A NEW SPATIOTEMPORAL FILTERING METHOD FOR SINGLE-TRIAL ERP SUBCOMPONENT ESTIMATION

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ABSTRACT

A new spatiotemporal filtering method for single trial estimation of event related potential (ERP) subcomponents is presented here. The method is an extension of the recent method for estimation of ERP components developed by Li et al. [1]. This method can be used for estimation of ERP subcomponents when they have temporal overlap and are often viewed as one component. Using the scalp projection of the subcomponents, a new constraint is added to the cost function which can deflate the other subcomponents. Therefore, the proposed method is capable of estimating the ERP subcomponents effectively. The method is applied to both simulated and real data and has been shown to perform very well even in low signal to noise ratios. The approach can be especially useful in mental fatigue analysis where the relative variability of the P300 subcomponents are the key factors in order to detect the level of fatigue.

Index Terms— Event related potential (ERP), subcomponent, spatiotemporal filtering, constraint.

1. INTRODUCTION

Event related potentials (ERPs) are voltage fluctuations in EEG induced within the brain and are time locked to sensory, motor, or cognitive events [2][3]. They are used mainly by clinicians in order to assess a number of neurological disorders and cognitive processes. An ERP wave consists of a sequence of labeled positive and negative amplitude components. These components reflect various sensory, cognitive and motor processes that are classified on the basis of their scalp distributions and responses to experimental stimuli.

One common approach to analyze the ERP components is to average the time-locked single-trial measurements. Averaging the EEG over a number of trials to obtain the ERP waveform results in loss of information related to trial-to-trial variability. The ERP variability across trials can be exploited in identification of many brain abnormalities. An effective analysis of ERPs should then be based on single trial estimation. Several methods such as Wiener [4], maximum a posterior (MAP) [5] and Kalman filtering approaches [6] have been used in single trial estimation. Other methods are proposed in [7] and [8] which are based on principal component analysis (PCA) and independent component analysis (ICA). These methods are not suitable in low signal to noise ratios and may fail in many situations because of very low signal to background noise power and inter-trial variability of the recorded ERPs.

Recently a method for spatiotemporal ERP component estimation from single trials is developed in [1]. The method is effective in ERP component estimation in negative signal to noise ratios. This method assumes that there is no temporal correlation or overlap between the components; in the case of having correlated components or subcomponents it can not achieve correct results. Therefore, in this paper we have extended and modified the method presented in [1] in order to have a single trial estimation of ERP subcomponents where there is inherent temporal correlation between the subcomponents.

Our proposed method for estimation of the ERP subcomponent descriptors defines two cost functions (in the case of having two subcomponents which have temporal correlation) and estimates the latency, amplitude, and scalp projections of both subcomponents. The method can be generalized and considered for the case that there are more than two subcomponents. However, because one important issue is to deal with the P300 subcomponents (i.e. P3a and P3b), the method can be used specifically for P300 subcomponent estimation. In this paper it is shown mathematically that the method in [1] for ERP component estimation can not be used for ERP subcomponent estimation because of temporal correlation between the subcomponents whereas, the new ERP subcomponent estimation method proposed here can result in a very good estimation of ERP subcomponent descriptors (latency, amplitude and scalp projection). The proposed method is highly demanded for some applications such as mental fatigue in which single trial estimation of P300 subcomponents and their variability can be utilized in detecting the level or degree of fatigue. The remainder of the paper is structured as follows. In section 2 linear generative EEG model is described. Then, in section 3 the new spatiotemporal filtering method is explained. In section 4 the results of applying the spatiotemporal filtering to both simulated and real data are provided. Finally, section 5 concludes the paper.

2. LINEAR GENERATIVE EEG MODEL

In this section a composite EEG model which includes two generally correlated subcomponents is provided and then a new approach for estimation of their parameters is proposed. To do this we start with writing the linear generative EEG model in the matrix form as:

$$\mathbf{X} = \mathbf{a}.\mathbf{s} + \sum_{i=1}^{N} \mathbf{b}_i \mathbf{n}_i \tag{1}$$

where $\mathbf{X}$ is a $D \times T$ matrix which represents the single-trial EEG data with D channels and T samples, $\mathbf{s}$ is a $1 \times T$ vector that can be the time course of ERP component, $\mathbf{n}_i$ denotes the noise in general and N is the number of noise components. The vectors $\mathbf{a}$ and $\mathbf{b}_i$ are of dimension $D \times 1$ and can be considered as the projection of the corresponding source to the electrodes on the scalp. $\mathbf{X}$ can also be modeled in terms of its constituent normalized components as:

$$\mathbf{X} = \sigma_{s0}\mathbf{a}_0.\mathbf{s}_0 + \sum_{i=1}^{N} \sigma_i \mathbf{b}_{0i}\mathbf{n}_{0i} \tag{2}$$

where $\mathbf{a}_0$, $\mathbf{s}_0$, and $\mathbf{n}_{0i}$ are the normalized versions of the their counterparts in (1). The scalars σ_{s0} and σ_i are the overall contributions of the sources to the multichannel EEG data. For a stable normalized scalp projection $\mathbf{a}_0$, it is expected that $\mathbf{a}_0$ is fixed for all the trials and the amplitude σ_{s0} may change across trials. Now consider the new formulation which decomposes the ERP component into its correlated subcomponents (we can assume that we are dealing with P300 that has two overlapped subcomponents as P3a and P3b):

$$\mathbf{X} = \sigma_1\mathbf{a}_1.\mathbf{s}_1 + \sigma_2\mathbf{a}_2.\mathbf{s}_2 + \sum_{i=1}^{N} \sigma_i \mathbf{b}_{0i}\mathbf{n}_{0i} \tag{3}$$

