

ADAPTIVE SPECTRUM SENSING AND LEARNING IN COGNITIVE RADIO NETWORKS

Abbas Taherpour[†], Saeed Gazor[‡], and Abolfazl Taherpour[†]

[†] Imam Khomeini International University, Qazvin, Iran

[‡] Queen's University, Kingston, ON., Canada
email:taherpour@ieee.org

ABSTRACT

In this paper, we propose a Primary User (PU) activity detection algorithm for a wideband frequency range which updates spectrum sensing parameters. We assume that the signal of PUs and noise are independent and jointly zero-mean Gaussian processes with unknown variances. We employ a Markov Model (MM) with two states to model the activity of PU which representing the presence and absence of the PU at each subband. By using such a MM, the proposed PU activity detector estimates the probabilities of PU presence in different subbands, recursively, in three steps. Our simulation results show that the proposed algorithm always performs better than the Energy Detector (ED) and despite its simple implementation has slightly better performance than the computationally complex Cyclostationarity Feature Detector (CFD) for practical values of the Signal-to-Noise Ratio (SNR).

1. INTRODUCTION

Recent measurements reveal that many portions of the licensed spectrum are not used over significant time periods [1]. Since the number of users and their data rates steadily increase, inefficient fixed spectrum policy is no longer a feasible approach. One proposal for alleviating the spectrum scarcity is allowing Secondary Users (SU)s to use the spectrum holes whenever is possible. The Cognitive Radio (CR) is the promising technology which is considered as the practical solution for implementation of dynamic spectrum sharing techniques.

One of the major challenges in implementing the CR technology is the spectrum sensing. In the spectrum sensing, the CRs must accurately monitor the presence or absence of the PUs with efficient methods over a particular part of the spectrum to find the available spectrum holes [2–6]. A common method for detection of an unknown signal in noise is using the ED (a.k.a. radiometry) [7].

The ED requires the noise variance to differentiate between the energies of noise and PU signal. In addition, it is well-known that the performance of the ED is susceptible to the noise power mismatches [8]. In the case of unknown noise variance, the cyclostationarity property of communication signals can be exploited for spectrum sensing [6]. In contrast to noise which is considered usually as a wide sense stationary process, the mean and autocorrelation of communication signals show built-in periodicity which can be used to differentiate the noise from modulated signal. A CFD performs better than the ED in discriminating PU signal against noise due to its robustness to the uncertainty in noise variance [9]. The drawback of CFD is that, it requires signifi-

cantly long observation time and is computationally complex for practical implementation.

On the other hand, the concept of the CR implies that it can learn and then adapt itself to the environment variations. The decisions in CRs are the results of extrapolations of the current observation based on reasoning or learning. The learning and using memory can be done within different functionalities of the CRs. In the context of the spectrum sensing, the CR must decide about the state of each frequency band at any given time. The CR in addition to the current observations can use previous information kept in its memory. In the most of previously reported works on spectrum sensing the process of learning and using the possible available information about the PU activity have been ignored.

In general, the available spectrum which is scanned for finding the spectrum hole is large and can be in order of GHz [3]. Thus, in this paper, we present a wideband spectrum sensing scheme that learn some unknown parameters from the environment. We study the spectrum sensing problem across a wide frequency range by dividing the scanning spectrum into multiple subbands. In order to address the learning process and using the previous observations, we use a two-state Markov chain with occupied (the channel is used by a PU) and vacant (the channel is available for the SU) states to model the usage of each subband. This model has been proposed and used for spectrum sensing and access in the context of CR [10, 11] and we try to use this model to bring the memory to proposed spectrum sensing algorithm. In [10] this model has been used for proposing a decentralized medium access control (MAC) approach for ad-hoc cognitive networks. In our work, the MM enable us to consider the previous decisions about the subband occupancy for current time decision making and to consider the available records for detection. Also, because of the variations in PU signal and noise variances due to the channel and the source variations, we assume that the PU signal and noise are random Gaussian distributed signals with unknown variances.

The remaining of the paper is organized as follows: In Section 2, we formulate the problem and present the MM for PU activity. In Section 3, we propose a three-step algorithm based on MM for the PU activity detector at each of the subbands. In Section 4, we evaluate the performance of the proposed detector based on computer simulation. Finally, Section 5 concludes the paper.

