WHY THE PHASED-MIMO RADAR OUTPERFORMS THE PHASED-ARRAY AND MIMO RADARS

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ABSTRACT

The phased-multiple-input multiple-output (MIMO) radar has been recently proposed as a new radar technique which combines the advantages of both the phased-array and MIMO radars [1]. Based on the beampattern and signal-to-noise ratio (SNR) analysis of the phased-MIMO radar and comparing it to the beampatterns and SNRs of the phased-array and MIMO radars, we explain, in this paper, why the phased-MIMO radar outperforms the phased-array and MIMO radars, and why it is the right candidate for future radar systems. Particularly, the phased-MIMO radar beampattern decouples into a product of three beampatterns each corresponding to a certain type of processing gain present in the radar system. Only some of such processing gains are present in the phased-array and MIMO radars. Moreover, the phased-MIMO radar improves the SNR gain compared to the MIMO radar by means of using transmit beamforming. Simulation results validate our theoretical developments on the superiority of the phased-MIMO radar.

1. INTRODUCTION

The multiple-input multiple-output (MIMO) radar has been recently developed based on the idea of employing multiple antennas to transmit multiple waveforms and multiple antennas to receive the echoes reflected by the target [2]–[4]. Although the MIMO radar has a number of advantages [4], including the capability of energy integration from different waveforms [5], it suffers from a significant disadvantage, that is, the absence of the coherent joint transmit/receive processing gain. This results in signal-to-noise ratio (SNR) gain loss as compared to the phased-array radar [6]–[7]. Focussing on the MIMO radar configuration with colocated antennas, it has been shown in [1] and [7] that the coherent processing gain can be added to the MIMO radar. The corresponding technique has been called *phased-MIMO radar*. The essence of this technique is that the transmitting array can be partitioned to a number of *overlapped subarrays* of smaller size or, more generally, transmit antenna element space can be transformed into the transmit beamspace. Then, one waveform can be transmitted coherently from each subarray (per each dimension of the transmit beamspace), while different orthogonal waveforms are transmitted from different subarrays (different dimensions of the transmit beamspace).

In this paper, we aim at showing by a basic example of employing uniform linear array (ULA) at the transmitter and using conventional nonadaptive beamforming why the phased-MIMO radar outperforms the phased-array and MIMO radars. Toward this end, we study the beampattern of the phased-MIMO radar and compare it to the beampatterns of the phased-array and MIMO radars. It serves us in claiming that the phased-MIMO radar is the right candidate for future radar systems. We show, particularly, that the beampattern of the phased-MIMO radar decouples into three components each corresponding to a certain type of processing gain present in the radar system, e.g., the transmit coherent processing gain, waveform diversity processing gain, and receive coherent processing gain. All these three processing gains are present in the phased-MIMO radar while only some types of the aforementioned gains are present in the phased-array and MIMO radars. A beampattern similar to the phased-MIMO radar beampattern can be archived by applying a beamspace transformation to the virtual data at the receiving end of the MIMO radar. However, in the latter case, no improvement to the SNR gain is achievable. The improvement of the SNR gain is enabled only due to introducing the transmit coherent processing capabilities, i.e., employing the beamspace transformation at the transmitter. Therefore, the SNR gains are also analyzed and compared to each other for all aforementioned modifications of radar techniques. Simulation results are used to validate our theoretical developments and demonstrate the improvements of the beampattern characteristics archived by the phased-MIMO radar.

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2. PHASED-MIMO RADAR MODEL

Consider a MIMO radar system of M_T transmit and M_R receive antennas, which are located close to each other in space so that they see targets at same directions and with same radar cross-sections (RCSs). In the phased-MIMO radar, the transmitting array of M_T antennas is partitioned into K ($1 \leq K \leq M_T$) overlapped subarrays [1] and [7]. The kth subarray is composed of the antennas located at the kth up to the $(M_T - K + k)$th positions. All elements of the kth subarray are used to coherently transmit the signal $\phi_k(t)$ so that a beam is formed towards the direction of the target. At the same time, different waveforms are transmitted by different subarrays.

The signal at the output of the antennas belonging to kth subarray is modeled as

$$\mathbf{s}_k(t) = \sqrt{\frac{M_T}{K}}\phi_k(t)\mathbf{w}_k^*, \quad k = 1, \dots, K \tag{1}$$

where $\mathbf{w}_k$ is the unit-norm complex vector of beamforming weights associated with the kth subarray, $\phi_k(t)$ $(k = 1, \dots, K)$

are the orthogonal waveforms, $(\cdot)^*$ denotes the conjugate operator, and M_T/K is the power normalization coefficient which ensures that the transmitted energy for the phased-MIMO radar within one radar pulse equals to M_T.

