SENSITIVIY OF SAIC AND MAIC CONCEPTS TO RESIDUAL FREQUENCY OFFSETS

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ABSTRACT

This paper presents a performance analysis of the single antenna interference cancellation (SAIC) and the multiple antenna interference cancellation (MAIC) techniques in the presence of non null frequency offsets of both the signal of interest (SOI) and interference, that are neither corrected nor compensated, in terms of signal to interference plus noise ratio (SINR) and bit error rate (BER). General theoretical expressions of the SINR and BER are given for BPSK SOI and interference. To obtain engineering insight, the particular case of the SAIC with no return to zero (NRZ) pulse shape filters is considered, where simple expressions are given and analyzed. It is proved in particular that the performance more deteriorates for non null frequency offset of the interference than for non null frequency offset of the SOI. Finally illustrative examples are presented in order to specify the validity domain of our approximations and to quantify the obtained results in the context of the global system for mobile communication (GSM) standard.

1. INTRODUCTION

For more than a decade, there has been an increasing interest in optimal widely linear (WL) processing [1] in radiocommunication contexts involving rectilinear signals, such as binary phase shift keying (BPSK) signals or quasi rectilinear signals such as continuous phase modulation (CPM) with modulation index $1/2$, or such as offset quadrature amplitude modulation (OQAM).

In particular, it has been pointing out in [4] that SAIC may be performed by WL filters in the context of rectilinear SOI and interference. Then it has been shown in [3], that the Gaussian minimum shift keying (GMSK) modulation whose linearized approximation was introduced in [2], may be interpreted as a BPSK modulation after a simple algebraic operation of derotation, displaying the great interest of optimal WL filtering for cochannel interference mitigation in the GSM network.

Concerning the performance of these SAIC and MAIC techniques, only a few contributions have appeared in the literature. Among them, [3] have given some bounds on the maximum likelihood sequence estimation (MLSE) for cochannel interference cancellation within the current GSM standard and [4] have presented some enlightening results about the behavior, properties and performance of the SAIC and MAIC techniques for the reception of a BPSK, MSK or GMSK SOI corrupted by interference of the same kind. In this latter case, no performance analysis has been given concerning the loss in performance in terms of SINR and BER in the presence of residual frequency offsets of the SOI and interference that are practically unavoidable. Therefore, it is of paramount importance to specify how these residual frequency offsets degrade the performance of the SAIC and MAIC receivers.

The purpose of this paper is to quantify this sensitivity. The paper is organized as follows. After the introduction of the observation model and data statistics given in Section 2, Section 3 reviews the optimal BPSK SAIC and MAIC receivers. A performance analysis of these receivers in the presence of residual frequency offsets of the BPSK SOI and interference is presented in Section 4 with a particular attention paid to SAIC with NRZ pulse shape filter. Finally, illustrative examples are given in order to specify the validity domain of our approximations and to quantify the obtained results in the context of the GSM standard in Section 5. Section 6 contains the conclusion.

2. HYPOTHESES AND DATA STATISTICS

Let us consider an array of N narrow-band sensors. Each sensor is assumed to receive a BPSK SOI corrupted by a noncircular total noise composed of a BPSK interference and background noise. Note that in cellular radiocommunication networks, such an interference may be generated by the network itself (e.g., from signals coming from neighboring cells using the same carrier frequency). Hence, this interference have the same waveform and modulation as the SOI. The complex envelopes of the SOI and the interference are given respectively by

$$s(t) = \sum_n a_n \upsilon(t - nT),$$

and

$$j(t) = \sum_n b_n \upsilon(t - nT - t_j),$$

where $a_n = \pm 1$ and $b_n = \pm 1$ are independent sequences of independent equiprobable symbols, T is the symbol duration, $t_j \in [0, T)$ is the time origin of the interference assuming an optimal sampling time for the SOI at $t = kT, k \in \mathbb{Z}$ and $\upsilon(t)$ is a square root raised cosine Nyquist filter. The vector $\mathbf{x}(t)$ of complex amplitudes of the signals at the output of these sensors is given by

$$\mathbf{x}(t) = \mu_s s(t) e^{i2\pi\Delta f_s t} \mathbf{h}_s + \mu_j j(t) e^{i2\pi\Delta f_j t} \mathbf{h}_j + \mathbf{n}(t), \quad (1)$$

where $\mathbf{h}_s$ and $\mathbf{h}_j$ are the channel impulse response vectors of the SOI and interference respectively. Note that model (1) assumes propagation channel with no delay spread, which

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occurs for example, for flat fading channel or ideal propagation. $\mathbf{h}_s = e^{i\phi_s}\mathbf{s}$ and $\mathbf{h}_j = e^{i\phi_j}\mathbf{j}$, where ϕ_s and ϕ_j, $\mathbf{s}$ and $\mathbf{j}$ correspond to the phases and the steering vectors (such that their first components is one) of the SOI and interference, respectively. Δf_s and Δf_j, are the frequency offsets of the SOI and interference respectively. μ_s and μ_j control the power of the SOI and interference. The background noise $\mathbf{n}(t)$ is assumed independent of the SOI and interference, zero-mean Gaussian circular, temporally and spatially white.