2. SYSTEM MODEL

We assume that the whole spectrum bandwidth is divided into K subbands, where each subband may be vacant or used

by a PU. We consider a simple two-state discrete (binary) MM for PU activity for each subband and at each sensing time as shown in Figure 1. The two states are vacant (PU absence) and occupied (PU presence). We assume that the probability transition matrix of this binary MM for subbands k^{th} is as follows:

$$\Pi_k = \begin{bmatrix} \alpha_k & 1 - \beta_k \\ 1 - \alpha_k & \beta_k \end{bmatrix} \quad (1)$$

where $\alpha_k \in (0, 1)$ denotes the probability of vacant subband when the subband was vacant at the previous sensing time and $\beta_k \in (0, 1)$ denotes the probability of presence of a PU when the PU was present at the previous sensing time. By using this notation, the probability of appearing PU when the subband was vacant is $1 - \alpha_k$ and the probability of vanishing PU conditioned that it was present in the previous sensing time will be $1 - \beta_k$. In the context of dynamic spectrum sharing, the false alarm probability P_{fa} indicates the probability that a spectrum holes be falsely detected as an occupied band, i.e., P_{fa} represents the percentage of the spectrum holes which are not used. Therefore, the SUs must reduce the false alarm probability P_{fa} as much as possible. On the other hand, the missed detection probability $P_m = 1 - P_d$ determines the probability that an occupied subband be mistakenly detected as a spectrum hole. Such a missed detection induces unauthorized interference for PU. Thus, the missed detection probability should be smaller than some small threshold to avoid perceptible performance loss for the PU. The matrix Π_k can be determined by considering the activity history of the PUs across the scanning subbands. This matrix regulates two trade-offs in the response of the algorithm. A greater value for α_k results to higher spectral usage efficiency and less PU interference protection level. In the other hand, a greater value for β_k results to the more conservative scenario for PU protection and lower spectral usage efficiency for SUs. In this model, the activity factor of PU is defined as proportion of the average occupied time slots to average of vacant ones and using the geometric distribution average can be written as follows:

$$\rho_k = \frac{\frac{1}{1-\beta_k}}{\frac{1}{1-\alpha_k} + \frac{1}{1-\beta_k}} = \frac{1 - \alpha_k}{2 - \alpha_k - \beta_k} \quad (2)$$

The activity factor can be obtained by using the historical data of each subband and is about $\rho_k = 0.375$ for voice applications and $\rho_k = 0.1$ or less for multimedia and data applications [12].

Let L denote the number of samples from each subband. The problem of spectrum sensing at $k^{\text{th}} \in \{1, \dots, K\} \triangleq \mathcal{S}$ subband and sensing time m can be written as follows:

$$\mathbf{X}_k(m) \sim \begin{cases} \mathcal{CN}(\mathbf{0}, \sigma_k^2(m)\mathbf{I}_L), & \text{if } \mathcal{H}_0^k \\ \mathcal{CN}(\mathbf{0}, \sigma_k^2(m)\mathbf{I}_L), & \text{if } \mathcal{H}_1^k \end{cases} \quad (3)$$

where $\mathbf{X}_k(m) \triangleq [x_{k,1}(m), \dots, x_{k,L}(m)]^T \in \mathbb{C}^L$ is the observed vector at the subband $k^{\text{th}} \in \mathcal{S}$ and \mathbf{I}_L is L -dimensional identity matrix. In this model, the additive noise and PU signal are assumed as complex Gaussian distributed random variables with variances $\sigma^2(m)$ and $\{\sigma_k^2(m)\}_{k=1}^K$ at the sensing time m , respectively. The value of the parameters $\sigma^2(m)$ and $\{\sigma_k^2(m)\}_{k=1}^K$ depend on the powers of noise and transmitted

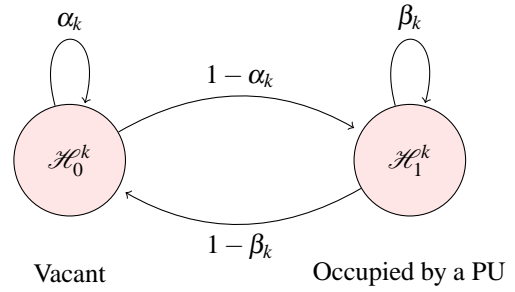


Figure 1: The PU activity modeling by using MM at k^{th} subband.