The reflected signal of a target located at direction θ in the far-field can be modeled as

$$r(t,\theta) = \sqrt{\frac{M_T}{K}}\beta(\theta)\left(\mathbf{c}(\theta)\odot\mathbf{d}(\theta)\right)^T\phi_K(t) \qquad (2)$$

where $\beta(\theta)$ is the target reflection coefficient which is assumed to be constant during the whole pulse but varies from pulse to pulse, $\mathbf{c}(\theta) \triangleq \left[\mathbf{w}_1^H\mathbf{a}_1(\theta),\dots,\mathbf{w}_K^H\mathbf{a}_K(\theta)\right]^T$ is the $K\times 1$ transmit coherent processing vector, $\mathbf{a}_k(\theta)$ is the steering vector associated with the kth subarray, $\mathbf{d}(\theta) \triangleq \left[e^{-j\tau_1(\theta)},\dots,e^{-j\tau_K(\theta)}\right]^T$ is the $K\times 1$ waveform diversity vector, $\tau_k(\theta)$ is the time required for the wave to travel from the first element of the first subarray to the first element of the kth subarray, $\phi_K(t) = [\phi_1(t),\dots,\phi_K(t)]$ is the $K\times 1$ vector of waveforms, and $\odot$ and $(\cdot)^T$ stand for the Hadamard product and transposition, respectively.

The $M_R\times 1$ received complex vector of array observations can be written as

$$\mathbf{x}(t) \;=\; r(t,\theta_s)\mathbf{b}(\theta_s)+\sum_{i=1}^{D} r(t,\theta_i)\mathbf{b}(\theta_i)+\tilde{\mathbf{n}}(t) \qquad (3)$$

where D is the number of interferences, $r(t,\theta_i)$ $(i=1,\dots,D)$ are the reflected interference signals coming from the directions θ_i $(i=1,\dots,D)$, $\mathbf{b}(\theta)$ is the $M_R\times 1$ receive steering vector associated with direction θ, and $\tilde{\mathbf{n}}(t)$ is the spatially and temporally white zero-mean noise with variance σ_n^2.

The returns due to the kth transmitted waveform are then recovered by match filtering the signal $\mathbf{x}(t)$ to each of the waveforms $\phi_k(t)$ $(k=1,\dots,K)$, that is,

$$\mathbf{x}_k \triangleq \int_{T_0}\mathbf{x}(t)\phi_k^*(t)dt, \quad k=1,\dots,K. \qquad (4)$$

Therefore, the so obtained $KM_R\times 1$ virtual data vector is

$$\mathbf{y}\triangleq[\mathbf{x}_1^T\cdots\mathbf{x}_K^T]^T=\sqrt{\frac{M_T}{K}}\beta_s\mathbf{u}(\theta_s)+\sum_{i=1}^{D}\sqrt{\frac{M_T}{K}}\beta_i\mathbf{u}(\theta_i)+\mathbf{n} \qquad (5)$$

where $\beta_s=\beta(\theta_s)$ and $\beta_i=\beta(\theta_i)$ $(i=1,\dots,D)$ are the reflection coefficients of the target and the interferences, respectively, $\mathbf{u}(\theta)\triangleq(\mathbf{c}(\theta)\odot\mathbf{d}(\theta))\otimes\mathbf{b}(\theta)$ is the $KM_R\times 1$ virtual steering vector associated with direction θ, $\mathbf{n}$ is the $KM_R\times 1$ noise term with covariance $\mathbf{R_n}=\sigma_n^2\mathbf{I}_{KM_R}$, and $\otimes$ stands for the Kroneker product.

The structure of the phased-MIMO radar is flexible and is characterized by the number of transmitted orthogonal waveforms K and the size of transmitting subarrays M_T-K+1. It is easy to see that:

(a) The phased-array radar structure is a special case of (5) for $K=1$. Then, the $M_R\times 1$ received data vector is

$$\mathbf{y} \;=\; \sqrt{M_T}\beta_s\mathbf{u}(\theta_s)+\sum_{i=1}^{D}\sqrt{M_T}\beta_i\mathbf{u}(\theta_i)+\mathbf{n} \qquad (6)$$

where $\mathbf{u}(\theta)=\left[\mathbf{w}^H\mathbf{a}(\theta)\right]\cdot\mathbf{b}(\theta)$ is the $M_R\times 1$ steering vector and $\mathbf{w}^H\mathbf{a}(\theta)$ is the uplink coherent processing gain of the conventional phased-array radar towards the direction θ.

