filters to generate the desired beampattern. The filter coefficients are derived adaptively from the received signals. One classic design example is the Frost Beamformer [1]. However, in order to have a similar beampattern over the entire frequency range, a large number of sensors and filter taps are needed.

Recently, many algorithms have been generated to synthesize frequency invariant (FI) beampattern [2–7] for broadband signal. It is proved that the FI beamformer has less computational complexity than conventional broadband methods [7, 8]. In this paper, a broadband beamspace adaptive approach that constructs a multi-beam network is proposed. Several Frequency Invariant (FI) beams were formed pointing to different directions. The outputs of these beams were then combined adaptively by a single weight for each of them.

This proposed method were devised based on the structure of General Sidelobe Canceler (GSC). Conventionally, the GSC performs a projection of the data onto an unconstrained subspace by means of a blocking matrix and a quiescent vector. Our method replaced the blocking matrix and the quiescent vector with the beampattern synthesis method proposed earlier [5] with nulling formed at the blocking direction. The adaptive weights were calculated using adaptive algorithm such as Least Mean Square (LMS) and Normalized Least Mean Square (NLMS). Due to the reason that the spatial response is frequency independent, a broadband signal is seen as a narrowband signal from the beamformer's perspective. This method transformed a broadband problem into a narrowband one, hence single weight is required at each of the adaptive path.

A similar idea was proposed in [9], in which the frequency invariant property was required in the whole field of view of each beam. In our method, the frequency invariant property was focused on the direction of interest only. By focusing the FI property on the desired direction, and relaxing that on the other directions, the system is expected to have better performance due to the extra degrees of freedom. Simulation results show that with the frequency invariant beampattern synthesize method used, our proposed method achieves faster convergence rate.

This paper is organized in the following way: in section II, the adaptive multi-beam structure is introduced; in section III, the implementation of the proposed beamforming design is described; numerical results are given in section IV. Finally, conclusion is drawn in section V.

2. ADAPTIVE BEAMFORMING USING MULTI-BEAM STRUCTURE

2.1 GSC

A Linearly Constrained Minimum Variance(LCMV) beamformer with K number of microphones and J number of taps performs the minimization of the output signal's variance with respect to some given spatial and spectral constraints [1]. It can be formulated as

$$\min_{\mathbf{w}} \ \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w} \qquad \text{subject to} \quad \mathbf{C}^H \mathbf{w} = \mathbf{f} \tag{1}$$

where $\mathbf{R}_{xx}$ is the covariance matrix of the received signal $\mathbf{x} = [x_0[n] \; x_1[n] \; \cdots x_{K-1}[n]]$, $\mathbf{w}$ is the weight vector, $\mathbf{C} \in {}^{\mathscr{K}\mathscr{J}\times\mathscr{J}}$ is the constraint matrix and $\mathbf{f}$ is the response vector.

The constrained optimization of the LCMV problem in (1) can be conveniently solved using a GSC structure as shown in Fig. 1. The GSC performs a projection of the data onto an unconstrained subspace by means of a blocking matrix $\mathbf{B}$ and a quiescent vector $\mathbf{w}_q$. Thereafter, the standard unconstrained optimization algorithm such as the least mean square(LMS) or recursive least square (RLS) algorithm can be applied [10].

Since $\mathbf{w}_q$ is designed to satisfy the specified constraints, the signal of interest will pass through the beamformer having a desired response independent of $\mathbf{w}_a$. In the lower branch, the blocking matrix is required to block the signal of interest so that only interference and noise exist. When adapting $\mathbf{w}_a$, the structure will tend to cancel the interference and noise component from $d[n]$, while minimizing the variance of the output signal $e[n]$.