PU signal and the channel gains which we consider them as the unknown parameters. We assume that the variances of the PU signal and noise are constant during the sensing time and change smoothly from each sensing time to another one. The LR function at the sensing time m and for $k^{\text{th}} \in \mathcal{S}$ subband can be written as following hypothesis testing problem:

$$L_k(m) = \frac{f(\mathbf{X}_k(m); \mathcal{H}_1^k)}{f(\mathbf{X}_k(m); \mathcal{H}_0^k)} = \left(\frac{\sigma^2(m)}{\sigma_k^2(m)} \right)^L \times \exp \left\{ \frac{\sigma_k^2(m) - \sigma^2(m)}{\sigma_k^2(m)\sigma^2(m)} \|\mathbf{X}_k(m)\|^2 \right\} \quad (4)$$

where $\|\cdot\|$ denotes the Euclidean norm. At the each sensing time, by observing the vectors $\{\mathbf{X}_k(m)\}_{k=1}^K$, we can calculate the LR functions by using the above equation and use this LR function, as explained in the next section, for PU presence detection at the each subband.

3. PROPOSED ALGORITHM

We define $P_{m|m-1}^{(k)}$ as the *a priori* probability of PU activity at k^{th} subband and at the sensing time m , i.e.,

$$P_{m|m-1}^{(k)} \triangleq P[\delta^{(k)}(m) = \mathcal{H}_1^{(k)} | \mathcal{O}^{(k)}(m-1)] \quad (5)$$

Where $\delta^{(k)}(m) = \mathcal{H}_1^{(k)}$ denote the event that subband k^{th} is occupied at the time m and $\mathcal{O}^{(k)}(m-1)$ denote the all available observations about the subband k^{th} up to time $m-1$. Note that *a priori* probability of PU activity, in the absence of any knowledge at time $m=1$, can be initialized as $P_{1|0}^{(k)} = \frac{1}{2}, \forall k = 1, \dots, K$. We develop three-step algorithm for PU activity detection by using these *a priori* probabilities:

Step I: LR Function Calculation

In this step, we calculate the LR function in (4) using the observed samples $\{\mathbf{X}_k(m)\}_{k=1}^K$ and the estimated variances in the previous sensing time. We assume that the CR knows an initial approximate estimations about the noise and PUs variances which are used to calculate the LR functions at $m=1$. After receiving some samples from the subbands, these approximate estimations can be updated as will be explained in the step 3.

Step II: Updating Posterior Probabilities

In this step, we combine the priori probabilities in (5) by LR functions in current time to obtain the *a posteriori* probabilities which are defined as follows:

$$P_{m|m}^{(k)} = P[\delta^{(k)}(m) = \mathcal{H}_1^{(k)} | \mathcal{O}^{(k)}(m)]. \quad (6)$$

$P_{m|m-1}^{(k)}$ and $P_{m|m}^{(k)}$ are the estimated probabilities of PU presence at the subband k^{th} and sensing time m , with and without the use of the current information provided by $\{\mathbf{X}_k(m)\}_{k=1}^K$. By using Bayes rule, we can derive the posterior probabilities $P_{m|m}^{(k)}$ by using *a priori* and current data about the PU presence as follows:

$$P_{m|m}^{(k)} = \frac{L_k(m)P_{m|m-1}^{(k)}}{L_k(m)P_{m|m-1}^{(k)} + (1 - P_{m|m-1}^{(k)})} \quad (7)$$

where $L_k(m)$ is the LR function of k^{th} subband at time m given in (4). In this step a hard decision rule is made as:

$$\mathcal{H}^{(k)}(m) = \begin{cases} \mathcal{H}_1^k, & \text{if } P_{m|m}^{(k)} \geq \eta \\ \mathcal{H}_0^k, & \text{otherwise} \end{cases} \quad (8)$$

where the threshold η , regulates the trade-off between the probability of false alarm P_{fa} and detection probability P_d . In this way, the PU is determined to be present or not at the subbands $k^{\text{th}} \in \mathcal{S}$ and at sensing time m and hence we can partition the the set of all indexes \mathcal{S} as $\mathcal{S} = \mathcal{S} \cup \overline{\mathcal{S}}$, where \mathcal{S} and $\overline{\mathcal{S}}$ denote the subsets containing the indexes of vacant and occupied subbands, respectively.