(b) The MIMO radar structure is also a special case of (5) for $K=M$. Then, the $M_TM_R\times 1$ virtual data vector is

$$\mathbf{y}=\beta_s\mathbf{a}(\theta_s)\otimes\mathbf{b}(\theta_s)+\mathbf{y}_{i+n} \qquad (7)$$

where $\mathbf{y}_{i+n}$ denotes the interference-plus-noise components.

3. WHY THE PHASED-MIMO RADAR IN BETTER THAN THE PHASED-ARRAY AND MIMO RADARS

In the case of non-adaptive beamforming, the transmit and receive beamforming vectors can be designed to maximize the output SNR gain of a single source signal observed in the background of white Gaussian noise. Then the corresponding conventional beamformer weight vectors are given for the kth transmitting subarray as

$$\mathbf{w}_k=\frac{\mathbf{a}_k(\theta_s)}{\|\mathbf{a}_k(\theta_s)\|}=\frac{\mathbf{a}_k(\theta_s)}{\sqrt{M_T-K+1}}, \quad k=1,\dots,K \qquad (8)$$

and for the receiving array as

$$\mathbf{w}_d=[\mathbf{c}(\theta_s)\odot\mathbf{d}(\theta_s)]\otimes\mathbf{b}(\theta_s). \qquad (9)$$

3.1 Beampattern improvements

Let $G(\theta)$ be the normalized beampattern

$$G(\theta) \;\triangleq\; \frac{\left|\mathbf{w}_d^H\mathbf{u}(\theta)\right|^2}{\left|\mathbf{w}_d^H\mathbf{u}(\theta_s)\right|^2}=\frac{\left|\mathbf{u}^H(\theta_s)\mathbf{u}(\theta)\right|^2}{\|\mathbf{u}(\theta_s)\|^4}. \qquad (10)$$

Considering the special case of a ULA, we have $\mathbf{a}_1^H(\theta_s)\mathbf{a}_1(\theta)=\ldots=\mathbf{a}_K^H(\theta_s)\mathbf{a}_K(\theta)$. Using (10), the beampattern of the phased-MIMO radar for ULA with partitioning to K transmit subarrays can be written as

$$G_K(\theta)=\frac{\left|\mathbf{a}_K^H(\theta_s)\mathbf{a}_K(\theta)\left[(\mathbf{d}(\theta_s)\otimes\mathbf{b}(\theta_s))^H(\mathbf{d}(\theta)\otimes\mathbf{b}(\theta))\right]\right|^2}{\|\mathbf{a}_K^H(\theta_s)\|^4\|\mathbf{d}(\theta_s)\otimes\mathbf{b}(\theta_s)\|^4}. \qquad (11)$$

After some algebra and using the facts that $\|\mathbf{a}_K(\theta_s)\|^2=M_T-K+1$, $\|\mathbf{d}(\theta_s)\|^2=K$, and $\|\mathbf{b}(\theta_s)\|^2=M_R$, the beampattern (11) can be rewritten as

$$G_K(\theta)=C_K(\theta)\cdot D_K(\theta)\cdot R(\theta) \qquad (12)$$

where $C_K(\theta)\triangleq|\mathbf{a}_K^H(\theta_s)\mathbf{a}_K(\theta)|^2/(M_T-K+1)^2$ is the transmit beampattern, $D_K(\theta)\triangleq|\mathbf{d}^H(\theta_s)\mathbf{d}(\theta)|^2/K^2$ is the waveform diversity beampattern, and $R(\theta)\triangleq|\mathbf{b}^H(\theta_s)\mathbf{b}(\theta)|^2/M_R^2$ is the receive beampattern. Therefore, the overall beampattern (12) of the phased-MIMO radar with transmitting ULA can be seen as the product of three individual beampatterns. Interestingly, the beampatterns of the phased-array, MIMO, and hybrid MIMO phased-array of [8], [9] radars do not enjoy all three processing gains as it follows.