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where σ_1, $\mathbf{a}_1$ and $\mathbf{s}_1$ are the amplitude, scalp projection, and time course of the first subcomponent, and σ_2, $\mathbf{a}_2$ and $\mathbf{s}_2$ are the amplitude, scalp projection, and time course of the second subcomponent of the combined component described by σ_{s0}, $\mathbf{a}_0$, $\mathbf{s}_0$ in equation (2). Suppose that the first subcomponent is estimated and normalized as $\mathbf{y}_1$. If we multiply its transpose to both sides of equation (3) we have the following relation (the cross term $\mathbf{n}_{0i}.\mathbf{y}_1^T$ almost vanishes because it is assumed that the noise is uncorrelated with the ERP subcomponents):

$$\mathbf{X}.\mathbf{y}_1^T = \sigma_1 \mathbf{a}_1.\mathbf{s}_1.\mathbf{y}_1^T + \sigma_2 \mathbf{a}_2.\mathbf{s}_2.\mathbf{y}_1^T \tag{4}$$

If we assume that the normalized estimated subcomponent $\mathbf{y}_1$ is exactly the same as the first source, the first term will be equal to $\sigma_1 \mathbf{a}_1$ (the normalization operation makes $\mathbf{s}_1.\mathbf{y}_1^T$ equal to 1). The main concern is the second term in which $\mathbf{s}_2.\mathbf{y}_1^T$ becomes a scale factor because of temporal correlation between the subcomponents. If $\mathbf{s}_2.\mathbf{y}_1^T$ were zero (in the case of having no temporal correlation), $\mathbf{X}.\mathbf{y}_1^T$ could result in an estimation of $\mathbf{a}_1$. Therefore, multiplication of one of the estimated subcomponents can not give us an estimate of its corresponding scalp projection because of the non zero nature of the second term in equation (4) when the subcomponents are temporally correlated.

This procedure is used in [1] in order to have an initial estimation of scalp projection $\mathbf{a}$ of the ERP component which is considered to be uncorrelated with the noise. But we showed that the method is not capable of estimating the scalp projections of the ERP subcomponents when there are some other correlated subcomponents. Hence, we need to extend the method to work for estimation of ERP subcomponents. The proposed method here is based on a cost function which can suppress one of the subcomponents. It is assumed that the noise is uncorrelated with both subcomponents and therefore, a filter is designed for estimation of the ERP subcomponents. Now consider the following constrained problem:

$$min \quad ||\mathbf{w}^T \mathbf{X} - \mathbf{r}_1||_2^2 \quad subject\ to \quad \mathbf{w}^T \mathbf{a}_2 = 0 \tag{5}$$

Where both $\mathbf{w}$ and $\mathbf{a}_2$ are $D \times 1$ vectors, $\mathbf{r}_1$ is an $1 \times T$ vector, and D is the number of channels. Using Lagrange multipliers, the constrained problem can be converted to an unconstrained problem:

$$\mathbf{F} = ||\mathbf{w}^T \mathbf{X} - \mathbf{r}_1||_2^2 + \mathbf{w}^T \mathbf{a}_2 q \tag{6}$$

where q is the Lagrange multiplier. The gradient of $\mathbf{F}$ with respect to $\mathbf{w}^T$ is:

$$\begin{aligned} \frac{\partial F}{\partial \mathbf{w}^T} &= \frac{\partial}{\partial \mathbf{w}^T}\{\mathbf{r}_1\mathbf{r}_1^T - 2\mathbf{r}_1\mathbf{X}^T\mathbf{w} + \mathbf{w}^T\mathbf{X}\mathbf{X}^T\mathbf{w} + \mathbf{w}^T\mathbf{a}_2 q\} \\ &= -2\mathbf{r}_1\mathbf{X}^T + 2\mathbf{w}^T\mathbf{X}\mathbf{X}^T + q\mathbf{a}_2^T \end{aligned} \tag{7}$$

By setting the above equation to zero and solving it for $\mathbf{w}^T$, we get the following equation:

$$\mathbf{w}^T = 0.5(2\mathbf{r}_1\mathbf{X}^T - q\mathbf{a}_2^T)\mathbf{C}_x^{-1} \tag{8}$$

where $\mathbf{C}_x = \mathbf{X}\mathbf{X}^T$. By substitution of (8) into the constraint in (5), we obtain:

$$\mathbf{w}^T\mathbf{a}_2 = 0.5(2\mathbf{r}_1\mathbf{X}^T - q\mathbf{a}_2^T)\mathbf{C}_x^{-1}\mathbf{a}_2 = 0; \tag{9}$$

The scale q is obtained as:

$$q = \frac{2\mathbf{r}_1\mathbf{X}^T\mathbf{C}_x^{-1}\mathbf{a}_2}{\mathbf{a}_2^T\mathbf{C}_x^{-1}\mathbf{a}_2} \tag{10}$$