Step III: Prediction and Parameter Estimation

A prediction of the priori probabilities for the next sensing time instant, $P_{m+1|m}^{(k)}$ is required to obtain the posterior probabilities in (7) as well as the hard decisions in (8). We use the MM in the Figure 3(a) to predict these probabilities. The predicted probabilities for the next sensing time $m+1$ are easily obtained based on the assumed MM as:

$$\begin{bmatrix} 1 - P_{m+1|m}^{(k)} \\ P_{m+1|m}^{(k)} \end{bmatrix} = \begin{bmatrix} \alpha_k & 1 - \beta_k \\ 1 - \alpha_k & \beta_k \end{bmatrix} \begin{bmatrix} 1 - P_{m|m}^{(k)} \\ P_{m|m}^{(k)} \end{bmatrix} \quad (9)$$

that is

$$P_{m+1|m}^{(k)} = (1 - \alpha_k)(1 - P_{m|m}^{(k)}) + \beta_k P_{m|m}^{(k)}. \quad (10)$$

Also in this step, by considering the subband which their index belong to the subset \mathcal{S} (vacant subbands), we can update the noise variance estimation for the sensing time $m+1$ as follows:

$$\widehat{\sigma}^2(m+1) = \begin{cases} \widehat{\sigma}^2(m+1), & \text{if } \mathcal{S} = \phi \\ \kappa_n \widehat{\sigma}^2(m) + (1 - \kappa_n) \left(\frac{1}{|\mathcal{S}|} \sum_{i \in \mathcal{S}} \frac{\|\mathbf{X}_i(m)\|^2}{L} \right) & \text{if } \mathcal{S} \neq \phi \end{cases} \quad (11)$$

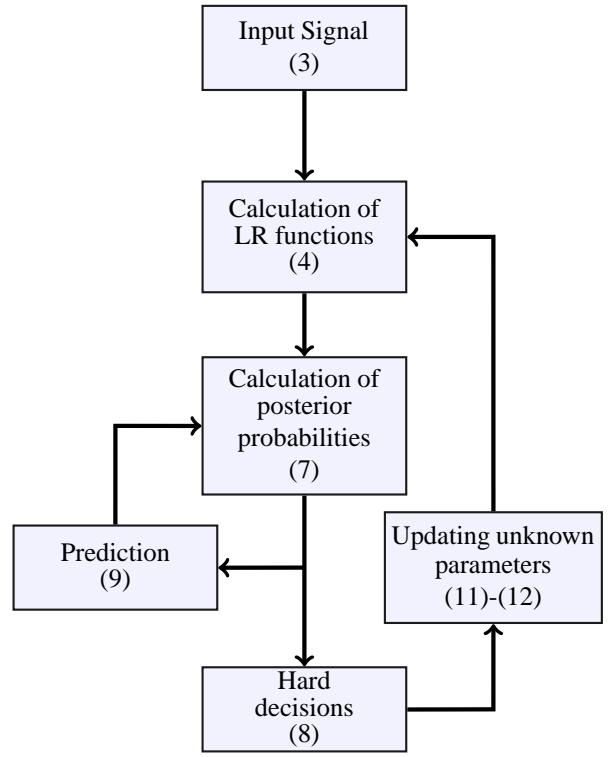


Figure 2: The block diagram of the proposed algorithm

similarly for the subbands which their indexes belong to $\overline{\mathcal{S}}$ (occupied subbands) the variance updating equation is as follows:

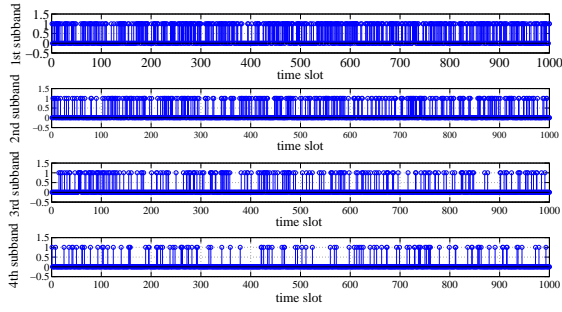
$$\widehat{\sigma}_k^2(m+1) = \begin{cases} \widehat{\sigma}_k^2(m+1), & \forall k \in \mathcal{S}, \text{ if } \overline{\mathcal{S}} = \phi \\ \kappa_s \widehat{\sigma}_k^2(m) + (1 - \kappa_s) \frac{\|\mathbf{X}_k(m)\|^2}{L}, & \forall k \in \overline{\mathcal{S}}, \text{ if } \overline{\mathcal{S}} \neq \phi \end{cases} \quad (12)$$