(a) The beampattern expression for the phased-array radar can be deduced form (12) by substituting $K=1$, that gives

$$G_{\mathrm{PH}}(\theta)=C_1(\theta)\cdot R(\theta) \qquad (13)$$

where $C_1(\theta)=|\mathbf{a}^H(\theta_s)\mathbf{a}(\theta)|^2/M_T^2$ and $D_1(\theta)=1$. Note that only the transmit and receive beampatterns are present in (13).

(b) The beampattern expression for the MIMO radar can be also deducted from (12) by substituting $K = M_T$, that gives

$$G_{\mathrm{MIMO}}(\theta) = D_M(\theta) \cdot R(\theta) \tag{14}$$

where $C_M(\theta) = 1$ and $D_M(\theta) = |\mathbf{a}^H(\theta_s)\mathbf{a}(\theta)|^2/M_T^2$. Note that only the waveform diversity and receive beampatterns are present in (14).

(c) The beampattern expression for the hybrid MIMO phased-array radar recently proposed in [8] and [9] can be deduced from (12) as well. Specifically, the partitioning of the transmitting array to K non-overlapped subarrays each of size M_T/K is adopted in [8] and [9]. Then, the kth element of vector $\mathbf{d}(\theta)$ corresponds to the $(kM_T/K+1)$th element of $\mathbf{a}(\theta)$ and the $M_T/K \times 1$ vectors $\mathbf{a}_1(\theta) = \ldots = \mathbf{a}_K(\theta)$ contain the first M_T/K elements of $\mathbf{a}(\theta)$. Hence, $|\mathbf{a}_K^H(\theta_s)\mathbf{a}_K(\theta)|^2|\mathbf{d}^H(\theta_s)\mathbf{d}(\theta)|^2 = |\mathbf{a}^H(\theta_s)\mathbf{a}(\theta)|^2$, and the beampattern (12) becomes

$$G_K(\theta) = \frac{\left|\mathbf{a}^H(\theta_s)\mathbf{a}(\theta)\right|^2 \left|\mathbf{b}^H(\theta_s)\mathbf{b}(\theta)\right|^2}{M_T^2 M_R^2}. \tag{15}$$

It easy to see that in all of these three special cases, we obtain the same beampattern, while the beampatter (12) is different.

The following theorem is instrumental for supporting the statement of the paper. We only present the theorem formulation, while the detailed proof can be found in [7].

Theorem 1*: *The highest sidelobe level of the transmit/receive beampattern of the phased-MIMO radar with $1 < K < M_T$ overlapped subarrays is lower than the highest sidelobe level of the transmit/receive beampattern of the phased-array radar and the highest sidelobe level of the overall beampattern of the MIMO radar.

This theorem accentually states that the phased-MIMO radar enjoys better robustness against interfering targets located in the sidelobe area as compared to the phased-array and MIMO radars.

3.2 SNR gain improvement

The output SNR of the phased-MIMO radar is defined as

$$\mathrm{SNR}_{\mathrm{PH-MIMO}} \triangleq \frac{\frac{M_T}{K}\sigma_s^2|\mathbf{w}_d^H\mathbf{u}(\theta_s)|^2}{\mathbf{w}_d^H\mathbf{R_n}\mathbf{w}_d} \tag{16}$$

where $\sigma_s^2 = \mathrm{E}\{|\beta_s|^2\}$ is the variance of the target reflection coefficient.

Using (9), it is easy to find that

$$\mathbf{w}_d^H\mathbf{R_n}\mathbf{w}_d = \sigma_n^2(M_T - K + 1)KM_R. \tag{17}$$

Moreover, using (9) again and the fact that $\mathbf{w}_d = \mathbf{u}(\theta_s)$, it can be found that

$$|\mathbf{w}_d^H\mathbf{u}(\theta_s)|^2 = |\mathbf{c}^H(\theta_s)\mathbf{c}^H(\theta_s)|^2|\mathbf{d}^H(\theta_s)\mathbf{d}^H(\theta_s)|^2 \times|\mathbf{b}^H(\theta_s)\mathbf{b}^H(\theta_s)|^2 = (M_T - K + 1)^2K^2M_R^2. \tag{18}$$

Substituting (17) and (18) into (16), the SNR of the phased-MIMO radar with non-adaptive transmit/receive beamforming can be expressed as

$$\mathrm{SNR}_{\mathrm{PH-MIMO}} = M_R M_T (M_T - K + 1)\frac{\sigma_s^2}{\sigma_n^2}. \tag{19}$$

Therefore, the SNR gain for the phased-MIMO radar with non-adaptive transmit/receive beamforming equals $M_R M_T (M_T - K + 1)$.