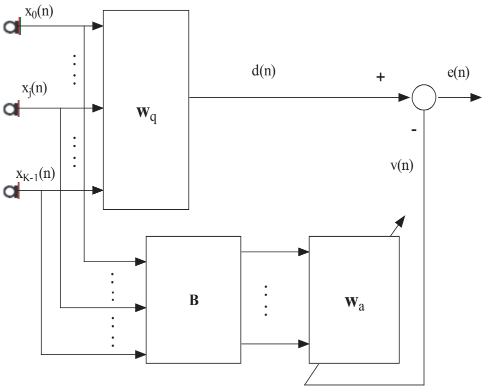

Figure 1: System structure of the Generalized Sidelobe Canceler

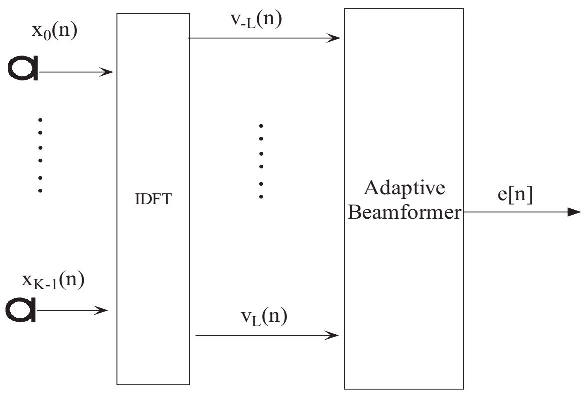

Figure 2: The proposed system structure

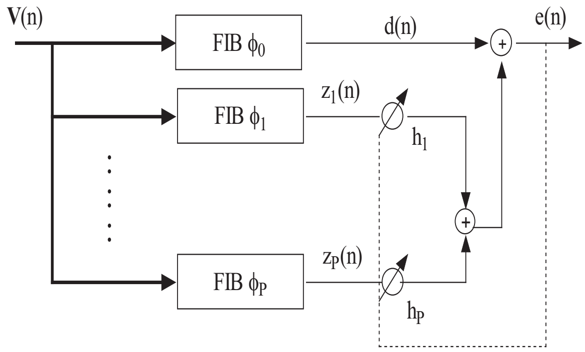

Figure 3: Adaptive array structure using frequency invariant beamformer

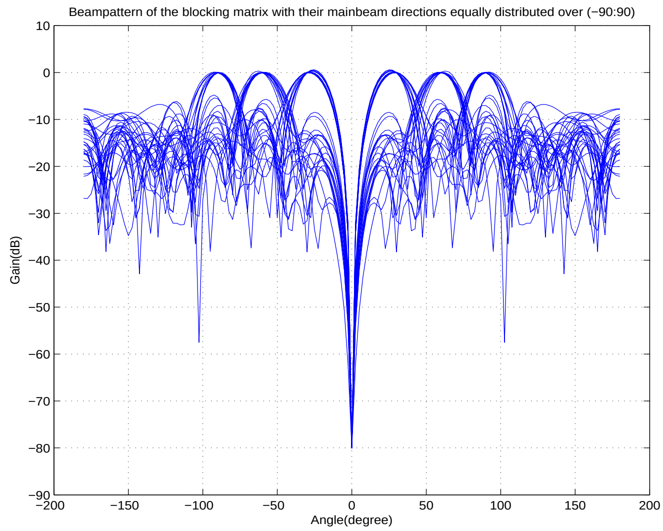

Figure 4: Beampattern of the blocking matrix which point to direction $\theta \in (-\pi/2 : \pi/2)$

2.2 Proposed Adaptive Structure

Conventional GSC structure converts the constrained adaptive beamformer to unconstrained one, however, it has its shortcomings. If the blocking matrix is not perfect, signal will leak into the lower branch, which in turn affect the cancelation of noise and interference from the desired signal. In this section, we are going to proposed an adaptive beamformer structure whose blocking matrix and quiescent vector are replaced by a 2-D frequency invariant beamformer.

As shown in Fig 2, signal received at each sensor $\mathbf{x}(n) = [x_0(n)\ x_1(n)\ \cdots\ x_{K-1}(n)]$ is first transformed into phase mode via IDFT, which is denoted by $\mathbf{v}(n) = [v_{-L}(n)\ v_{-L+1}\ \cdots\ v_L(n)]$. Assuming M is the total number of phases and it is an odd number, hence, $L = (M-1)/2$. These phase mode data are then passed into an adaptive beamformer whose structure is shown in Fig 3.

In this structure, the frequency-invariant mainbeam is pointing towards ϕ_0 which is the direction of signal of interest. The auxiliary beam is pointing towards $\phi_1, \phi_2, \ldots, \phi_P$ and having a zero response at ϕ_0 at the same time. This null formed at ϕ_0 is equivalent of the blocking matrix in the GSC structure. By this arrangement, the broadband beamforming problem is transformed into a narrowband beamforming problem. For adaptive narrowband problem, single adaptive weight is required at each frequency invariant beamformer(FIB) output. Thus, the total number of adaptive weights is reduced significantly.

The multi-beam beamforming network is constructed by the weight vectors computed using the following beampattern synthesis methods (2), and (3). The output of the main beam is $d(n)$. A group of adaptive weights h_i are multiplied by the outputs of the auxiliary beams $z_i(n)$. The output of the beamspace adaptive array is $e(n) = d(n) - \mathbf{h}^T\mathbf{z}(n)$, where $\mathbf{h} = [h_1\ h_2\ \cdots\ h_P]^T$ and $\mathbf{z}(n) = [z_1(n)\ \ z_2(n)\ \ \cdots\ \ z_P(n)]^T$. The real-valued weight vector $\mathbf{h}$ is adjusted by the error signal $e(n)$ with adaptive algorithms.