The sampled observation vector $\mathbf{x}_\upsilon(kT) \stackrel{\text{def}}{=} \mathbf{x}(t) \otimes \upsilon(-t)^*/_{t=kT}$ (where $\otimes$ is the convolution operation) obtained after a matched filtering operation to the pulse shape filter $\upsilon(t)$ and a decimation operation at the symbol rate, is given by

$$\begin{aligned}\mathbf{x}_\upsilon(kT) &= \mu_s a_k e^{i2\pi\Delta f_s kT} I_0 \mathbf{h}_s + \mu_s \sum_{n\neq k} a_n e^{i2\pi\Delta f_s kT} I_{k-n}\mathbf{h}_s \\ &+ \mu_j \sum_n b_n e^{i2\pi\Delta f_j kT} J_{k-n}(t_j)\mathbf{h}_j + \mathbf{n}_\upsilon(kT), \quad (2)\end{aligned}$$

where

$$\begin{aligned}I_n &\stackrel{\text{def}}{=} \int \upsilon^*(-\tau)\upsilon(nT-\tau)e^{-i2\pi\Delta f_s \tau}d\tau \\ J_n(t_j) &\stackrel{\text{def}}{=} \int \upsilon^*(-\tau)\upsilon(nT-\tau-t_j)e^{-i2\pi\Delta f_j \tau}d\tau. \quad (3)\end{aligned}$$

The extended model is given by $\widetilde{\mathbf{x}}_\upsilon(kT) = [\mathbf{x}_\upsilon(kT)^T, \mathbf{x}_\upsilon(kT)^H]^T$ with

$$\begin{aligned}\widetilde{\mathbf{x}}_\upsilon(kT) &= \mu_s a_k \widetilde{\mathbf{h}}_s(k,k) + \mu_s \sum_{n\neq k} a_n \widetilde{\mathbf{h}}_s(k,n) \\ &+ \mu_j \sum_n b_n \widetilde{\mathbf{h}}_j(k,n) + \widetilde{\mathbf{n}}_\upsilon(kT), \quad (4)\end{aligned}$$

where

$$\begin{aligned}\widetilde{\mathbf{h}}_s(k,n) &\stackrel{\text{def}}{=} [e^{i2\pi\Delta f_s nT} I_{k-n}\mathbf{h}_s^T, e^{-i2\pi\Delta f_s nT} I_{k-n}^*\mathbf{h}_s^H]^T, \\ \widetilde{\mathbf{h}}_j(k,n) &\stackrel{\text{def}}{=} [e^{i2\pi\Delta f_j (nT+t_j)} J_{k-n}(t_j)\mathbf{h}_j^T, \\ &\qquad e^{-i2\pi\Delta f_j (nT+t_j)} J_{k-n}^*(t_j)\mathbf{h}_j^H]^T, \\ \widetilde{\mathbf{n}}_\upsilon(kT) &\stackrel{\text{def}}{=} [\mathbf{n}_\upsilon(kT)^T, \mathbf{n}_\upsilon(kT)^H]^T.\end{aligned}$$

The second order statistics of the data considered in this paper are defined by $\mathbf{R}_{\widetilde{\mathbf{x}}}(k) \stackrel{\text{def}}{=} \mathrm{E}[\widetilde{\mathbf{x}}_\upsilon(kT)\widetilde{\mathbf{x}}_\upsilon^H(kT)]$ and we assume that $\mathrm{E}[\widetilde{\mathbf{n}}_\upsilon(kT)\widetilde{\mathbf{n}}_\upsilon(kT)^H] = \eta_2\mathbf{I}$ where η_2 is the mean power of the background noise per sensor and $\mathbf{I}$ is the $2N\times 2N$ identity matrix.

3. SAIC AND MAIC RECEIVERS

3.1 Optimal receiver

Under the assumption of equiprobable SOI symbol sequences, without any residual offset and stationary noncircular Gaussian distributed total noise of extended covariance matrix denoted specifically here by $\mathbf{R}_{\widetilde{\mathbf{n}}}$, the MLSE receiver which minimizes the output sequence error rate is given [4] by a WL filtering of $\mathbf{x}_\upsilon(kT)$. This one is the so-called WL spatial matched filter $\widetilde{\mathbf{w}}_{\text{MLSE}}$ defined by

$$\widetilde{\mathbf{w}}_{\text{MLSE}} \stackrel{\text{def}}{=} \mathbf{R}_{\widetilde{\mathbf{n}}}^{-1}\widetilde{\mathbf{h}}_s, \quad (5)$$

with $\widetilde{\mathbf{h}}_s \stackrel{\text{def}}{=} [\mathbf{h}_s^T, \mathbf{h}_s^H]^T$, whose output $y(kT)$ is real-valued, followed by a zero threshold detector. This filter is proportional to the filter $\widetilde{\mathbf{w}}_{\text{MMSE}}$ which minimizes the mean square error (MSE) between the output $\widetilde{\mathbf{w}}^H\widetilde{\mathbf{x}}_\upsilon(kT)$ of the WL filter $\widetilde{\mathbf{w}} = [\mathbf{w}^T, \mathbf{w}^H]^T$ and a_k and is given by