As special cases of (19), the following SNRs are instrumental for further studies and comparisons.

(a) Substituting $K = M_T$ into (19), the SNR for the MIMO radar can be found as

$$\mathrm{SNR}_{\mathrm{MIMO}} = M_R M_T \frac{\sigma_s^2}{\sigma_n^2} \tag{20}$$

Therefore, the SNR gain for the MIMO radar equals $M_R M_T$ that is $(M_T - K + 1)$ times smaller than the SNR gain of the phased-MIMO radar with non-adaptive transmit/receive beamforming.

(b) Similarly, substituting $K = 1$ into (19), the SNR for the phased-array radar with non-adaptive transmit/receive beamforming simplifies to

$$\mathrm{SNR}_{\mathrm{PH}} = M_R M_T^2 \frac{\sigma_s^2}{\sigma_n^2} = M_T \cdot \mathrm{SNR}_{\mathrm{MIMO}} \tag{21}$$

It is easy to see from (21) that the SNR gain of the phased-array radar equals $M_R M_T^2$ that is M_T times larger than the SNR gain of the MIMO radar. Therefore, the phased-array radar is more robust to background noise than the MIMO radar.

Finally, the SNR for the phased-MIMO radar can be expressed through the SNR for the phased-array radar as

$$\mathrm{SNR}_{\mathrm{PH-MIMO}} = \eta \cdot \mathrm{SNR}_{\mathrm{PH}} \tag{22}$$

where $1/M_T \leq \eta \triangleq (M_T - K + 1)/M_T \leq 1$ is the ratio of the phased-MIMO radar SNR gain to the phased-array radar SNR gain. Since η in (22) depends on K, the SNR gain of the phased-MIMO radar linearly decreases with K. At the same time, larger K's provide larger dimension of the extended virtual array for the phased-MIMO radar. Therefore, there is a tradeoff between the SNR gain and the high angular resolution capabilities for the phased-MIMO radar.

On top of the aforementioned argument based on the SNR gain analysis, it is shown in [7] that the phased-MIMO radar provides significantly better interference suppression capabilities as compared to the phased-array and MIMO radars. Such analysis is carried on using the SINRs under dominant interference powers of all three radar techniques.

3.3 Joint beampattern, SNR gain, and angular resolution improvements

As have been shown above the phased-MIMO radar provides joint improvements to the beampattern, SNR gain, and angular resolution as compared to the phased-array and MIMO radars. However, to complete the argument on the superiority of the phased-MIMO radar, the following two cases are also of interest.

(a) Tapering technique can be used at the transmitter of the phased-array radar. This is equivalent to partitioning the transmit array into a number of overlapping subarrays just as in the phased-MIMO radar, but different from the phased-MIMO radar, the same waveform is transmitted from each subarray. Applying the same beamforming weights per each transmit subarray as in phased-MIMO radar, i.e., using (8), it is straightforward to see that the beampattern and SNR for such phased-array radar with tapering can be also expressed as in (12) and (22), respectively. However, in this case, the virtual array aperture, i.e., the dimension of the virtual data vector (5), reduces to M_R only since $K = 1$. Therefore, the phased-array radar with tapering is not capable of improving the angular resolution, while the phased-MIMO radar provides such capabilities.

(b) The beampattern of (12) can also be achieved using the beamspace transformation at the receiver of the MIMO radar. Specifically, let us introduce the following $M_T \times K$ matrix $\mathbf{W} \triangleq \frac{1}{\sqrt{M_T-K+1}}[\tilde{\mathbf{w}}_1 \ldots, \tilde{\mathbf{w}}_K]^H$, where $\tilde{\mathbf{w}}_k \triangleq [\mathbf{0}^T_{[k-1]}, \mathbf{a}^T_k(\theta_s), \mathbf{0}^T_{[K-k]}]^T$ is the $M_T \times 1$ weight vectors with $\mathbf{0}^T_{[k-1]}$ denoting the vector of $k-1$ zeros. Then, (7) reduces to the $KM_R \times 1$ virtual data vector