Without loss of generality, assuming that the signal of interest comes from broadside, the main beam should point to the direction of $\phi = 0$. The blocking matrix is arranged such that its column vectors form P FIBs with their main beam directions equally distributed over $[-\pi/2 : \pi/2]$. Since the first FIB and the P^{th} FIB actually point to the same direction, therefore we only need $P-1$ FIBs. All of the $P-1$ FIBs should have zero response at broadside. Fig 4 shows the beampattern of the blocking matrix. As seen from Fig 4, a deep null is formed at the direction of signal of interest.

2.3 Design of Frequency Invariant Beamformer

The following beam synthesis methods are used to form the multi-beam beamforming network.

2.3.1 Selective Frequency-Invariant Beamformer

$$\begin{array}{ll} \min & \int\limits_{\omega}\int\limits_{\phi} \|G(\omega,\phi)\|^2 d\omega d\phi \\ s.t & \|G(\omega,\phi_0)-1\| \le \delta, \omega \in [\omega_l,\omega_u] \end{array} \quad (2)$$

where $G(\omega,\phi)$ is the spatial response of the beamformer and is given in [8]. ω_l and ω_u are the lower and upper limit of the desired frequency range. ϕ_0 is the desired direction. δ is a predefined variable that controls the mainbeam ripples. The purpose is to minimize the square of the array gain across all frequencies and all angles, while constraining the gain at the desired angle to be approximately one.

In the design of beampattern for the auxiliary beam, an additional constraint will be included. This is because the blocking matrix in the GSC structure is replaced by FI Beamformer with zero response imposed at the direction of signal of interest. The design for auxiliary beam is shown below:

2.3.2 Selective Frequency-Invariant Beamformer for auxiliary beam design

$$\begin{array}{ll} \min & \int\limits_{\omega}\int\limits_{\phi} \|G(\omega,\phi)\|^2 d\omega d\phi \\ s.t & \left\{ \begin{array}{l} \|G(\omega,\phi_i)-1\| \le \delta, \;\; \omega \in [\omega_l,\omega_u] \\ G(\omega,\phi_0)=0, \;\; \omega \in [\omega_l,\omega_u] \end{array} \right. \end{array} \quad (3)$$

where $G(\omega,\phi)$ is the spatial response of the beamformer, ω_l and ω_u are the lower and upper limit of the desired frequency range. ϕ_0 is the direction of signal of interest, and ϕ_i is the pointing directions in the auxiliary beam. δ is a predefined variable that controls the mainbeam ripples.

3. IMPLEMENTATION OF THE PROPOSED SYSTEM

In this section, we will describe the implementation of the proposed system from a mathematical point of view.

Consider a circular array with K omnidirectional microphones located $\{r\cos\phi_k, r\sin\phi_k\}$ as shown in Fig 5 where r is the radius and $\phi_k = 2\pi k/K (k = 0 \cdots K-1)$ is the angular location of k^{th} microphone with respect to the reference axis. The microphones are uniformly distributed along the circle, and the inter-microphone spacing is fixed at $\lambda_{min}/2$, where λ_{min} denotes the smallest wavelength of the broadband signal of interest. For a plane wave (i.e., far field) arriving at azimuth angle ϕ and elevation angle θ ($\theta = 90°$), the steering vector of a circular array is

$$\mathbf{a}(\omega,\phi) = \left[e^{j\omega\tilde{r}\varepsilon\cos(\phi-\phi_0)}, \ldots, e^{j\omega\tilde{r}\varepsilon\cos(\phi-\phi_{K-1})} \right]^T \quad (4)$$

where $\tilde{r}$ is the normalized radius with respect to minimum wavelength and ε is the ratio between sampling frequency and maximum frequency.

Fig 2 and 3 show the system structure for the circular array. The received signals are first transformed into phase mode via Inverse Discrete Fourier Transform (IDFT). The transformed data are then feed into the proposed adaptive beamformer structure. Assuming $X[n] = [x_0(n), x_1(n), \ldots, x_{K-1}(n)]^T$ is the signal received at instance n, the transformed data is then denoted by $\mathbf{V}[n] = \mathbf{W}_{M,K} \cdot \mathbf{x}[n]$, where $\mathbf{W}_{M,K}$ is an $M \times K$ IDFT matrix whose mk^{th} element is $e^{\frac{j2\pi km}{K}}$ and