$$\begin{aligned}\widetilde{\mathbf{w}}_{\text{MMSE}} &\stackrel{\text{def}}{=} \mathbf{R}_{\widetilde{\mathbf{x}}}^{-1}\mathbf{r}_{\widetilde{\mathbf{x}},a} \quad (6) \\ &= [\mu_s/(1+\mu_s^2\widetilde{\mathbf{h}}_s^H\mathbf{R}_{\widetilde{\mathbf{n}}}^{-1}\widetilde{\mathbf{h}}_s]\mathbf{R}_{\widetilde{\mathbf{n}}}^{-1}\widetilde{\mathbf{h}}_s \stackrel{\text{def}}{=} \beta\widetilde{\mathbf{w}}_{\text{MLSE}},\end{aligned}$$

where $\mathbf{R}_{\widetilde{\mathbf{x}}} \stackrel{\text{def}}{=} \mathrm{E}[\widetilde{\mathbf{x}}_\upsilon(kT)\widetilde{\mathbf{x}}_\upsilon^H(kT)]$, $\mathbf{r}_{\widetilde{\mathbf{x}},a} \stackrel{\text{def}}{=} \mathrm{E}[\widetilde{\mathbf{x}}_\upsilon(kT)a_k]$. Note that this WL MMSE gives the same output SINR that the MLSE receiver (5).

3.2 Implementation

Without any knowledge about $\mathbf{h}_s$, but if a training sequence $(a_k)_{k=1,\ldots,K}$ is available after a synchronisation process, we use the following estimated WL MMSE receiver.

$$\widehat{\widetilde{\mathbf{w}}}(K) \stackrel{\text{def}}{=} \widehat{\mathbf{R}}_{\widetilde{\mathbf{x}}}^{-1}(K)\widehat{\mathbf{r}}_{\widetilde{\mathbf{x}},a}(K), \quad (7)$$

with $\widehat{\mathbf{R}}_{\widetilde{\mathbf{x}}}(K) \stackrel{\text{def}}{=} \frac{1}{K}\sum_{k=1}^K \widetilde{\mathbf{x}}_\upsilon(kT)\widetilde{\mathbf{x}}_\upsilon^H(kT)$ and $\widehat{\mathbf{r}}_{\widetilde{\mathbf{x}},a}(K) \stackrel{\text{def}}{=} \frac{1}{K}\sum_{k=1}^K \widetilde{\mathbf{x}}_\upsilon(kT)a_k$.

Then a sequence $(a_k)_{k=K+1,\ldots,K+L}$ of information symbols is transmitted[1] for which the output $y(kT)$ of the estimated WL MMSE filter is given from (4) for $k = K+1,\ldots,K+L$ by

$$\begin{aligned}y(kT) &= \widehat{\widetilde{\mathbf{w}}}(K)^H\widetilde{\mathbf{x}}_\upsilon(kT), \quad k = K+1,\ldots,K+L, \quad (8) \\ &= \mu_s a_k \widehat{\widetilde{\mathbf{w}}}(K)^H\widetilde{\mathbf{h}}_s(k,k) + \mu_s \sum_{n\neq k} a_n \widehat{\widetilde{\mathbf{w}}}(K)^H\widetilde{\mathbf{h}}_s(k,n) \\ &+ \mu_j \sum_n b_n \widehat{\widetilde{\mathbf{w}}}(K)^H\widetilde{\mathbf{h}}_j(k,n) + \widehat{\widetilde{\mathbf{w}}}(K)^H\widetilde{\mathbf{n}}_\upsilon(kT). \quad (9)\end{aligned}$$

4. PERFORMANCE ANALYSIS

4.1 Assumptions

The performance analysis of such a scheme is challenging because $y(kT)$ given by (9) is a random variable (RV) depending on the RVs $(a_k, b_k, \mathbf{n}_\upsilon(kT))_{k=1,\ldots,K}$ of the training period through the estimate $\widehat{\widetilde{\mathbf{w}}}(K)$ and on the RVs $(a_k, b_k, \mathbf{n}_\upsilon(kT))_{k=K+1,\ldots,K+L}$ of the information period. To simplify the performance analysis, we assume that for K "sufficiently large", the loss of SINR with respect to those obtained with

$$\widetilde{\mathbf{w}}(K) \stackrel{\text{def}}{=} \mathbf{R}_{\widetilde{\mathbf{x}}}^{-1}(K)\mathbf{r}_{\widetilde{\mathbf{x}},a}(K) \quad (10)$$

is "very weak", where

$$\mathbf{R}_{\widetilde{\mathbf{x}}}(K) \stackrel{\text{def}}{=} \frac{1}{K}\sum_{k=1}^K \mathbf{R}_{\widetilde{\mathbf{x}}}(k) \text{ and } \mathbf{r}_{\widetilde{\mathbf{x}},a}(K) \stackrel{\text{def}}{=} \frac{1}{K}\sum_{k=1}^K \mathbf{r}_{\widetilde{\mathbf{x}},a}(k)$$

with $\mathbf{r}_{\widetilde{\mathbf{x}},a}(k) \stackrel{\text{def}}{=} \mathrm{E}[\widetilde{\mathbf{x}}_\upsilon(kT)a_k]$. Note that in the absence of frequency offset for which $\mathbf{x}_\upsilon(kT)$ is stationary, this assumption is valid for relatively small values of K. More precisely,