$$\mathbf{y} = \beta_s[\mathbf{W} \otimes \mathbf{I}_{M_R}] \cdot [\mathbf{a}(\theta_s) \otimes \mathbf{b}(\theta_s)] + \mathbf{y}_i + \bar{\mathbf{n}} \qquad (23)$$

where $\mathbf{y}_i$ denotes the interference term and $\bar{\mathbf{n}} \triangleq [\mathbf{W} \otimes \mathbf{I}_{M_R}]\mathbf{n}$. Noting that $[\mathbf{W} \otimes \mathbf{I}_{M_R}] \cdot [\mathbf{a}(\theta_s) \otimes \mathbf{b}(\theta_s)] = \mathbf{W}\mathbf{a}(\theta_s) \otimes \mathbf{b}(\theta_s)$, (23) can be rewritten as

$$\begin{aligned}\mathbf{y} &= \beta_s \mathbf{W}\mathbf{a}(\theta_s) \otimes \mathbf{b}(\theta_s) + \mathbf{y}_i + [\mathbf{W} \otimes \mathbf{I}_{M_R}]\mathbf{n} \\ &= \beta_s \sqrt{M_T/K}(\mathbf{c}(\theta) \odot \mathbf{d}(\theta_s)) \otimes \mathbf{b}(\theta_s) + \mathbf{y}_i + \bar{\mathbf{n}}. \qquad (24)\end{aligned}$$

Comparing (24) and (5), we can see that they are identical up to the noise term. Indeed, while the noise in (5) is zero-mean spatially and temporally white, the noise $\bar{\mathbf{n}}$ in (24) is colored. Therefore, both models (5) and (24) yield the same beampattern, however, the SNR gain for (24) reduces because of the noise amplification. Indeed, the covariance matrix of the colored noise $\bar{\mathbf{n}}$ can be found as

$$\begin{aligned}\mathbf{R}_{\bar{\mathbf{n}}} &= \sigma_n^2[\mathbf{W} \otimes \mathbf{I}_{M_R}][\mathbf{W} \otimes \mathbf{I}_{M_R}]^H \\ &= \sigma_n^2(M_T-K+1)\|\mathbf{d}^H(\theta_s)\mathbf{W}\|^2\|\mathbf{b}(\theta_s)\|^2 \\ &= \sigma_n^2 M_R\left((M_T-2K)K^2 + 2\sum_{k=1}^{K} k^2\right) \\ &= \sigma_n^2 M_R\left((M_T-2K)K^2 + \frac{K(K+1)(2K+1)}{3}\right). \qquad (25)\end{aligned}$$

In (25), we have used the facts that $\mathbf{w}_d^H[\mathbf{W} \otimes \mathbf{I}_{M_R}] = [\mathbf{a}_K^H(\theta_s)\mathbf{a}_K(\theta_s)]^{1/2}[\mathbf{d}(\theta_s) \otimes \mathbf{b}(\theta_s)]^H[\mathbf{W} \otimes \mathbf{I}_{M_R}] = (M_T-K+1)[\mathbf{d}^H(\theta_s)\mathbf{W}] \otimes [\mathbf{b}^H(\theta_s)\mathbf{I}_{M_R}]$, $\|\mathbf{d}^H(\theta_s)\mathbf{W}\| = M_R\left((M_T-2K)K^2 + 2\sum_{k=1}^{K} k^2\right)/(M_T - K + 1)$, and $\sum_{k=1}^{K} k^2 = K(K+1)(2K+1)/6$. Substituting (18) and (25) into (16), we finally obtain the SNR for the the MIMO radar with beamspace transformation at the receiver

$$\mathrm{SNR}_{\mathrm{RBC}} = \frac{KM_R(M_T-K+1)^2}{(M_T-2K)K^2 + \frac{K(K+1)(2K+1)}{3}} \frac{\sigma_s^2}{\sigma_n^2} \leq \mathrm{SNR}_{\mathrm{MIMO}} \qquad (26)$$

For $\mathrm{SNR}_{\mathrm{RBC}} \leq \mathrm{SNR}_{\mathrm{MIMO}}$ see also Fig. 4.

4. SIMULATIONS

Assume a ULA of $M = 10$ omnidirectional antennas spaced half a wavelength apart from each other and used for transmitting the baseband waveforms $s_k(t) = e^{j2\pi\frac{k}{T_0}t}$ ($k = 1, \ldots, K$). Also assume a ULA of $N = 10$ omnidirectional antennas spaced half a wavelength apart from each other at the receiving end. The additive noise is modeled as a complex Gaussian zero-mean spatially and temporally white random sequence with identical variances in each array sensor. Assume two interfering targets located at directions $-30°$ and $-10°$ and one target of interest located at direction $\theta_s = 10°$.