Now consider the main cost function in (5):

$$\mathbf{G} = ||\mathbf{w}^T\mathbf{X} - \mathbf{r}_1||_2^2 \tag{11}$$

If we set $\mathbf{G}$ to zero, the optimum solution given by $\mathbf{w}_{opt}^T$ to extract $\mathbf{r}_1$ is:

$$\mathbf{w}_{opt}^T = \mathbf{r}_1\mathbf{X}^T\mathbf{C}_x^{-1} \tag{12}$$

If we multiply $\mathbf{w}_{opt}^T$ to both sides of equation (3), it can be seen that in order to have an estimation of $\mathbf{s}_1$, by having an appropriate reference source for $\mathbf{s}_1$ that is $\mathbf{r}_1$, $\mathbf{w}_{opt}^T\mathbf{a}_1$ will become $1/\sigma_1$ and $\mathbf{w}_{opt}^T\mathbf{a}_2$ will become zero. Therefore, we can conclude that in the case of having the exact reference for $\mathbf{s}_1$, $\mathbf{w}_{opt}^T\mathbf{a}_1 = 1/\sigma_1$ and $\mathbf{w}_{opt}^T\mathbf{a}_2 = 0$. Substitute $\mathbf{w}_{opt}^T$ from equation (12) into $\mathbf{w}_{opt}^T\mathbf{a}_2 = 0$, the following equation is achieved:

$$\mathbf{r}_1\mathbf{X}^T\mathbf{C}_x^{-1}\mathbf{a}_2 = 0 \tag{13}$$

Equation (13) is equal to the numerator of equation (10) and the Lagrange multiplier will be equal to zero. This is expected because this makes the cost functions $\mathbf{F}$ and $\mathbf{G}$ equal to each other. In other words, when it is reasonable that $\mathbf{w}_{opt}^T\mathbf{a}_2 = 0$ there is no need to add an extra constraint $\mathbf{w}^T\mathbf{a}_2 = 0$ because as explained, this is hold implicitly. Now, consider the following cost function:

$$\widetilde{\mathbf{F}} = ||\mathbf{w}^T\mathbf{X} - \mathbf{r}_1||_2^2 + \mathbf{w}^T\widetilde{\mathbf{a}}_2\widetilde{q} \tag{14}$$

using equation (10) we have the following solution for the Lagrange multiplier:

$$\widetilde{q} = \frac{2\mathbf{r}_1\mathbf{X}^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2}{\widetilde{\mathbf{a}}_2^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2} \tag{15}$$

with the help of equation (8) we find $\mathbf{w}^T$ as follows:

$$\begin{aligned} \mathbf{w}^T &= \mathbf{r}_1\mathbf{X}^T\mathbf{C}_x^{-1} - \frac{\mathbf{r}_1\mathbf{X}^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2}{\widetilde{\mathbf{a}}_2^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2}\widetilde{\mathbf{a}}_2^T\mathbf{C}_x^{-1} \\ \mathbf{w} &= \mathbf{C}_x^{-1}\mathbf{X}\mathbf{r}_1^T - \frac{\mathbf{r}_1\mathbf{X}^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2}{\widetilde{\mathbf{a}}_2^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2}\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2 \end{aligned} \tag{16}$$

we multiply both sides of the above equation by $\mathbf{X}^T$ to obtain the following equation:

$$\mathbf{X}^T\mathbf{w} = \mathbf{X}^T\mathbf{C}_x^{-1}\mathbf{X}\mathbf{r}_1^T - \frac{\mathbf{r}_1\mathbf{X}^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2}{\widetilde{\mathbf{a}}_2^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2}\mathbf{X}^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2 \tag{17}$$

In this paper, a suitable choice is made for $\widetilde{\mathbf{a}}_1$ and $\widetilde{\mathbf{a}}_2$ as:

$$\begin{aligned} \widetilde{\mathbf{a}}_1 &= \mathbf{X}\mathbf{r}_1^T \\ \widetilde{\mathbf{a}}_2 &= \mathbf{X}\mathbf{r}_2^T \end{aligned} \tag{18}$$

Therefore, it is simple to derive the following equations:

$$\begin{aligned} \widetilde{\mathbf{a}}_1 &= \sigma_1\mathbf{a}_1\mathbf{s}_1\mathbf{r}_1^T + \sigma_2\mathbf{a}_2\mathbf{s}_2\mathbf{r}_1^T \\ \widetilde{\mathbf{a}}_2 &= \sigma_1\mathbf{a}_1\mathbf{s}_1\mathbf{r}_2^T + \sigma_2\mathbf{a}_2\mathbf{s}_2\mathbf{r}_2^T \\ \widetilde{\mathbf{a}}_1 &= \sigma_1\mathbf{a}_1\mathbf{s}_1\mathbf{r}_1^T + \frac{(\widetilde{\mathbf{a}}_2 - \sigma_1\mathbf{a}_1\mathbf{s}_1\mathbf{r}_2^T)}{\mathbf{s}_2\mathbf{r}_2^T}\mathbf{s}_2\mathbf{r}_1^T \\ \widetilde{\mathbf{a}}_1 &= \sigma_1\mathbf{a}_1\mathbf{s}_1\mathbf{r}_1^T + \widetilde{\mathbf{a}}_2\frac{\mathbf{s}_2\mathbf{r}_1^T}{\mathbf{s}_2\mathbf{r}_2^T} - \sigma_1\mathbf{a}_1\frac{\mathbf{s}_1\mathbf{r}_2^T(\mathbf{s}_2\mathbf{r}_1^T)}{\mathbf{s}_2\mathbf{r}_2^T} \end{aligned} \tag{19}$$