[1] In practice in each burst, the training sequence is preceded and followed by this information sequence with e.g., $K = 26$ and $L = 58$ in the GSM standard.

it is proved in [5, Chap. 6.1.2] that to obtain a loss of SINR lower than 3dB, $K = 2N$ samples are needed for not too small output SINR. We will consider throughout our performance analysis that $\widehat{\widetilde{\mathbf{w}}}(K)$ can be replaced by $\widetilde{\mathbf{w}}(K)$ given by (10), without affecting the SINR at the output $y(kT)$.

4.2 Theoretical SINR, SNR and INR

With $\widehat{\widetilde{\mathbf{w}}}(K)$ given by (10) in (8) and with the independence of a_k, b_k and $\widetilde{\mathbf{n}}_\upsilon(kT)$, for $k = K+1,\ldots,K+L$, the SINR, the interference to noise ratio (INR) and the signal to noise ratio (SNR) at the output of the estimated WL MMSE filter are given by

$$\begin{aligned}\mathrm{SINR}(kT) &= \mu_s^2[\widetilde{\mathbf{w}}^H(K)\widetilde{\mathbf{h}}_s(k,k)]^2/\mu_s^2\sum_{n\neq k}[\widetilde{\mathbf{w}}^H(K)\widetilde{\mathbf{h}}_s(k,n)]^2 \\ &+ \mu_j^2\sum_n[\widetilde{\mathbf{w}}^H(K)\widetilde{\mathbf{h}}_j(k,n)]^2+\eta_2||\widetilde{\mathbf{w}}(K)||^2, \quad (11)\\ \mathrm{INR}(kT) &= \mu_s^2\sum_{n\neq k}[\widetilde{\mathbf{w}}^H(K)\widetilde{\mathbf{h}}_s(k,n)]^2 \\ &+ \mu_j^2\sum_n[\widetilde{\mathbf{w}}^H(K)\widetilde{\mathbf{h}}_j(k,n)]^2/\eta_2||\widetilde{\mathbf{w}}(K)||^2, \quad (12)\\ \mathrm{SNR}(kT) &= \mu_s^2[\widetilde{\mathbf{w}}^H(K)\widetilde{\mathbf{h}}_s(k,k)]^2/\eta_2||\widetilde{\mathbf{w}}(K)||^2. \quad (13)\end{aligned}$$

4.3 Theoretical BER

But the relevant criterion to evaluate the loss in performance is the output BER. Using (9)

$$y(kT) = a_k\alpha_k + \sum_{n\neq k}a_n\alpha_{n,k} + \sum_n b_n\beta_{n,k} + n_k, \quad (14)$$

with $\alpha_k \stackrel{\text{def}}{=} \mu_s\widetilde{\mathbf{w}}^H(K)\widetilde{\mathbf{h}}_s(k,k)$, $\alpha_{n,k} \stackrel{\text{def}}{=} \mu_s\widetilde{\mathbf{w}}^H(K)\widetilde{\mathbf{h}}_s(k,n)$, $\beta_{n,k} \stackrel{\text{def}}{=} \mu_j\widetilde{\mathbf{w}}^H(K)\widetilde{\mathbf{h}}_j(k,n)$ and $n_k \stackrel{\text{def}}{=} \widetilde{\mathbf{w}}^H(K)\widetilde{\mathbf{n}}_\upsilon(kT)$ which is a real valued zero-mean Gaussian RV of variance $\eta_2||\widetilde{\mathbf{w}}(K)||^2$ and conditioning on specific values of the sequence $(a_n)_{n\neq k}$, (b_n), the BER at time kT is clearly given by $\mathrm{BER}(kT) = \mathrm{Q}\left(\frac{\alpha_k-\sum_{n\neq k}a_n\alpha_{n,k}-\sum_n b_n\beta_{n,k}}{\eta_2^{1/2}||\widetilde{\mathbf{w}}(K)||}\right)$, where $\mathrm{Q}(v) \stackrel{\text{def}}{=} \int_v^{+\infty}\frac{1}{\sqrt{2\pi}}e^{-u^2/2}du$. This gives from the total probability formula and from the assumption of equiprobable sequences $(a_n)_{n\neq k}$, (b_n) contained in (14), the following BER

$$\mathrm{BER}(kT) = \frac{1}{2^{I+J}}\sum_{s=1}^{2^{I+J}}\mathrm{Q}\left(\frac{\alpha_k-\sum_{i\neq k}a_i^s\alpha_{i,k}-\sum_j b_j^s\beta_{j,k}}{\eta_2^{1/2}||\widetilde{\mathbf{w}}(K)||}\right) \quad (15)$$

where $(\ldots,a_{k-1}^s,a_{k+1}^s,\ldots,b_{k-1}^s,b_k^s,b_{k+1}^s,\ldots)$ denotes the 2^{I+J} different $(I+J)$-uplets[2] of binary symbols $(a_i)_{i\in I},(b_i)_{j\in J}$ with $(a_i)_{i\in I}$ are the SOI inter symbol interference and $(b_i)_{j\in J}$ are the interference symbols associated with the SOI symbol a_k.