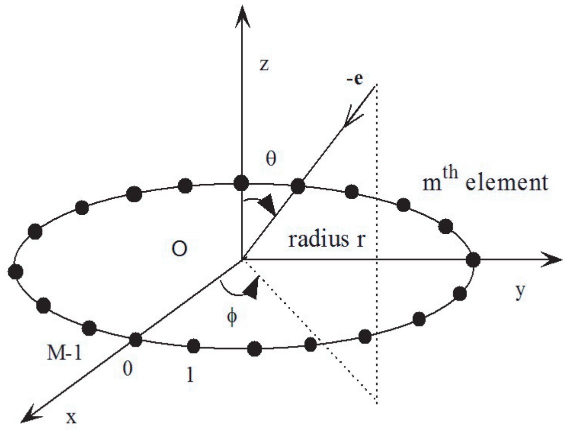

Figure 5: Circular Array Configuration

$$\{\mathbf{V}[n]\}_m = v_m(n) = \sum_{k=0}^{K-1} x_k[n] e^{\frac{j2\pi km}{K}}. \quad (5)$$

Here, M is the total number of phases, and assumed to be an odd number. $L = (M-1)/2$, and $m = -L, \cdots, L$.

Assuming there is only one source signal $s(n)$ with spctrum $S(\omega)$, taking the Discrete-time Fourier Transform (DTFT) of (5), we have,

$$\begin{aligned} V_m(\omega) &= \sum_{k=0}^{K-1} X_k(\omega) e^{j\frac{2\pi km}{K}} \\ &= S(\omega) \cdot \sum_{k=0}^{K-1} e^{j\omega\tilde{r}\varepsilon\cos(\phi-\phi_k)} e^{j\frac{2\pi mk}{K}} \\ &= S(\omega) \cdot a_m(\omega,\phi) \end{aligned} \quad (6)$$

where $X_k(\omega)$ is the Fourier Transform of $x_k[n]$.

Consider I broadband signals $\mathbf{s}[n] = [\, s_1[n], \; \cdots, \; s_I[n] \,]^T$ which impinge the circular array at azimuth angles $\phi_i, i = 1, \; \cdots, \; I$ respectively. Let $\mathbf{S}(\omega) = [\, S_1(\omega), \; \cdots, \; S_I(\omega) \,]$ denote the spectrum of the impinging signal $\mathbf{s}(n)$, hence,

$$V_m(\omega) = \mathbf{a}_m^T(\omega,\Phi)\mathbf{S}(\omega) + N_m(\omega) \quad (7)$$

where $\mathbf{a}_m^T(\omega,\Phi) = [a_m(\omega,\phi_1), \; \cdots, \; a_m(\omega,\phi_I)]^T$.

The transformed data $V_m(\omega)$ are then feed into a structure as shown in Fig 3. The upper path is a fixed FI beamformer, while the lower path consists of $P-1$ FI beamformer followed by a single-tap adaptive beamformer. The upper FI beamformer forms a main beam that is steered towards the signal of interest. The $P-1$ FI beamformers in the lower path form zero response in the direction of signal of interest, which acts as a blocking matrix that prevents the desired signal at the look direction from entering the adaptive part.

Consequently, the input to the adaptive part mainly consists of the undesirable signals. The adaptive coefficient **h** is then continuously updated in order to remove any undesired signals other than that of at the looking direction from appearing at the array output. This is achieved by minimizing the output energy of the beamformer using either LMS or NLMS adaptive filtering algorithms.

Let $b^0_m[n]$ be the coefficient of the upper FI beamformer $B^0_m(\omega)$ i.e., $B^0_m(\omega) = \sum_{n=0}^{N} b^0_m[n]e^{-jn\omega}$,where N is the filter order, similarly, the coefficients of the $P-1$ FI beamformers in the lower path are represented as $b^1_m[n], b^2_m[n], \cdots, b^{P-1}_m[n]$. The array output of the system in spectral domain can be expressed as

$$\begin{aligned} E(\omega) &= \sum_{m=-L}^{L} B^0_m(\omega)V_m(\omega) \\ &\quad - \sum_{i=1}^{P-1} \left\{ \sum_{m=-l}^{L} B^i_m(\omega)V_m(\omega) \right\} h_i \end{aligned} \tag{8}$$