Using equation (17), the following equations can be derived:

$$\begin{aligned} \mathbf{X}^T\mathbf{w} &= \mathbf{X}^T\mathbf{C}_x^{-1}\mathbf{X}\mathbf{r}_1^T - \frac{\widetilde{\mathbf{a}}_1^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2}{\widetilde{\mathbf{a}}_2^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2}\mathbf{X}^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2 \\ &= \mathbf{X}^T\mathbf{C}_x^{-1}\mathbf{X}\mathbf{r}_1^T \\ &\quad - \frac{[\sigma_1\mathbf{s}_1\mathbf{r}_1^T\mathbf{a}_1^T + \frac{\mathbf{s}_2\mathbf{r}_1^T}{\mathbf{s}_2\mathbf{r}_2^T}\widetilde{\mathbf{a}}_2^T - \sigma_1\frac{\mathbf{s}_1\mathbf{r}_2^T(\mathbf{s}_2\mathbf{r}_1^T)}{\mathbf{s}_2\mathbf{r}_2^T}\mathbf{a}_1^T]\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2}{\widetilde{\mathbf{a}}_2^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2}\mathbf{X}^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2 \end{aligned} \tag{20}$$

Now we want to show that $\mathbf{a}_1^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2 = 0$. We use equation (13) with swapped indices as:

$$\begin{aligned} \mathbf{r}_2\mathbf{X}^T\mathbf{C}_x^{-1}\mathbf{a}_1 &= 0 \\ \widetilde{\mathbf{a}}_2^T\mathbf{C}_x^{-1}\mathbf{a}_1 &= 0 \\ \mathbf{a}_1^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2 &= 0 \end{aligned} \tag{21}$$

Therefore, equation (20) can be simplified to:

$$\begin{aligned} \mathbf{X}^T\mathbf{w} &= \mathbf{X}^T\mathbf{C}_x^{-1}\mathbf{X}\mathbf{r}_1^T - \frac{[\frac{\mathbf{s}_2\mathbf{r}_1^T}{\mathbf{s}_2\mathbf{r}_2^T}\widetilde{\mathbf{a}}_2^T]\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2}{\widetilde{\mathbf{a}}_2^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2}\mathbf{X}^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2 \\ &= \mathbf{X}^T\mathbf{C}_x^{-1}\mathbf{X}\mathbf{r}_1^T - [\frac{\mathbf{s}_2\mathbf{r}_1^T}{\mathbf{s}_2\mathbf{r}_2^T}]\mathbf{X}^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2 \\ &= \mathbf{X}^T\mathbf{C}_x^{-1}[\sigma_1\mathbf{a}_1\mathbf{s}_1\mathbf{r}_1^T + \sigma_2\mathbf{a}_2\mathbf{s}_2\mathbf{r}_1^T] \\ &\quad - \mathbf{X}^T\mathbf{C}_x^{-1}[\frac{\mathbf{s}_2\mathbf{r}_1^T}{\mathbf{s}_2\mathbf{r}_2^T}][\sigma_1\mathbf{a}_1\mathbf{s}_1\mathbf{r}_2^T + \sigma_2\mathbf{a}_2\mathbf{s}_2\mathbf{r}_2^T] \\ &= \mathbf{X}^T\mathbf{C}_x^{-1}[\sigma_1\mathbf{a}_1\mathbf{s}_1\mathbf{r}_1^T + \sigma_2\mathbf{a}_2\mathbf{s}_2\mathbf{r}_1^T] \\ &\quad - \mathbf{X}^T\mathbf{C}_x^{-1}[\frac{\mathbf{s}_2\mathbf{r}_1^T}{\mathbf{s}_2\mathbf{r}_2^T}\sigma_1\mathbf{a}_1\mathbf{s}_1\mathbf{r}_2^T + \sigma_2\mathbf{a}_2\mathbf{s}_2\mathbf{r}_1^T] \end{aligned} \tag{22}$$

$\mathbf{X}^T\mathbf{w}$ can be simplified more into:

$$\begin{aligned} \mathbf{X}^T\mathbf{w} &= \mathbf{X}^T\mathbf{C}_x^{-1}[\sigma_1\mathbf{a}_1\mathbf{s}_1\mathbf{r}_1^T] - \mathbf{X}^T\mathbf{C}_x^{-1}[\frac{\mathbf{s}_2\mathbf{r}_1^T}{\mathbf{s}_2\mathbf{r}_2^T}\sigma_1\mathbf{a}_1\mathbf{s}_1\mathbf{r}_2^T] \\ &= \mathbf{X}^T\mathbf{C}_x^{-1}\sigma_1\mathbf{a}_1[\mathbf{s}_1\mathbf{r}_1^T - \frac{\mathbf{s}_2\mathbf{r}_1^T}{\mathbf{s}_2\mathbf{r}_2^T}(\mathbf{s}_1\mathbf{r}_2^T)] \end{aligned} \tag{23}$$

Then, we can derive the following equation:

$$\mathbf{X}\mathbf{X}^T\mathbf{w} = \mathbf{X}\mathbf{X}^T\mathbf{C}_x^{-1}\sigma_1\mathbf{a}_1[\mathbf{s}_1\mathbf{r}_1^T - \frac{\mathbf{s}_2\mathbf{r}_1^T}{\mathbf{s}_2\mathbf{r}_2^T}(\mathbf{s}_1\mathbf{r}_2^T)] \tag{24}$$