4.4 Particular case of SAIC with NRZ pulse shape filter

To give an interpretation of the loss in performance from the different theoretical expressions (11), (12), (13) and (15) that are lacking of engineering insights, we consider the particular case of SAIC ($N = 1$) with $t_j = 0$ and with the following NRZ filter

$$\upsilon(t) = \begin{cases} 1/\sqrt{T} & \text{if} -\frac{T}{2}\leq t\leq\frac{T}{2} \\ 0 & \text{elsewhere}\end{cases}$$

for which the values of I_n and $J(t_j)$ defined in (3), are given by

$$I_n = \frac{\sin(\pi\Delta f_s T)}{\pi\Delta f_s T}\delta(n) \quad \text{and} \quad J_n \stackrel{\text{def}}{=} J_n(0) = \frac{\sin(\pi\Delta f_j T)}{\pi\Delta f_j T}\delta(n).$$

Consequently $\widetilde{\mathbf{x}}_\upsilon(kT)$ given by (4) reduces to

$$\widetilde{\mathbf{x}}_\upsilon(kT) = \mu_s a_k\widetilde{\mathbf{h}}_s(k,k) + \mu_j b_k\widetilde{\mathbf{h}}_j(k,k) + \widetilde{\mathbf{n}}_\upsilon(kT), \quad (16)$$

with no interference inter symbol coming from the SOI and interference, where $\widetilde{\mathbf{h}}_s(k,k) = I_0(e^{i2\pi\Delta f_s kT}e^{i\phi_s}, e^{-i2\pi\Delta f_s kT}e^{-i\phi_s})^T$ and $\widetilde{\mathbf{h}}_j(k,k) = J_0(e^{i2\pi\Delta f_j kT}e^{i\phi_j}, e^{-i2\pi\Delta f_j kT}e^{-i\phi_j})^T$. This gives

$$\begin{aligned}\mathbf{R}_{\widetilde{\mathbf{x}}}(K) &= \mu_s^2 I_0^2\begin{pmatrix}1 & \alpha_s(K)e^{2i\phi_s}\\ \alpha_s^*(K)e^{-2i\phi_s} & 1\end{pmatrix}\\ &+ \mu_j^2 J_0^2\begin{pmatrix}1 & \alpha_j(K)e^{2i\phi_j}\\ \alpha_j^*(K)e^{-2i\phi_j} & 1\end{pmatrix} + \eta_2\mathbf{I}\\ \mathbf{r}_{\widetilde{\mathbf{x}},a}(K) &= \mu_s I_0\begin{pmatrix}\alpha_s'(K)e^{i\phi_s}\\ \alpha_s'^*(K)e^{-i\phi_s}\end{pmatrix},\end{aligned}$$

where $\alpha_s(K) \stackrel{\text{def}}{=} \frac{1}{K}\sum_{k=1}^K e^{i4\pi\Delta f_s kT}$, $\alpha_j(K) \stackrel{\text{def}}{=} \frac{1}{K}\sum_{k=1}^K e^{i4\pi\Delta f_j kT}$ and $\alpha_s'(K) \stackrel{\text{def}}{=} \frac{1}{K}\sum_{k=1}^K e^{i2\pi\Delta f_s kT}$.

Using these expressions of $\mathbf{R}_{\widetilde{\mathbf{x}}}(K)$ and $\mathbf{r}_{\widetilde{\mathbf{x}},a}(K)$ in (10), the inversion matrix lemma gives after straightforward but cumbersome algebraic derivations $\widetilde{\mathbf{w}}(K) = (w(K), w(K)^*)^T$ with

$$\begin{aligned}w(K) &\propto (1+\varepsilon_s+\varepsilon_j)\alpha_s'(K)e^{i\phi_s}\\ &- [\varepsilon_s\alpha_s(K)e^{2i\phi_s}+\varepsilon_j\alpha_j(K)e^{2i\phi_j}]\alpha_s'^*(K)e^{-i\phi_s}, \quad (17)\end{aligned}$$

where $\propto$ denotes proportional up to a real-valued constant and $\varepsilon_s \stackrel{\text{def}}{=} \frac{\mu_s^2 I_0^2}{\eta_2}$ and $\varepsilon_j \stackrel{\text{def}}{=} \frac{\mu_j^2 J_0^2}{\eta_2}$.