Substitute (7) into (8),

$$\begin{aligned} E(\omega) &= \sum_{m=-L}^{L} B^0_m(\omega)(\mathbf{a}^T_m\mathbf{S}(\omega)+N_m(\omega)) \\ &\quad - \sum_{i=1}^{P-1} \left\{ \sum_{m=-l}^{L} B^i_m(\omega)(\mathbf{a}^T_m\mathbf{S}(\omega)+N_m(\omega)) \right\} \cdot h_i \\ &= \sum_{m=-L}^{L} B^0_m(\omega)\mathbf{a}^T_m(\omega,\Phi)\mathbf{S}(\omega) + B^0_m(\omega)N_m(\omega) - \\ &\quad \sum_{i=1}^{P-1} \left\{ \sum_{m=-l}^{L} B^i_m(\omega)\mathbf{a}^T_m(\omega,\Phi)\mathbf{S}(\omega) + B^i_m(\omega)N_m(\omega) \right\} \\ &\quad \cdot h_i \end{aligned} \tag{9}$$

Since $B^0_m(\omega)\mathbf{a}^T_m(\omega,\Phi)$ is designed to be frequency-invariant at the direction of signal of interest from previous section, hence,

$$B^0_m(\omega)\mathbf{a}^T_m(\omega,\Phi) \approx \mathbf{a}^T_{m'}(\Phi) \tag{10}$$

Also $B^i_m(\omega)\mathbf{a}^T_m(\omega,\Phi)$ are designed to have zero responses at the direction of signal of interest, hence, taking IDFT of (9), one gets the time-domain expression as,

$$\begin{aligned} e(n) &= \sum_{m=-L}^{L} \mathbf{a}^T_{m'}(\Phi)s[n] + \eta^0_m[n] \\ &\quad - \sum_{i=1}^{P} \sum_{m=-L}^{L} \eta^i_m[n]h_i \end{aligned} \tag{11}$$

where $\eta_m[n]$is the noise at the output of the FI beamformer.

The adaptive weights are updated using NLMS algorithm.

$$\mathbf{h}[n] = \mathbf{h}[n-1] + \mu \cdot \mathbf{z} \cdot e(n)/|\mathbf{z}|^2 \tag{12}$$

where μ is a stepsize parameter and $\mathbf{z} = [z_1, \cdots, z_p]$ are the inputs to the adaptive part and $\mathbf{h}[n] = h_1[n], \cdots, h_P[n]$.

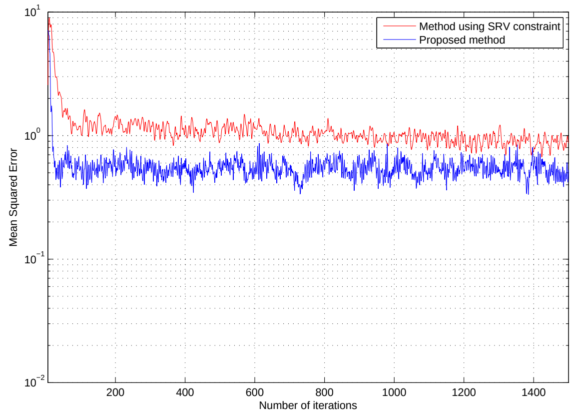

Figure 6: Learning curve of the proposed method averaged over 200 trials. The step size is 0.01 in the NLMS algorithm

4. NUMERICAL RESULTS

In our simulations, 20 microphones are arranged as shown in Fig 5. For each FI beamformer, the number of phase mode M is 17, the filter order for the frequency compensation network is 16, and the interested frequency bandwidth is $[0.3\pi, 0.95\pi]$. The blocking matrix of the proposed GSC is formed by 9 FI beamformers over $[-\pi : \pi]$. One adaptive weight is used for each of the FI beamformers. Assume there are two non-coherent signals impinging on the array at angles 0° and 30°, respectively. Here, we assume the desired signal arrives at the array at angle 0° and the interference signal impinges on the array at angle 30°. The desired signal is composed of 30 sinusoidal signals with frequencies ranging from 800 Hz to 3800 Hz at an interval of 100 Hz. The interfering signal is also composed of 30 sinusoidal signals with frequencies ranging from 500 Hz to 3500 Hz at an interval of 100 Hz. The additive white Gaussian noise at each sensor is assumed to have the same power. The Signal-to-Interference-and-Noise-Ratio (SINR) is -20 dB, and the number of snapshot is 1450.

The performance of the proposed method using (2) and (3) is evaluated and compared with the method proposed in [9]. As shown in Fig 6, in the stated frequency range, our method achieved faster convergence speed, and it converges to smaller mean squared error.

5. CONCLUSION

A broadband beamspace adaptive array that constructs a multi-beam network is proposed in this paper. The main idea is to replace the blocking matrix and the quiescent vector in the conventional General Sidelobe Canceler (GSC) with a frequency invariant beampattern synthesis method. A broadband signal problem is therefore transformed into a narrowband signal problem. Simulation results show that our proposed algorithm has faster convergence rate than an existing algorithm.

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