Considering $\mathbf{C}_x = \mathbf{X}\mathbf{X}^T$ and $\mathbf{X}\mathbf{X}^T\mathbf{C}_x^{-1} = I$, the following equation is derived:

$$\mathbf{X}\mathbf{X}^T\mathbf{w} = \sigma_1\mathbf{a}_1[\mathbf{s}_1\mathbf{r}_1^T - \frac{\mathbf{s}_2\mathbf{r}_1^T}{\mathbf{s}_2\mathbf{r}_2^T}(\mathbf{s}_1\mathbf{r}_2^T)] \tag{25}$$

If we normalize $\mathbf{X}\mathbf{X}^T\mathbf{w}$, $\mathbf{a}_1$ will be obtained. Therefore, following (18) if we use $\widetilde{\mathbf{a}}_2 = \mathbf{X}\mathbf{r}_2^T$ and solve equation (14) and multiply the resulted $\mathbf{w}$ by $\mathbf{X}\mathbf{X}^T$, we can have an estimate of $\mathbf{a}_1$ which is the scalp projection of the first subcomponent whose corresponding temporal reference signal is given by $\mathbf{r}_1$. Therefore, solving equation (14) is effective and useful in estimation of the scalp projection of one of the subcomponents which is the main contribution of this paper. One significant achievement in this paper which has also been confirmed by the simulated results, is that when there is a mismatch between the reference signal and the actual source the normalized vector of $\mathbf{X}\mathbf{X}^T\mathbf{w}$ does not change or because of different noise level it changes very slightly. In other words, the estimation of scalp projection of one of the ERP subcomponents is independent of the choice of reference signals. In the case of having a mismatch between the actual source and the reference signal, still we can have a very good approximation of scalp projections of the subcomponents. This is because the mismatch results in the change of the scale ($[\mathbf{s}_1\mathbf{r}_1^T - \frac{\mathbf{s}_2\mathbf{r}_1^T}{\mathbf{s}_2\mathbf{r}_2^T}(\mathbf{s}_1\mathbf{r}_2^T)]$) in equation (25) in the estimation. After normalization however, this has no effect.

Estimation of scalp projection of the correlated subcomponents is very useful for localization of the ERP subcomponent in the brain. However, in order to have the temporal estimation for each subcomponent, we need to solve equation (11) and find $\mathbf{w}_{opt}^T$ given in (12) and then multiply it by $\mathbf{X}$. Therefore, having a reference signal for each subcomponent, $\mathbf{w}_{opt}^T\mathbf{X}$ can be the temporal estimation of that subcomponent.

The pseudocode for the new spatiotemporal filtering methodology is provided in Algorithm 1. ERP subcomponents are modeled using parametric functions in many studies and among them Gaussian waveform is very common [9][10]. Although real ERP subcomponents may not look exactly Gaussian the modeling results in a robust and fast estimation of the peak parameters (latency and amplitude) that neurophysiologists and cognitive scientists are primary concerned. Like the method in [1] we used Gamma wave as an approximation to ERP subcomponents because any desired shapes can be easily obtained by tuning its parameters. Based on the given algorithm, first, we generate two references $(\mathbf{r}_1, \mathbf{r}_2)$ representing the first and second subcomponents using Gamma functions expressed as:

$$\mathbf{r}(t) = \mathbf{c}t^{k-1}exp(-\frac{t}{\theta}) \tag{26}$$

Where $k > 0$ is a shape parameter, $\theta > 0$ is a scale parameter, and c is a normalizing constant. After generating Gamma wave as a reference signal for each ERP subcomponent, we slide each wave in a range of valid latencies or order to generate more reference signals for both subcomponents. The peak latency of the reference is denoted as τ. After generating the references, the value of $\widetilde{\mathbf{a}}_1$ and $\widetilde{\mathbf{a}}_2$ can be obtained using equation (18). Then, it is possible to solve the constrained problem given in (14) considering each reference and $\widetilde{\mathbf{a}}_i(i{=}1,2)$ that is obtained by considering the reference for another subcomponent. Next, we estimate $\mathbf{w}_1(\tau_1)$ and $\mathbf{w}_2(\tau_2)$ using equation (16) as shown in Algorithm 1. Then, considering three cost functions ($\mathbf{J}_1(\tau_1)$, $\mathbf{J}_2(\tau_2)$, and $\widetilde{\mathbf{J}}(\tau_1,\tau_2)$), the reference signals, for which the sum of the three cost functions is minimum, are selected. Then we are able to estimate $\mathbf{a}_1$, $\mathbf{a}_2$, σ_1, and σ_2. These estimates are shown in Algorithm 1. Practically, it is expected to get better results in peak latency estimation if we use $\widetilde{\mathbf{J}}(\tau_1,\tau_2)$ cost function. This is not surprising since for temporal estimation of the subcomponents it is reasonable to use $\mathbf{w}_{opt}$ given in equation (12) and for the estimation of scalp projections and the amplitude it is better to use the constrained problem given in (14). Therefore, for peak latency estimation we use $\mathbf{w}_{opt}$ resulted when using the reference signals for subcomponents and considering $\widetilde{\mathbf{J}}(\tau_1,\tau_2)$ cost function.