Consequently the general expressions of (11), (12), (13) and (15) reduce to

$$\begin{aligned}\mathrm{SINR}(kT) &= \frac{2\varepsilon_s[\Re(w^*e^{i\psi_{s,k}}e^{i\phi_s})]^2}{2\varepsilon_j[\Re(w^*e^{i\psi_{j,k}}e^{i\phi_j})]^2+|w(K)|^2}, \quad (18)\\ \mathrm{INR}(kT) &= \frac{2\varepsilon_j[\Re(w^*e^{i\psi_{j,k}}e^{i\phi_j})]^2}{|w(K)|^2}, \quad (19)\\ \mathrm{SNR}(kT) &= \frac{2\varepsilon_s[\Re(w^*e^{i\psi_{s,k}}e^{i\phi_s})]^2}{|w(K)|^2}, \quad (20)\end{aligned}$$

with $\psi_{s,k} \stackrel{\text{def}}{=} 2\pi\Delta f_s kT$ and $\psi_{j,k} \stackrel{\text{def}}{=} 2\pi\Delta f_j kT$ and [4]

$$\begin{aligned}\mathrm{BER}(kT) &= \frac{1}{2}\Big\{\mathrm{Q}(\sqrt{\mathrm{SNR}(kT)}+\sqrt{\mathrm{INR}(kT)})\\ &+ \mathrm{Q}(\sqrt{\mathrm{SNR}(kT)}-\sqrt{\mathrm{INR}(kT)})\Big\} \quad (21)\end{aligned}$$

[2] where $I = J-1$ with J is such that JT represents the length of the "significant" part of $\int\upsilon(\tau)\upsilon^*(\tau-t)d\tau$.

We assume to get engineering insight that the residual frequency offsets are sufficiently weak[3] such that $(K+L)|\Delta f_s|T \ll 1$ and $(K+L)|\Delta f_j|T \ll 1$ which implies that $\varepsilon_s \approx \frac{\mu_s^2}{\eta_2}$ and $\varepsilon_j \approx \frac{\mu_j^2}{\eta_2}$. Furthermore, we assume that the power of the interference is large with respect to the power of the background noise, i.e., $\varepsilon_j \gg 1$. We note [4] that in the absence of residual offset and for $\psi \stackrel{\text{def}}{=} \phi_j - \phi_s = 0$, the SAIC receiver has no rejection capability, i.e., the INR at the output of the optimal SAIC receiver is INR$= 2\varepsilon_j = \frac{2\mu_j^2}{\eta_2}$. So we assume that ψ is such that $|\psi| \gg (K+L)|\Delta f_j|T$ and $|\psi| \gg (K+L)|\Delta f_s|T$. To proceed on, we must specialize the case where there is residual offset of interference or SOI only.

4.4.1 *Frequency offset of interference only*

In this case $\Delta f_s = 0$, and thus $\alpha_s(K) = \alpha_s^{'}(K) = 1$ and $\psi_{s,k} = 0$. With respect to the SINR, INR and SNR given by the optimal SAIC without residual offset, we obtain from (18), (19) and (20)

$$\text{SINR}(kT) \approx \frac{\text{SINR}}{1+2\varepsilon_j[\pi\{2k-K-1\}\Delta f_j T]^2}, \qquad (22)$$

whereas $\text{INR}(kT) \approx 2\varepsilon_j[\pi\{2k-K-1\}\Delta f_j T]^2$ with $\text{INR} \approx \frac{1}{2\varepsilon_j \tan^2(\psi)} \ll 1$ and $\text{SNR}(kT) \approx \text{SNR} \approx 2\varepsilon_s \sin^2(\psi)$ for $k = K+1,\dots,K+L$. We see that the SNR is preserved in contrast to the INR and SINR which strongly degrade in the presence of residual offset of interference for $\varepsilon_j \gg 1$. This loss in performance naturally increases with $|\Delta f_j|T$ and (using $2k-K = 2(k-K)+K$) with the position $k-K$ of the information symbol, but also strongly with the input INR ε_j and the size K of the training sequence.

4.4.2 *Frequency offset of SOI only*

In this case $\Delta f_j = 0$, and thus $\alpha_j(K) = 1$ and $\psi_{j,k} = 0$. The simplification of the expressions of SINR(kT), INR(kT) and SNR(kT) is more complex because we must distinguish $\psi = \pm\pi/2$ from the case where ψ is not in the neighborhood of $\pm\pi/2$. For $\phi = \pm\pi/2$ we have

$$\text{SINR}(kT) \approx \text{SINR}\left(1 - \frac{(2\pi\Delta f_s T)^2((K+1)^2+4k^2)}{8}\right), \qquad (23)$$

whereas $\text{INR}(kT) \approx \frac{1}{2\varepsilon_j}(\pi(K+1)\Delta f_s T)^2$ with $\text{INR} = 0$ and $\text{SNR}(kT) \approx \text{SNR}\left(1 - 4(\pi k \Delta f_s T)^2\right)$ with SNR$= 2\varepsilon_s$ for $k = K+1,\dots,K+L$.

For ψ not in the neighborhood of $\pm\pi/2$, we have

$$\text{SINR}(kT) \approx \text{SINR}\left(1 - \frac{4\pi k \Delta f_s T}{\tan(\psi)}\right), \qquad (24)$$

whereas $\text{INR}(kT) \approx \text{INR}\left(1 + \frac{4\pi(K+1)\Delta f_s T}{\sin(2\psi)}\right)$ with $\text{INR} \approx \frac{1}{2\varepsilon_j \tan^2(\psi)} \ll 1$ and $\text{SNR}(kT) \approx \text{SNR}\left(1 - \frac{4\pi k \Delta f_s T}{\tan(\psi)}\right)$ with SNR$= 2\varepsilon_s \sin^2(\psi)$ for $k = K+1,\dots,K+L$. In a similar way for $\psi = \pm\pi/2$, the loss in performance increases with $|\Delta f_s|T$, with the position $k-K$ of the information symbol and the size K of the training sequence, but no longer with ε_j. But for ψ not in the neighborhood of $\pm\pi/2$, depending of the sign of $\Delta f_s T/\tan(\psi)$, the SINR can locally decrease or increase for $(K+L)|\Delta f_s|T \ll 1$, independently of ε_j.