Compare the proposed phased-MIMO radar (5) with the phased-array radar (6) and the MIMO radar (7). For the phased-MIMO radar $K = 5$ overlapped subarrays are used.

Figs. 1 and 2 show the transmit beampatterns and the waveform diversity beampatterns, respectively, for all three radar techniques tested, while Fig. 3 shows the overall transmit/receive beampatterns for the same techniques. It can be seen in Fig. 1 that the phased-array radar has the typical conventional beampattern with mainlobe (of width π/M) centered at θ_s, while the MIMO radar has flat (0 dB) transmitting gain. However, the phased-MIMO transmit beampattern is characterized by the aperture of the individual subarrays, i.e., $M_T - K + 1$, which is smaller than the aperture of the whole array used in the transmit beampattern of the phased-array radar. The reduction in the subarray aperture results in the beampattern of the phased-MIMO radar with a

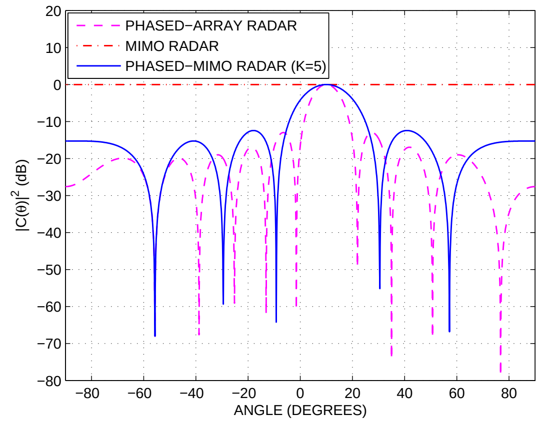

Figure 1: Transmit beampatterns.

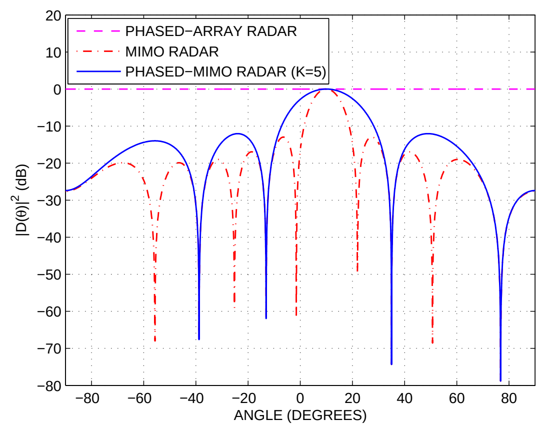

Figure 2: Waveform diversity beampatterns.

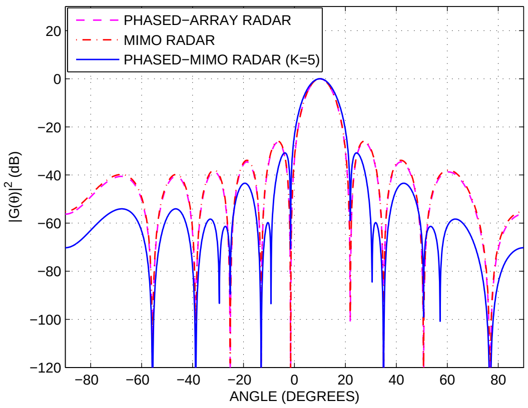

Figure 3: Overall beampatterns.

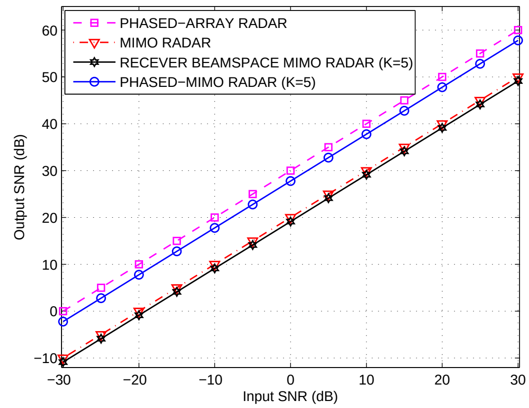

Figure 4: Output SNRs versus σ_s^2/σ_n^2.