where $\kappa_n \in (0, 1)$ and $\kappa_s \in (0, 1)$ determine the effect of the previous estimations of noise and the PU signal variances and the current observations, respectively. The values of the parameters κ_n and κ_s depend on the environment which the CR is used. Actually, in the proposed algorithm the parameters α_k and β_k depends on the PU service and model the probability of the PU transition between active and idle states. For instance lower values of α_k and β_k show a PU with the bursty transmission. Also to capture the variation of the environment we use the the paramors κ_n and κ_s . These parameters should be selected near 1 for a quickly varying environment or 0 otherwise. In the special case, we can chose these parameters equal, i.e., $\kappa_n = \kappa_s = \kappa$.

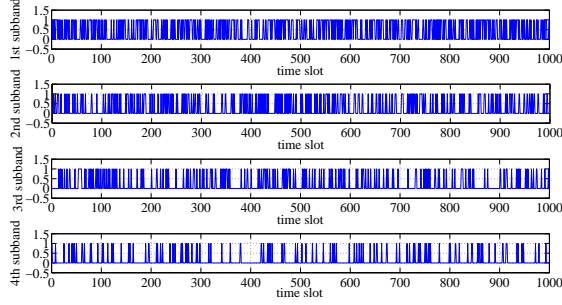
By using the above three-step algorithm, we can decide about the status of the each subband at each sensing time, recursively. Figure 3(b) shows the block diagram of the proposed algorithm for the PU activity detection.

4. SIMULATION RESULTS

In this section, we present some numerical results for performance evaluation of the proposed algorithm and then compare with those of the previously presented algorithms. For a given average SNR and false alarm probability P_{fa} , we generate the decision statistic randomly according to the assumed



(a) The initial status of four subbands



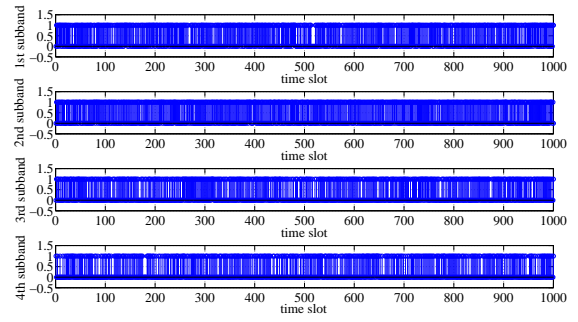
(b) The hard decision output of the proposed algorithm

Figure 3: The initial status and decision outputs of the proposed algorithm for $L = 8$, $K = 4$, and average SNR $\bar{\gamma} = 5$ dB

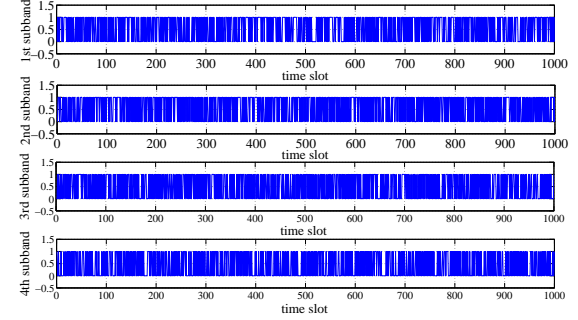
distributions for 10^6 independent trials (in the absence of PU signal at the subband) and choose the detection threshold as $100P_{fa}$ percentile of the generated data, i.e., for $P_{fa} = 10^{-3}$, $100 \times 10^{-3} = 0.1\%$ of the generated decision statistic (out of 10^6) are above the determined threshold. By considering an occupied subband and using obtained threshold, we can compute the probability of detection, i.e., P_d .