4.4.3 *Comparisons*

Comparing the loss in performance due to residual offsets of the interference or the SOI from the aforementioned expressions is not easy. But comparing (22), (23) and (24) using the assumption that $\varepsilon_j \gg 1$, we see that a loss in SINR greater than e.g. 3dB is obtained for weaker value of $|\Delta f_j|T$ than of $|\Delta f_s|T$, and larger is ε_j, weaker is this ratio $|\Delta f_j|/|\Delta f_s|$. Consequently, under the assumption that $\varepsilon_j \gg 1$, the SAIC is less sensitive to residual offsets of the SOI than of the interference. Note that this property is similar to the sensitivity to steering errors in spatial beamforming for which the loss in SINR is much more sensitive to interference steering error than to SOI steering error. This property will be specified and confirmed in the next section in other scenarios.

5. ILLUSTRATIONS

Throughout this section we use $\varepsilon_s = 10$dB and $\varepsilon_j = 20$dB with $K = 26$ and $L = 58$ (GSM standard).

5.1 Validation of the assumptions

Here, $\psi \stackrel{\text{def}}{=} \phi_j - \phi_s$ is fixed to $\pi/3$. Figs.1 and 2 show the theoretical (exact (21) and approximate (22) and (24)) and empirical (Monte Carlo with 10000 runs) SINR as a function of the position $k-K$ of the information symbol for $\Delta f_s = 0$, $\Delta f_j T = 5\ 10^{-4}$ and $\Delta f_j = 0$, $\Delta f_s T = 5\ 10^{-4}$ respectively, for the SAIC receiver with NRZ pulse shape filter. The SINRs at the middle of the burst (30-th symbol) are plotted in Figs.3 and 4 as a function of $\Delta f_j T$ and $\Delta f_s T$ for $\Delta f_s T = 0$ and $\Delta f_j T = 0$, respectively.

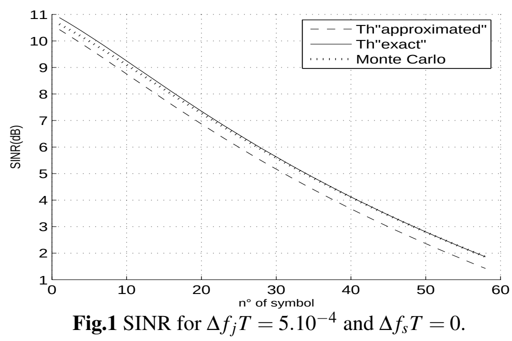

Fig.1 SINR for $\Delta f_j T = 5.10^{-4}$ and $\Delta f_s T = 0$.

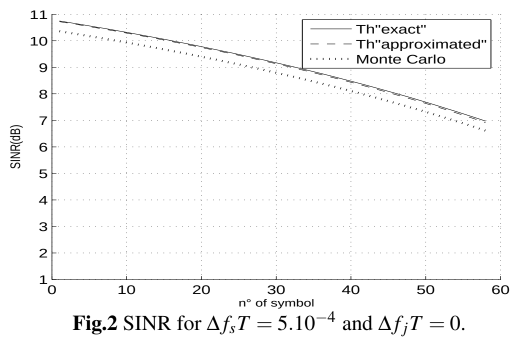

Fig.2 SINR for $\Delta f_s T = 5.10^{-4}$ and $\Delta f_j T = 0$.

[3] Note that when $(K+L)|\Delta f_s|T \sim 1$ or $(K+L)|\Delta f_j|T \sim 1$, the SOI or the interference are seen by the SAIC receiver as second order circular and no rejection capability is possible.

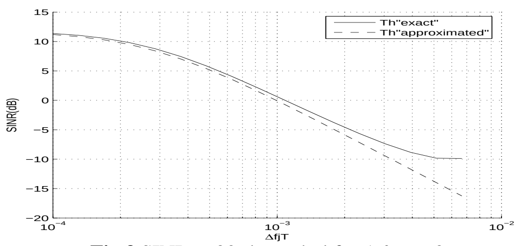

Fig.3 SINR at 30-th symbol for $\Delta f_s T = 0$.

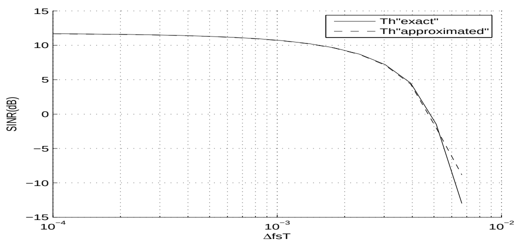

Fig.4 SINR at 30-th symbol for $\Delta f_j T = 0$.