wider main beam and a little higher sidelobe levels as compared to the beampattern of the phased-array radar. This small loss in beampattern shape is repaid at a greater gain in the waveform diversity beampattern of the phased-MIMO radar as shown in Fig. 2. Particularly, the phased-array radar has no waveform diversity gain (0 dB flat pattern), while the waveform diversity beampatterns of the MIMO and phased-MIMO radars are equivalent to conventional beampatterns offered by an M_T and K elements arrays, respectively. The waveform diversity beampattern of the phased-MIMO radar has a wider mainlobe and higher sidelobe levels as compared to the waveform diversity beampattern of the MIMO radar since $K \leq M_T$. However, it can be seen in Fig. 3 that the overall transmit/receive beampattern shape for the phased-MIMO radar is significantly improved as compared to the shapes of the phased-array and MIMO radar beampatterns. It is because the overall beampattern of the phased-MIMO radar is proportional to the multiplication of the transmit and the waveform diversity beampatterns, while the phased-array and MIMO radars overall transmit/receive beampatterns are exactly the same that agrees with (13) and (14).

Finally, Fig. 4 shows the output SNR versus σ_s^2/σ_n^2. It can be seen that in agreement with our analytical results the phased-array radar output SNR is 10 times higher than the MIMO radar output SNR. Moreover, the output SNR of the phased-MIMO radar is very close to that of the phased-array radar, while the MIMO radar with beamspace transformation at the receiver has even lower SNR than the MIMO radar. In addition, the phased-MIMO radar enjoys the waveform diversity benefits. This shows the superiority of the phased-MIMO radar and depicts the tradeoff between the SNR gain and the high angular resolution capabilities.

5. CONCLUSION

Answering the question why the phased-MIMO radar is the right candidate for future radar systems, we have shown that the phased-MIMO radar provides jointly (i) improvements to the beampattern as compared to the phased-array and MIMO radar; (ii) improvements to the SNR gain as compared to the MIMO radar and the MIMO radar with beamspace transformation at the receiver; (iii) improvements to angular resolution as compared to the phased-array radar and the phased-array radar with tapering. Simulation results precisely approve our theoretical developments on the superiority of the phased-MIMO radar.

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REFERENCES

[1] A. Hassanien and S. A. Vorobyov, "Transmit/receive beamforming for MIMO radar with colocated antennas," in *Proc. IEEE Int. Conf. on Acoustic, Speech, and Signal Processing*, Taipei, Taiwan, Apr. 2009, pp. 2089-2092.

[2] E. Fishler, A. Haimovich, R. Blum, L. Cimini, D. Chizhik, and R. Valenzuela, "Spatial diversity in radars: Models and detection performance," *IEEE Trans. Signal Processing*, vol. 54, pp. 823–838, Mar. 2006.

[3] A. Haimovich, R. Blum, and L. Cimini, "MIMO radar with widely separated antennas," *IEEE Signal Processing Magaz.*, vol. 25, pp. 116–129, Jan. 2008.

[4] J. Li and P. Stoica, "MIMO radar with colocated antennas," *IEEE Signal Processing Magaz.*, vol. 24, pp. 106–114, Sept. 2007.

[5] A. De Maio, M. Lops, and L. Venturino, "Diversity-integration tradeoffs in MIMO detection," *IEEE Trans. Signal Processing*, vol. 56, pp. 5051–5061, Oct. 2008.

[6] F. Daum and J. Huang, "MIMO radar: Snake oil or good idea," *IEEE Aerosp. Electron. Syst. Magazine*, pp. 8–12, May 2009.

[7] A. Hassanien and S. A. Vorobyov, "Phased-MIMO radar: A tradeoff between phased-array and MIMO radars," *IEEE Trans. Signal Processing*, vol. 58, pp. 3137–3151, Jun. 2010.

[8] J. P. Browning, D. R. Fuhrmann, and M. Rangaswamy, "A hybrid MIMO phased-array concept for arbitrary spatial beampattern synthesis," in *Proc. IEEE Digital Singal Processing and Signal Processing Education Workshop*, Marco Island, FL, Jan. 2009, pp. 446–450.

[9] D. R. Fuhrmann, J. P. Browning, and M. Rangaswamy, "Signaling strategies for the hybrid MIMO phased-array radar," *IEEE J. Select. Topics Signal Processing*, vol. 4, no. 1, pp. 66-78, Feb. 2010.