Figure 3(a) depicts our simulation scenario in order to evaluate the performance of the proposed algorithm with four different cases for PU activity over four different subbands. We denote the vacant and occupied status of subbands with 0 and 1, respectively. It is assumed that $[\alpha_1, \alpha_2, \alpha_3, \alpha_4] = [0.6, 0.7, .8, 0.9]$ and $[\beta_1, \beta_2, \beta_3, \beta_4] = [0.4, 0.3, 0.2, 0.1]$ which lead to the PU activity factor of $[\rho_1, \rho_2, \rho_3, \rho_4] = [0.4, 0.3, 0.2, 0.1]$. Also we assume that average SNR $\bar{\gamma} = 5$ dB, $L = 8$ and $\kappa_n = \kappa_s = 0.9$, for forgetting factors in (11) and (12). We use the proposed algorithm to detect the presence or absence of PU in this scenario. Figure 3(b) illustrates the simulation results of hard detection $\mathcal{H}^{(k)}(m)$ in (8) by using the proposed algorithm. For all subbands, the threshold has been computed for $P_{fa} = 0.01$. As can be seen the proposed algorithm detect the PU activity in these scenario with high probability so that respectively for four subbands in the Figure 3(b), in 99.4%, 98.3%, 97.8% and 98.8% of times, the proposed detector detects the states of the subbands, correctly.

In order to evaluate the performance of the proposed algorithm when there are some errors between the actual and the used transition probabilities, we use the scenario in Figure 4(a). It is assumed that $[\alpha_1, \alpha_2, \alpha_3, \alpha_4] = [0.6, 0.7, .8, 0.9]$ and $[\beta_1, \beta_2, \beta_3, \beta_4] = [0.8, 0.9, 0.7, 0.6]$. We assume that there



(a) The initial status of four subbands



(b) The hard decision output of the proposed algorithm

Figure 4: The performance of the proposed algorithm in the case of uncertainty in transition probabilities for $L = 8$, $K = 4$, and average SNR $\bar{\gamma} = 5$ dB

is 10% difference between the actual and the used probabilities in the proposed algorithm. The other simulation parameters are the same as Figure 3. Figure 4(b) shows the results of the simulation based on the proposed algorithm. As can be seen, in this case the performance of the proposed algorithm is degraded slightly so that the detection probabilities are 0.916, 0.929, 0.90.7 and 0.918. Our further simulation shows that for a given constant activity factor, i.e., ρ_k , the more the value of transition probabilities, i.e., α_k and β_k , the less the effect of parameters uncertainty on the performance of the proposed algorithm. In the other words, for a given constant false alarm probability, the sensitivity of the proposed algorithm increases with the decrease in the value of transition probabilities. Thus, in the case of having some uncertainty in MM parameters, the proposed detects the status of the subbands correctly at most of times.

Figure 5 illustrates the detection probability P_d versus average SNR $\bar{\gamma}$ for different false alarm probabilities P_{fa} and the number of samples $L = 32$. As can be observed by increasing $\bar{\gamma}$ or P_{fa} , the performance of the proposed algorithm improves. It is notable that in practice by increasing the number of samples L , the average SNR increases and therefore by considering the mentioned behavior, the performance will improve. Unfortunately, we can not increase L arbitrarily since L determines the acquisition time (the waiting time-lag before a decision can be made). Thus in practice, we have to make a trade-off between P_{fa} (the spectrum usage efficiency), P_d (PU interference protection level) and L (the acquisition time). In Table 1, the proposed MM based Detector (MMD) is compared with the ED [2] and CFD [6]. We assume that

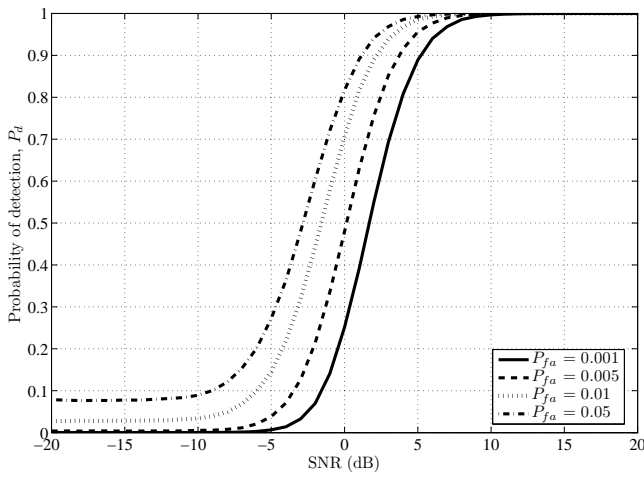


Figure 5: The probability of detection P_d versus SNR for different false alarm probabilities P_{fa} and $L = 32$

the ED knows the average SNR value $\bar{\gamma}$. We see that both MMD and CFD always have better performance than ED. When the SNR is low the CFD performs better than MMD. But when the SNR value increases the performance of the MMD will be similar to that of the CFD and both of them have approximately the same probability of detection. As noted before, the CFD is very complex for practical implementation and the CR requires to know some features of PU signals that may not be possible in practice. Also the sensing time for CFD must be large enough to detect the cyclostationarity property of observed signal. By considering the mentioned trade-offs between performance and sensing time, the proposed MMD can be performed when there are some new samples from each of the subbands.