We see from these figures that all the theoretical SINR fit the empirical ones. However, we note in Fig.1 that the empirical SINR perfectly fits the theoretical SINR in contrast to Fig.2. This shows that the assumptions of Subsection 4.1 is better justified in case of frequency offset of interference than of SOI. This is likely due to the estimated filter $w(K)$ in (17) that is more perturbed to frequency offset of SOI than of interference. Furthermore our simplified expressions (22) and (24) are roughly valid up to $\Delta f_j T = 10^{-3}$ and $\Delta f_s T = 5\ 10^{-3}$ respectively.

5.2 Practical applications

We consider now the practical case in which the phases of the SOI and interference for the SAIC receiver, and the phases and the directions of arrival for the MAIC receiver are totally unknown. Figs.5 shows the theoretical BER given by (21) and the empirical BER (Monte Carlo with 20000 runs) averaged over the phases ϕ_s and ϕ_j, and the L information symbols for the SAIC receiver with an NRZ pulse shape filter, as a function of $\Delta f_j T$ for $\Delta f_s T = 0$ and $\Delta f_s T$ for $\Delta f_j T = 0$. This BER is compared to those obtained with a raise cosine pulse shape filter with a roll-off of 0.22 in Fig.6. Note that it is derived by 20000 Monte Carlo runs because the derivation from (21) is too computationally demanding due to the truncation of $\upsilon(t)$ to 15 samples.

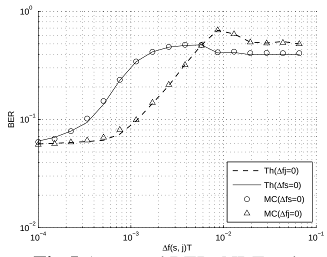

Fig.5 Averaged BER, NRZ pulse

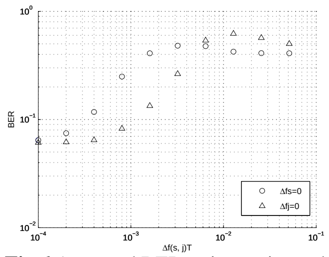

Fig.6 Averaged BER, raise cosine pulse

We see from Fig.5 that the averaged theoretical BER fits the empirical ones and that the raise cosine pulse shape filter roughly does not modify the loss in performance. Note that the SAIC is less sensitive to residual offsets of the SOI than of the interference. More precisely, we note an increase of the BER of ten per cent for $\Delta f_j T = 1.5\ 10^{-4}$ or for $\Delta f_s T = 5\ 10^{-4}$, i.e., 40Hz or 135Hz for the GSM standard ($T = 1/270$) ms, respectively. We note that for $\varepsilon_j = 10$dB and $\varepsilon_j = 30$dB these values become $\Delta f_j T = 4\ 10^{-4}$ and $\Delta f_j T = 1.2\ 10^{-4}$, whereas the value of $\Delta f_s T$ keeps the same value as predicted by our approximations (22) and (23).

Finally Figs.7 shows the averaged theoretical BER given by (21) and the empirical BER (Monte Carlo with 10000 runs) with respect to the phases ϕ_s and ϕ_j, the directions of arrival θ_s and θ_j and the L information symbols for the MAIC receiver (with $N = 2$ omnidirectional sensors equispaced half a wavelength apart) with an NRZ pulse shape filter as a function of $\Delta f_j T$ for $\Delta f_s T = 0$ and $\Delta f_s T$ for $\Delta f_j T = 0$. We see that MAIC receiver roughly presents the same sensitivity than the SAIC receiver.

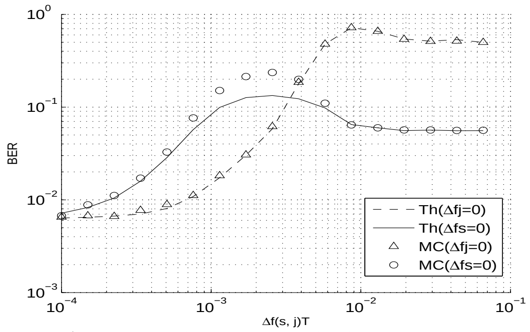

Fig.7 Averaged BER, NRZ pulse for MAIC ($N = 2$)

6. CONCLUSION

In this paper, we have presented a theoretical performance analysis of the loss in performance of the SAIC and MAIC receiver in the presence of residual frequency offsets of BPSK SOI and interference. We have proved for NRZ pulse shape filters than the SAIC receiver is less sensitive to residual offset of the SOI than of the interference for strong interference w.r.t. background noise and some rules of thumb about the normalized tolerable residual offsets have been given. In particular for a burst structure similar to those of the GSM, an increasing of BER of ten per cent is obtained for $\Delta f_j = 40$Hz or $\Delta f_s = 135$Hz. These properties of sensitivity have been extended by Monte Carlo experiments to raise cosine pulse shape filters, MAIC receivers and extensive scenarios of phases and directions of arrival.

Extension of this work to MSK and GMSK signals will be considered in a future contribution to analyze the behavior of the SAIC and MAIC receivers for cochannel mitigation with nonnull residual offsets in the GSM network.

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