3. RESULTS

In this section, the results of applying the proposed method to both simulated and real data are provided. The goal of the simulation study is to evaluate the ability of the method in peak latency, amplitude, and scalp projection estimation in different SNR levels. The method is then applied to real data.

3.1 Simulation Study

In order to evaluate the method and quantify the estimation error for ERP subcomponent descriptor (latency, amplitude) two Gamma waves are generated as ERP subcomponents. Therefore, the Gamma waves are used as approximations to ERP subcomponents because the desired shapes can be easily achieved by adjusting their parameters. For simplicity of writing, we call the first subcomponent P3a and the second subcomponent P3b. The simulated subcomponents are shown in Fig. 1 as the synthetic P3a and P3b subcomponents. Two scalp projections which are somehow orthogonal are generated for P3a and P3b. Then, using equation (3), a 20 channel dataset is generated. Each channel includes 40 trials. In all of the trials the latency of P3a is fixed at 150 ms and the latency of P3b is fixed at 200 ms. Therefore, the subcomponents are relatively highly correlated in time domain. The amplitudes of P3a and P3b changes in different trials. The variance of the noise is fixed at all the trials. However the noise changes slightly from trial to trial.

Algorithm 1 New spatiotemporal filtering method

-Generate $\mathbf{r}_1(\tau_1)$,$\mathbf{r}_2(\tau_2)$ *considering reasonable ranges of peak latencies*$[\tau_1 \in \mathbf{T}_{s1}, \tau_2 \in \mathbf{T}_{s2}]$ *for the first and second subcomponents using Gamma waves*

-Set $\widetilde{\mathbf{a}}_1(\tau_1) = \mathbf{X}\mathbf{r}_1(\tau_1)^T$ and $\widetilde{\mathbf{a}}_2(\tau_2) = \mathbf{X}\mathbf{r}_2(\tau_2)^T$

-Find $\mathbf{w}_1(\tau_1) = \mathbf{C}_x^{-1}\mathbf{X}\mathbf{r}_1(\tau_1)^T - \frac{\mathbf{r}_1(\tau_1)\mathbf{X}^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2(\tau_2)}{\widetilde{\mathbf{a}}_2^T(\tau_2)\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2(\tau_2)}\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_2(\tau_2)$

$\mathbf{w}_2(\tau_2) = \mathbf{C}_x^{-1}\mathbf{X}\mathbf{r}_2(\tau_2)^T - \frac{\mathbf{r}_2(\tau_2)\mathbf{X}^T\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_1(\tau_1)}{\widetilde{\mathbf{a}}_1^T(\tau_1)\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_1(\tau_1)}\mathbf{C}_x^{-1}\widetilde{\mathbf{a}}_1(\tau_1)$

-Set $\mathbf{J_1}(\boldsymbol{\tau_1}) = ||\mathbf{w}_1^T(\tau_1)\mathbf{X} - \mathbf{r_1}(\tau_1)||_2^2 + \mathbf{w}_1(\tau_1)^T\widetilde{\mathbf{a}}_2(\tau_2)\widetilde{q}$

$\mathbf{J_2}(\boldsymbol{\tau_2}) = ||\mathbf{w}_2^T(\tau_2)\mathbf{X} - \mathbf{r}_2(\tau_2)||_2^2 + \mathbf{w}_2(\tau_2)^T\widetilde{\mathbf{a}}_1(\tau_1)\widehat{q}$

$\widetilde{\mathbf{J}}(\boldsymbol{\tau_1}, \boldsymbol{\tau_2}) = ||(\mathbf{r}(\tau_1) + \mathbf{r}(\tau_2))[\mathbf{X}^T\mathbf{C}_x^{-1}\mathbf{X} - I]||_2^2 +$
$||\mathbf{r}(\tau_1)[\mathbf{X}^T\mathbf{C}_x^{-1}\mathbf{X} - I]||_2^2 + ||\mathbf{r}(\tau_2)[\mathbf{X}^T\mathbf{C}_x^{-1}\mathbf{X} - I]||_2^2$

$[\mathbf{l}_1, \mathbf{l}_2] = argmin_{\tau_1,\tau_2}(\mathbf{J_1}(\boldsymbol{\tau_1}) + \mathbf{J_2}(\boldsymbol{\tau_2}) + \widetilde{\mathbf{J}}(\boldsymbol{\tau_1}, \boldsymbol{\tau_2}))$

-Estimate $\mathbf{a}_1 = \frac{\mathbf{X}\mathbf{X}^T\mathbf{w}_1(\mathbf{l}_1)}{norm(\mathbf{X}\mathbf{X}^T\mathbf{w}_1(\mathbf{l}_1))}$

$\mathbf{a}_2 = \frac{\mathbf{X}\mathbf{X}^T\mathbf{w}_2(\mathbf{l}_2)}{norm(\mathbf{X}\mathbf{X}^T\mathbf{w}_2(\mathbf{l}_2))}$

$\sigma_1 = 1/[(\mathbf{r}_1(\mathbf{l}_1)\mathbf{X}^T\mathbf{C}_x^{-1})(\mathbf{a}_1)]$

$\sigma_2 = 1/[(\mathbf{r}_2(\mathbf{l}_2)\mathbf{X}^T\mathbf{C}_x^{-1})(\mathbf{a}_2)]$

-Find the peak latencies as $[\widetilde{\tau}_1, \widetilde{\tau}_2] = argmin_{\tau_1,\tau_2}\widetilde{\mathbf{J}}(\boldsymbol{\tau_1}, \boldsymbol{\tau_2})$