Table 1: The performance comparison of the ED [2], CFD [6] and proposed MMD for $L = 4$

SNR	P_{fa}	P_d		
		ED	MMD	CFD
-15 dB	0.1212	0.00432	0.3212	0.2984
-10 dB	0.0973	0.00921	0.5340	0.5179
-5 dB	0.0384	0.08582	0.8717	0.8702
0 dB	0.0192	0.2063	0.9219	0.9163
5 dB	0.0081	0.8902	0.9972	0.9966
10 dB	0.00097	0.9843	0.9997	0.9993

5. CONCLUSION

In this paper, we proposed a simple adaptive MM based algorithm to detect the absence or presence of PU in multiple subbands. The PU activity was modeled by a two-state Markov chain with the known transition probabilities. Also, the PU signal and noise are assumed to be complex Gaussian distributed random process with unknown variances. By observing new samples from subbands the LR functions are calculated and a three-step algorithm for PU activity detection is performed which results in a soft and hard detection outputs for each subbands. Also, the prior probabilities of PU activity at the subbands based on the MM model and noise and PU signal variances for the next sensing time is predicted.

The simulation results showed that the performance of both MMD and CFD is better than the ED and by increasing the average SNR the performance of MMD improves and becomes slightly better than that of the CFD. By considering the high complexity and also the large sensing time for CFD, the proposed MM based algorithm can be performed simply at each sensing time. Also this algorithm allows us to predict the status of the PU presence or absence within subbands which is useful for predicting the time amount of subband availability and hence CR signal transmission rate.

REFERENCES

- [1] Federal Communications Commissions, "Spectrum policy task force report (ET Docket No. 02-135)," Nov. 2002.
- [2] A. Taherpour, Y. Norouzi, M. Nasiri-Kenari, A. Jamshidi, and Z. Zeinalpour-Yazdi, "Asymptotically optimum detection of primary user in cognitive radio networks," *IET Communications*, vol. 1, pp. 1138–1145, Dec. 2007.
- [3] A. Taherpour, S. Gazor, and M. Nasiri-Kenari, "Invariant Wideband Spectrum Sensing Under Unknown Variances," *IEEE Transactions on Wireless Communications*, vol. 8, pp. 2182–2186, May 2009.
- [4] A. Taherpour, M. Nasiri-Kenari, and S. Gazor, "Multiple antenna spectrum sensing in cognitive radios," *IEEE Transactions on Wireless Communications*, vol. 9, pp. 814–823, Feb. 2010.
- [5] B. Wild and K. Ramchandran, "Detecting primary receivers for cognitive radio applications," in *DySPAN 2005*, pp. 124–130, Nov. 2005.
- [6] P. Sutton, K. Nolan, and L. Doyle, "Cyclostationary signatures in practical cognitive radio applications," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 1, pp. 13–24, 2008.
- [7] H. Urkowitz, "Energy detection of unknown deterministic signals," *Proceedings of the IEEE*, vol. 55, pp. 523–531, Apr. 1967.
- [8] A. Sonnenschein and P. M. Fishman, "Radiometric detection of spread-spectrum signals in noise of uncertain power," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 28, pp. 654–660, July 1992.
- [9] W. A. Gardner, "Signal interception: a unifying theoretical framework for feature detection," *IEEE Transactions on Communications*, vol. 36, pp. 897–906, Aug. 1988.
- [10] Q. Zhao, L. Tong, A. Swami, and Y. Chen, "Decentralized cognitive MAC for opportunistic spectrum access in ad hoc networks: A POMDP framework," *IEEE Journal on selected areas in communications*, vol. 25, no. 3, p. 589, 2007.
- [11] C. Clancy, J. Hecker, E. Stuntebeck, and T. O'Shea, "Applications of machine learning to cognitive radio networks," *IEEE Wireless Communications*, vol. 14, no. 4, pp. 47–52, 2007.
- [12] N. Tripathi, "Simulation based analysis of the radio interface performance of an IS-2000 system for various data services," in *IEEE VEH TECHNOL CONF*, vol. 4, pp. 2665–2669, 2001.