We have applied the method to the simulated data considering the reference signal as the actual synthetic data and the results are shown in Table 1. The first column corresponds to the average value of SNR for all 40 trials. The second and fifth columns are the mean and variance of the estimated latencies for P3a and P3b respectively. The mean and variance of the ratio between the actual and estimated amplitudes are available in the third and sixth columns. This value should be close to 1. The correlations between the estimated scalp projections and actual scalp projections are available in forth and seventh columns. The variance of the noise is changed in order to generate different levels of SNR and the algorithm is run for the new SNR level. The available noise power is measured by SNR in dB which is defined as:

$$\mathbf{SNR} = 10log(\frac{P_{signal}}{P_{noise}}) \tag{27}$$

Therefore, in the case of exact match, when the reference is the actual synthetic signal, with an increase in SNR the variances of the latency and amplitude estimations decrease and the means of the latency and amplitude tend toward the actual values. The correlation coefficients of the actual and estimated scalp projections increase and reach almost to 1. Also, we used two references for P3a and P3b which were not the exact synthetic signals. These presumed waves for P3a and P3b are shown in Fig. 1. The results of estimated latency, amplitude, and scalp projections are shown in Table 2. When the SNR increases, the variances of estimations for latency and amplitude decrease. But there is some error in latency estimation which is about 20 ms. This can not be significant in some applications. It is possible to enhance the method in order to achieve better results when there is a mismatch between the actual and presumed subcomponents. However, still we have good estimations for the scalp projections of both subcomponents which are significant and favorable.

3.2 Real Data

The EEG data were recorded using a Nihon Kohden model EEG-F/G amplifier and Neuroscan Acquire 4.0 software. EEG activity was recorded following the international 10 – 20 electrode setting system from 15 electrodes. The reference electrodes were linked to the earlobes. The impedance for all the electrodes was below $5k\Omega$, the sampling frequency Fs = 2 kHz, and the data were subsequently bandpass filtered (0.1 – 70 Hz). Subjects were required to sit alert and still with their eyes closed to avoid any interference. The stimuli were presented through ear plugs inserted in the ear. Forty rare tones (1 kHz) were randomly distributed amongst 160 frequent tones (2 kHz).

Table 1. Estimated latencies, amplitudes and scalp projections for simulated P3a and P3b in the case of exact match

SNR(dB)	P3a latency	P3a amplitude	P3a scalp projection	P3b latency	P3b amplitude	P3b scalp projection
-4.7985	153.8000 ± 2.0406	1.6039 ± 0.1285	0.9701	206.0500 ± 4.5907	1.4127 ± 0.0837	0.9792
-3.9647	154.3500 ± 2.0450	1.5071 ± 0.1219	0.9751	204.5250 ± 3.9222	1.3517 ± 0.1003	0.9818
-3.0178	153.5000 ± 1.7687	1.4662 ± 0.1022	0.9791	202.8500 ± 4.7907	1.2890 ± 0.0809	0.9853
-2.0325	153.6000 ± 1.5326	1.4165 ± 0.0947	0.9819	203 ± 4.3853	1.2306 ± 0.0576	0.9888
-0.8349	152.8750 ± 1.3623	1.3680 ± 0.0839	0.9858	201.4250 ± 4.4600	1.1866 ± 0.0556	0.9910
0.4663	152.6250 ± 1.1916	1.3294 ± 0.0751	0.9881	201.9750 ± 3.8263	1.1409 ± 0.0480	0.9936
2.0559	152.1000 ± 1.0077	1.2914 ± 0.0660	0.9906	201.4000 ± 2.8266	1.0993 ± 0.0603	0.9953
2.9637	151.6500 ± 0.6998	1.2636 ± 0.0700	0.9920	199.9250 ± 3.5833	1.0789 ± 0.0491	0.9964
3.9945	151.3250 ± 0.6558	1.2680 ± 0.0543	0.9933	200.3250 ± 2.7492	1.0629 ± 0.0363	0.9967
5.1388	151.1250 ± 0.6864	1.2413 ± 0.0484	0.9941	200.1000 ± 2.8627	1.0523 ± 0.0432	0.9978
6.4869	150.8500 ± 0.6222	1.2313 ± 0.0490	0.9945	200.9500 ± 2.5815	1.0295 ± 0.0319	0.9983
8.1083	150.5250 ± 0.5541	1.2131 ± 0.0441	0.9955	200.2750 ± 2.3964	1.0102 ± 0.0201	0.9988
10.0147	150.5000 ± 0.5064	1.2019 ± 0.0336	0.9955	200.4500 ± 1.5013	0.9940 ± 0.0275	0.9992
12.5061	150.0750 ± 0.2667	1.1768 ± 0.0345	0.9962	199.7250 ± 1.3202	0.9691 ± 0.0183	0.9995
16.0293	150.0250 ± 0.1581	1.1705 ± 0.0295	0.9964	200.2500 ± 0.8697	0.9537 ± 0.0121	0.9997

Table 2. Estimated latencies, amplitudes and scalp projections for simulated P3a and P3b in the case of Mismatch

SNR(dB)	P3a latency	P3a amplitude	P3a scalp projection	P3b latency	P3b amplitude	P3b scalp projection
-3.9329	166.7250 ± 5.5423	1.6461 ± 0.1211	0.9716	182.1500 ± 12.0268	1.3083 ± 0.0960	0.9809
-3.0555	167.6750 ± 4.9634	1.5950 ± 0.1095	0.9740	182.9000 ± 14.0307	1.2197 ± 0.0722	0.9852
-2.0338	167.3000 ± 4.3276	1.5141 ± 0.1036	0.9794	180.4500 ± 12.0829	1.1469 ± 0.0921	0.9883
-0.8666	167.4500 ± 4.2242	1.4607 ± 0.0858	0.9813	181.6750 ± 13.6107	1.0856 ± 0.0795	0.9903
0.5041	164.8750 ± 2.9368	1.4239 ± 0.0773	0.9852	183.1500 ± 15.3749	1.0325 ± 0.0430	0.9927
2.0499	164.1500 ± 2.2481	1.4002 ± 0.0670	0.9866	189.4750 ± 18.9804	0.9768 ± 0.0454	0.9952
2.9711	163.7000 ± 1.5722	1.3712 ± 0.0627	0.9875	192.8750 ± 20.0617	0.9541 ± 0.0530	0.9958
3.9868	163.0250 ± 1.3105	1.3432 ± 0.0619	0.9890	199.3000 ± 20.1357	0.9237 ± 0.0420	0.9968
5.1544	163.3000 ± 1.2445	1.3433 ± 0.0508	0.9906	207.4500 ± 17.7460	0.9074 ± 0.0286	0.9974
6.4777	164.3750 ± 1.1477	1.3173 ± 0.0512	0.9910	216.7500 ± 6.8827	0.8900 ± 0.0329	0.9982
7.9829	165.5250 ± 0.6789	1.3138 ± 0.0396	0.9910	219.2000 ± 0.8829	0.8725 ± 0.0261	0.9985
10.0087	166.5250 ± 0.6400	1.3013 ± 0.0423	0.9914	220.3500 ± 0.8022	0.8481 ± 0.0218	0.9990

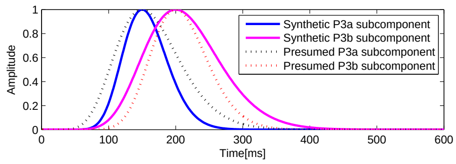
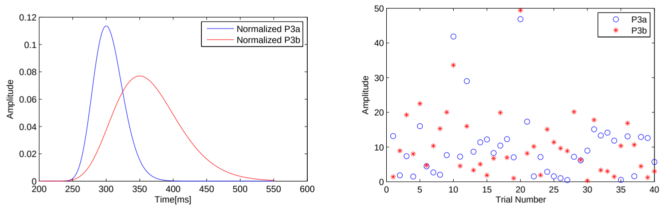
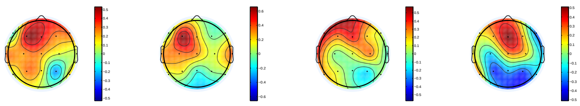


Fig. 1. Synthetic and presumed reference signals for P3a and P3b used in the simulation study.



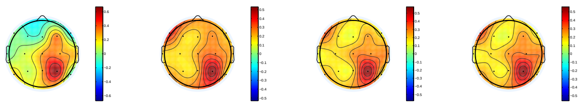
(a) (b)

Fig. 2. (a) Reference signals for P3a and P3b, (b) Estimated amplitudes for P3a and P3b in different trials.



(a) (b) (c) (d)

Fig. 3. Scalp projections of P3a in four selected progressive trials.



(a) (b) (c) (d)

Fig. 4. Scalp projections of P3b in four selected progressive trials.

The subjects were asked to press a button as soon as they heard a low tone (1 kHz). The task was designed to assess basic memory processes. ERP components measured in this task included N100, P200, N200, and P3a and P3b. It is selected 40 trials of the data and the presented method for estimation of latency, amplitude and scalp projection of P3a and P3b is applied. Based on the averaged ERP of 40 trials and also having some prior knowledge about the data, we have selected two reference signal for P3a and P3b which are shown in Fig. 2(a) . The amplitude variations of the P3a and P3b are shown in Fig. 2(b). The mean latency of P3a was 283.3ms and the mean latency of P3b was obtained as 367.6ms. The estimated scalp projections of P3a and P3b in four selected but progressive trials are shown respectively in Fig. 3 and Fig. 4. The P3a has a more fronto-central distribution as expected and P3b has more posterior distribution. The scalp projections are plotted using EEGlab [11].

4. DISCUSSION AND CONCLUSIONS

In this paper we proposed a new method for single trial estimation of the ERP subcomponents. The method defines a new cost function in which the scalp projection of each subcomponent can be estimated. The proposed method in this paper overcomes the problem of existence of temporal correlation between the subcomponents or even between components in the recently developed spatiotemporal filtering method. Simulated results show that the method is robust in estimation of latency, amplitude, and scalp projections of ERP subcomponents when there is a correlation between subcomponents. The scalp projections of both subcomponents can be estimated with high accuracy even in the case that there is a mismatch between the reference signal and the actual source. This is very important when we are dealing with localization of ERP subcomponents in the brain. Since the scalp projections can be effectively analyzed and used in order to give us the exact location of the source in the brain. Also It has been shown that the method is effective for single trial estimation of P300 subcomponents. This is very useful for some applications such as mental fatigue where the variability of P300 subcomponent descriptors can determine the fatigue state.

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